Neap:

Trial Examination 2014

VCE Mathematical Methods (CAS) Units 3&4

Written Examination 2

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 2 hours

Student's Name: ______________________________

Teacher's Name: ______________________________

Structure of Booklet

Section	Number of questions	Number of questions to be answered	Number of marks
1	22	22	22
2	4	4	58
			Total 80

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator (memory DOES NOT need to be cleared) and, if desired, one scientific calculator. For approved computer-based CAS, their full functionality may be used.

Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

Question and answer booklet of 23 pages and a sheet of miscellaneous formulas.
Answer sheet for multiple-choice questions.

Instructions

Write **your name** and **teacher's name** in the space provided above on this page.

All written responses must be in English.

At the end of the examination

Place the answer sheet for multiple-choice questions inside the front cover of this booklet.

Students are NOT permitted to bring mobile phones and/or any other electronic communication devices into the examination room.

SECTION 1

Instructions for Section 1

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Question 1

The average value of the function $f(x) = 3x - 3\cos\left(\frac{\pi x}{2}\right)$ for $0 \le x \le 3$ is equal to

A. $\frac{9\pi - 4}{2\pi}$

B. $\frac{6}{\pi} + \frac{27}{2}$

C. $\frac{9}{2} + \frac{2}{\pi}$

D. $\frac{-(9\pi + 4)}{2\pi}$

E. $\frac{-9}{2}$

Question 2

The number of undecayed atoms of carbon-14 in an ancient chunk of tree varies with time according to the rule $N(t) = 10\ 000 \times 2^{-\frac{t}{5730}}$, where t is the time measured in years, $t \ge 0$.

The average rate of change in the number of atoms over the first 10 000 years is closest to

A. –0.72

B. 0.702

C. 0.298

D. –0.298

E. –0.702

Question 3

Leo is a forward in the local soccer team. If he scores a goal, the probability that he will score a goal in the next game is 0.45. If he does not score a goal, the probability that he will score a goal in the next game is 0.8.

The probability that Leo will score a goal in the last game of the season (there are 30 games during the season and Leo plays in all) if he scores a goal in the 10th game is

A. 0.5217

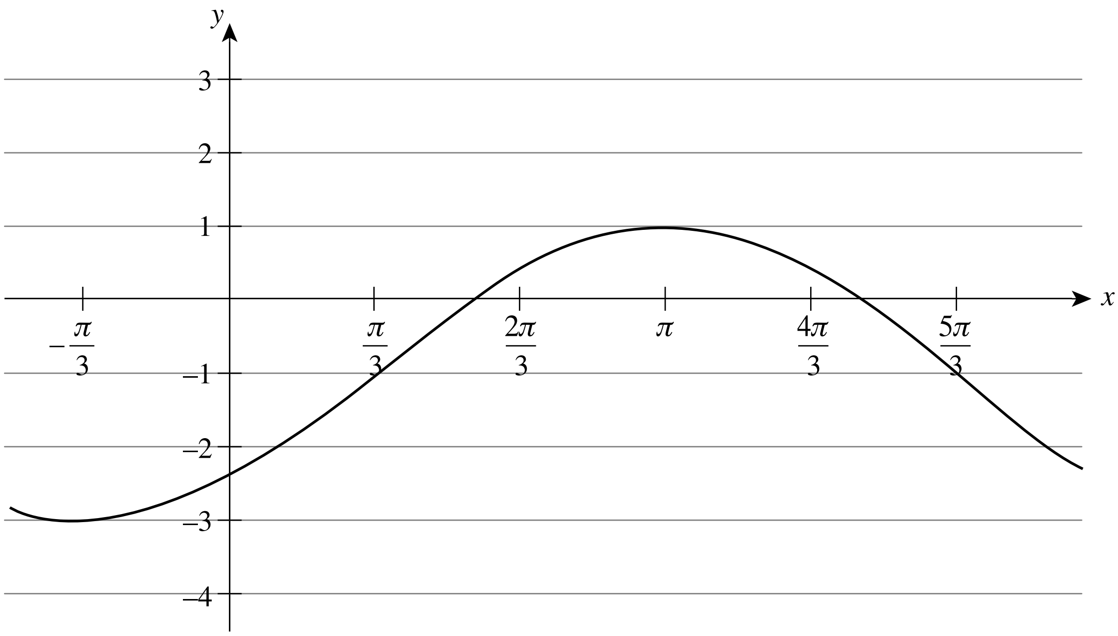
B. 0.4074

C. 0.4783

D. 0.05916

E. 0.5926

Question 4



The graph shown above could have equation

A. $y = 2\sin\left(\frac{3x}{4} + \frac{\pi}{4}\right) - 1$

B. $y = 2\cos\left(\frac{3x}{4} + \frac{\pi}{4}\right) - 1$

C. $y = -2\cos\left(\frac{3x}{4} + \frac{\pi}{2}\right) - 1$

D. $y = -2\cos\left(\frac{3x}{2} + \frac{\pi}{4}\right) - 1$

E. $y = -2\cos\left(\frac{3x}{4} + \frac{\pi}{4}\right) - 1$

Question 5

Consider the following simultaneous equations:

$$2x + 8y + 2kz = 12$$
$$6x - 3y + 6kz = 9$$
$$kx + 3y + z = 5$$

Which of the following statements is true?

A. There is a unique solution when $k^2 \neq 1$ and no solution when $k^2 = 1$.

B. There is a unique solution when $k^2 \neq 1$ and infinitely many solutions when $k^2 = 1$.

C. There is no solution when $k = 1$ and infinitely many solutions when $k = -1$.

D. There is no solution when $k = -1$ and infinitely many solutions when $k = 1$.

E. There is no unique solution when $k^2 = 1$ and no solution when $k = 1$.

Question 6

The derivative of $f(x) = \log_e(\cos(2x^2 - 3x))$ is

A. $\dfrac{4x - 3}{\cos(2x^2 - 3x)}$

B. $\dfrac{-x(4x - 3)\sin(2x - 3)}{\cos(2x - 3)}$

C. $\dfrac{(3 - 4x)\sin(2x^2 - 3x)}{\cos(2x^2 - 3x)}$

D. $(3 - 4x)\tan(2x^2 + 3x)$

E. $-x(4x - 3)\tan(2x - 3)$

Question 7

If Z has a standard normal distribution with $\Pr(Z > a) = A$ and $\Pr(Z > b) = B$, where $a \in [0, 3)$, $b \in (0, 3]$, $A \in [0, 1]$, $B \in [0, 1]$ and $b > a$, then $\Pr(-b < Z < -a)$ is equal to

A. $B - A$

B. $A - B$

C. $1 - (A - B)$

D. $1 - (A + B)$

E. $A + B - 0.5$

Question 8

There are 25 students in a class. The probability that a randomly chosen student is a blonde girl is 0.2, the probability that a randomly chosen student is a boy is 0.6 and the probability that a randomly chosen student is not blonde is $\frac{18}{25}$.

Therefore the probability that a randomly chosen boy is blonde is

A. $\frac{2}{15}$

B. $\frac{2}{25}$

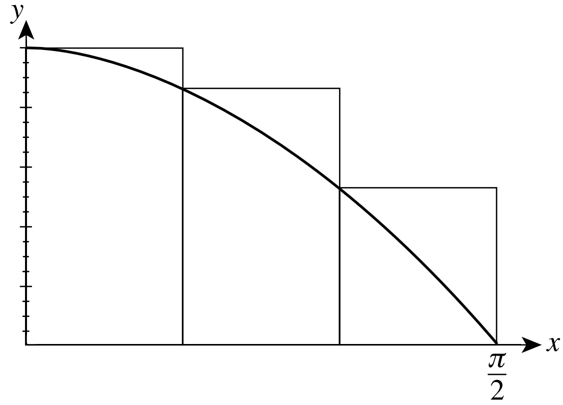
C. $\frac{7}{25}$

D. $\frac{13}{25}$

E. $\frac{3}{5}$

Question 9

The area of the three rectangles, each of equal width, shown below can be used to approximate the area bounded by the curve $y = \cos(x)$, the x-axis and the line $x = \frac{\pi}{2}$.



The value of this approximation is closest to

A. $\frac{(\sqrt{3}-1)\pi}{6}$

B. $\frac{(\sqrt{3}-1)\pi}{12}$

C. $\frac{(\sqrt{3}-3)\pi}{6}$

D. $\frac{(\sqrt{3}+3)\pi}{12}$

E. 1.24

Question 10

The general solution to $2\sin(2x) + \sqrt{3} = 0$ is

A. $x = \frac{\pi(6k-1)}{6}$ or $x = \frac{\pi(3k-1)}{3}$, $k \in R$

B. $x = \frac{\pi(6k+1)}{6}$ or $x = \frac{\pi(3k-1)}{6}$, $k \in Z$

C. $x = \frac{\pi(6k-1)}{6}$ or $x = \frac{\pi(3k-1)}{3}$, $k \in Z$

D. $x = \frac{\pi(6k-1)}{3}$ or $x = \frac{\pi(3k-1)}{3}$, $k \in Z$

E. $x = \frac{\pi(6k-1)}{6}$ or $x = \frac{\pi(1-3k)}{6}$, $k \in Z$

Question 11

A discrete random variable is known to be distributed binomially.

The expression $1-(0.8^{12}+12\times 0.2\times 0.8^{11})$ represents the probability of

A. at least one success in twelve trials with a 0.2 probability of a success.
B. at least one success in twelve trials with a 0.8 probability of a success.
C. at least two successes in twelve trials with a 0.8 probability of a success.
D. at least two successes in twelve trials with a 0.2 probability of a success.
E. more than two successes in twelve trials with a 0.2 probability of a success.

Question 12

For the function $f:[-\pi, 2\pi]\rightarrow R,\ f(x)=|\cos x|$, the derivative is positive on the interval

A. $\left(-\frac{\pi}{2}, 0\right)\cup\left[\frac{\pi}{2}, \pi\right)\cup\left[\frac{3\pi}{2}, 2\pi\right)$

B. $\left[-\frac{\pi}{2}, 0\right]\cup\left[\frac{\pi}{2}, \pi\right]\cup\left[\frac{3\pi}{2}, 2\pi\right]$

C. $(-\pi, 0)\cup(\pi, 2\pi)$

D. $\left(-\frac{\pi}{2}, 0\right)\cup\left(\frac{\pi}{2}, \pi\right)\cup\left(\frac{3\pi}{2}, 2\pi\right)$

E. $\left(-\frac{\pi}{2}, 0\right]\cup\left(\frac{\pi}{2}, \pi\right]\cup\left(\frac{3\pi}{2}, 2\pi\right]$

Question 13

If $f(x)=3x^3-ax^2+bx$ has a stationary point at (–1, 4), then the values of a and b respectively are

A. –5 and 2
B. 2 and –5
C. –5 and –2
D. –2 and –5
E. –5 and 8

Question 14

There are 15 pictures in a box: 7 are triangles, 6 are squares and 2 are parallelograms. 3 of the pictures are randomly drawn without replacement.

The probability of getting at least 2 triangles if you know that at least 1 triangle has been drawn is

A. $\frac{29}{65}$

B. $\frac{24}{65}$

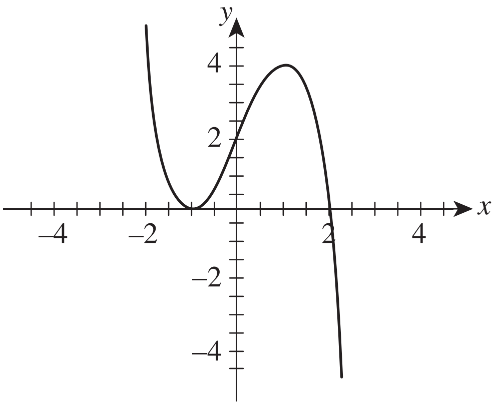
C. $\frac{8}{57}$

D. $\frac{57}{65}$

E. $\frac{29}{57}$

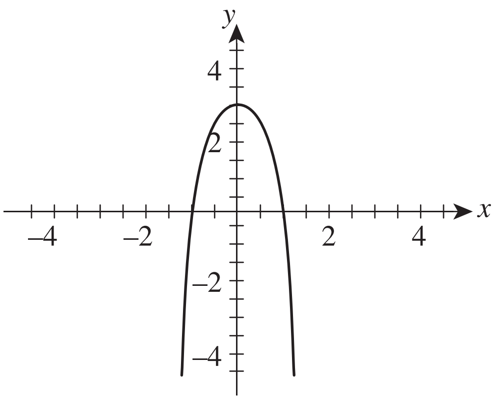
Question 15

The graph of the function $f'(x)$ is shown below.

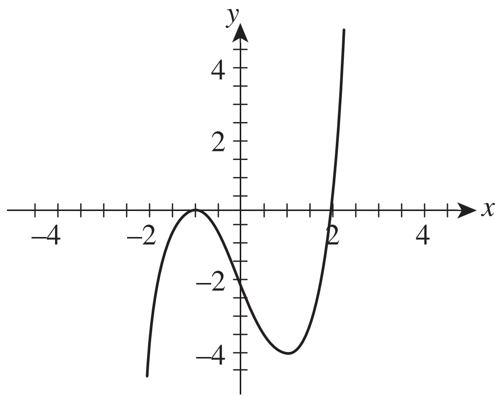


The graph of the function $f(x)$ could be

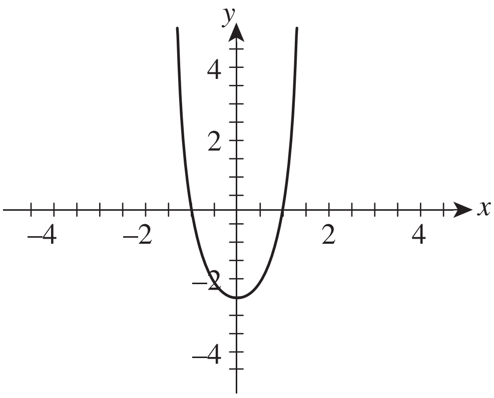
A.



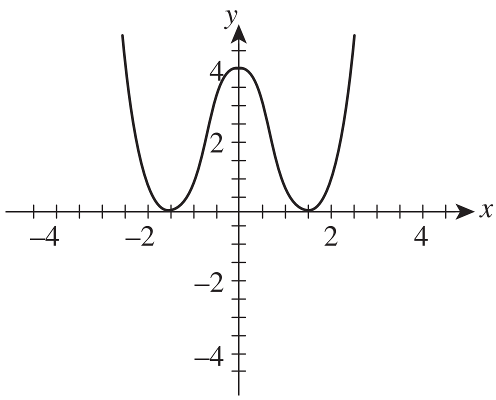
B.



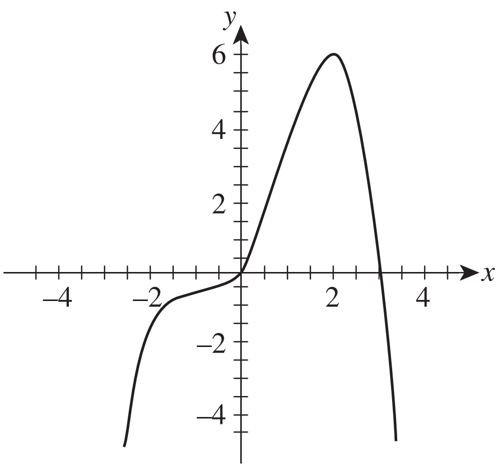
C.



D.



E.



Question 16

The equation of the tangent to the curve with equation $y = e^{-2x} - 2$ at the point where the curve crosses the x-axis is

A. $y = -4x - 2\log_e 2$

B. $y = 4x + 2\log_e 2$

C. $y = 2x - 4e^2$

D. $y = -2x + 2e^2$

E. $y = 4x - 2\log_e 2$

Question 17

For the function $f: R \to R,\ f(x) = x^4 - \frac{4}{3}x^3 + \frac{1}{2}x^2 + a,$ the stationary point of inflexion has coordinates $\left(0.5, \frac{-47}{48}\right)$.

The value of a is

A. –2
B. 0
C. –1
D. any real number
E. 1

Question 18

Let $y = f(g(x)),$ where $g(x) = \sin(2x)$.

Therefore $\frac{dy}{dx}$ is equal to

A. $f'(\sin(2x))\cos(2x)$

B. $2f(\sin(2x))\cos(2x)$

C. $2f'(\sin(x))\cos(x)$

D. $2f(\sin(2x))\cos(2x) + \sin(2x)f'(\cos(2x))$

E. $2f'(\sin(2x))\cos(2x)$

Question 19

If $f(x) = \log_e(x),$ then $f(x^3) = f(y) - f(x),$ where y is equal to

A. $4x$

B. x^4

C. $\log_e(4x)$

D. e^{4x}

E. $4e^x$

Question 20

Oil is leaking from a cone-shaped funnel at a rate of 200 cm^3 per minute. The cone has a base radius of 20 cm and a height of 50 cm. The depth of the oil at time t minutes is d cm.

The rate of change of d in cm/minute is given by

A. $-\frac{400\pi d^2}{2}$

B. $\frac{1250}{\pi d^2}$

C. $\frac{375}{\pi d^2}$

D. $-\frac{1250}{\pi d^2}$

E. $\frac{-5000}{\pi d^2}$

Question 21

If $1 + \frac{f(x)}{f(y)} = y^2(f(x) + f(y))$, then $f(x)$ is

A. $\frac{1}{x}$

B. e^x

C. $\frac{1}{x^2}$

D. x^2

E. x

Question 22

The transformation $T: R^2 \rightarrow R^2$ that could be used to obtain the graph of $y = 1 - \frac{1}{(2x-3)^2}$ from the graph of $y = \frac{1}{x^2}$ is

A. $T\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} \frac{3}{2} \\ 1 \end{bmatrix}$

B. $T\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -3 \\ -1 \end{bmatrix}$

C. $T\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

D. $T\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -\frac{3}{2} \\ -1 \end{bmatrix}$

E. $T\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} \frac{3}{2} \\ 1 \end{bmatrix}$

END OF SECTION 1

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

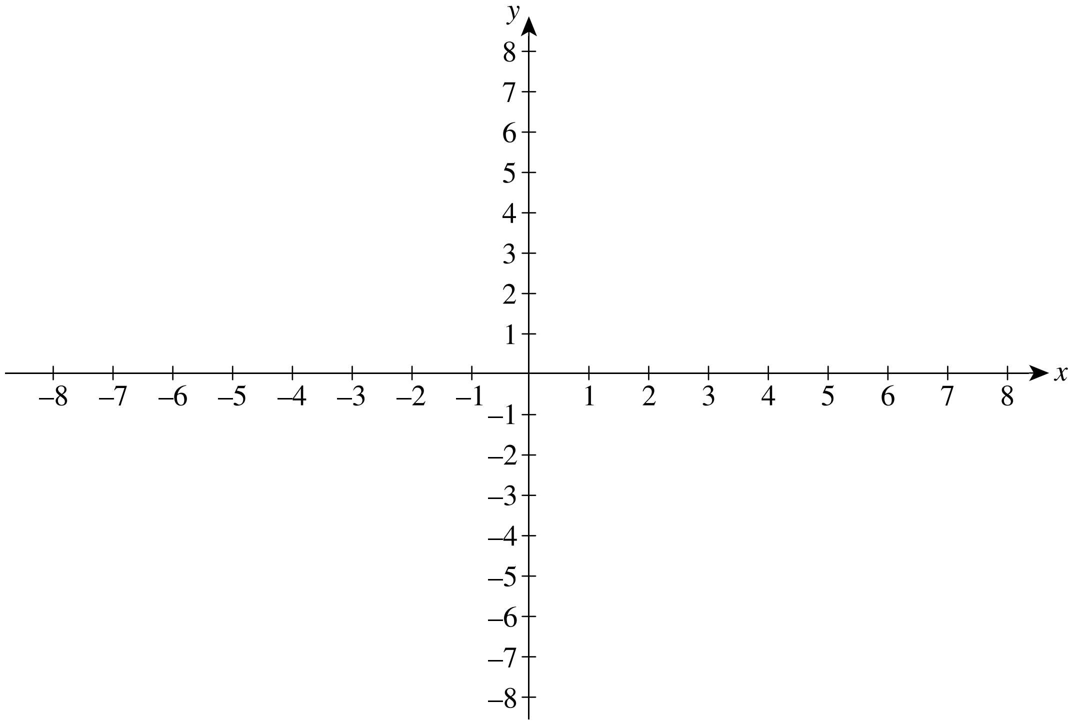
Question 1 (10 marks)

a. The function $f(x) = \frac{1}{x^3}$ is dilated from the *y*-axis by a factor of $\frac{1}{2}$, translated 1 unit to the left and 3 units down. The function obtained as a result of those transformations is $g(x)$.

Find the rule of the inverse function $g^{-1}(x)$ and state its domain and range. 3 marks

b. Find the equation of the normal to the graph of $g(x)$ at the right point of its intersection with the graph $g^{-1}(x)$. Give value of *y*-intercept correct to 4 decimal places. 3 marks

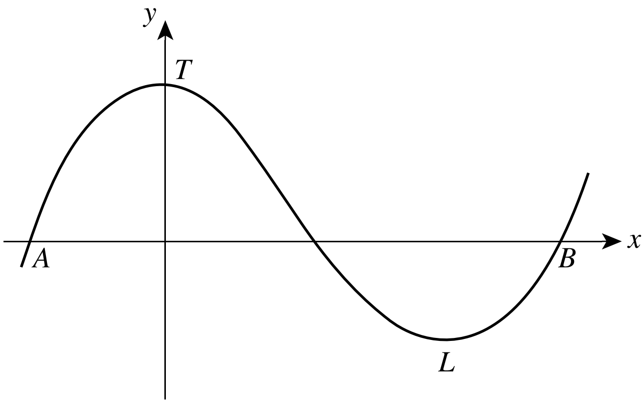
c. Sketch the graphs of $g(x)$ and $g^{-1}(x)$ on the same axis below. List coordinates of any axes intercepts, and points of intersection correct to 4 decimal places. 2 marks



d. Find the area enclosed between the graphs of $g(x)$, $g^{-1}(x)$ and the y-axis, correct to 4 decimal places 2 marks

Question 2 (18 marks)

Phil Hotham is a construction engineer working to build a new ski resort at Fool's Creek. Part of the plan of the main ski run on the mountain is shown on the diagram below.



This section of the mountain can be mapped by the curve with equation

$$f(x) = \frac{x^3}{480} - \frac{7x^2}{48} + 75$$

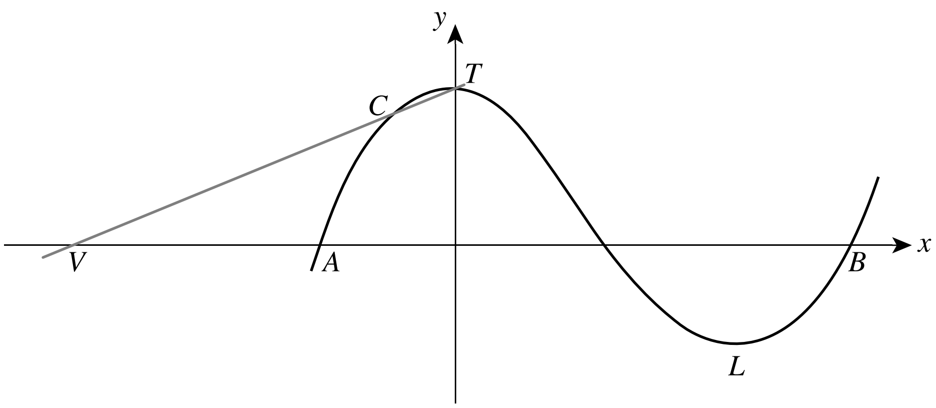
from A to B. The x-axis represents the level at which the village will be built. T is the highest point of the ski run and L is the lowest point. All distances are measured in metres.

a. Find the coordinates of A and B. 2 marks

b. Find the height of the top of the mountain above the village, T. 2 marks

c. Find the average gradient of the ski run from T to L. 3 marks

Phil is planning a chairlift to take the skiers to the top of the mountain from the village. He plans to build the village at the point V, which is 40 m from A. The path of the chairlift is shown on the diagram below from V to T.



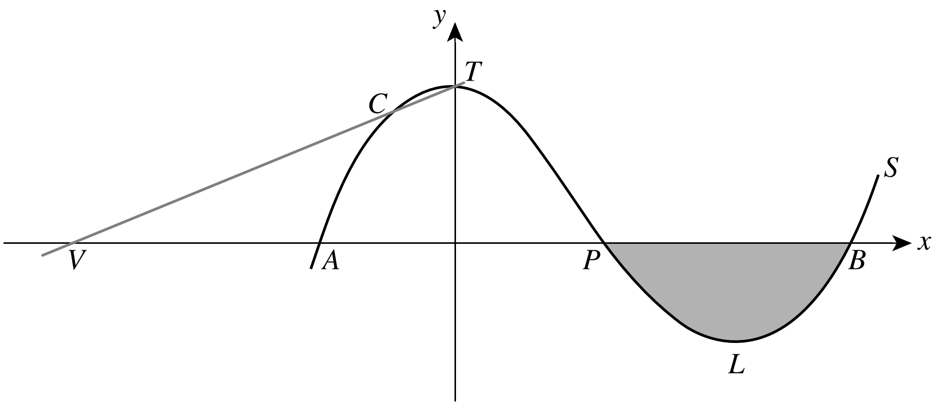
d. Find the equation of the line which models the path of the chairlift. 1 mark

Phil has found that the chairlift will reach the mountainside at the point C before it gets to the top of the mountain. He decides to build a tunnel to complete the journey to the top, T.

e. Find the coordinates of point C. 2 marks

f. Find the length of the tunnel required to get to the top of the mountain from C to T. State your answer to the nearest centimetre. 2 marks

The developers of Fool's Creek would like to entice visitors to the resort all year round. Phil discovers that when the snow melts, a pond is formed in the valley between P and B, as shown in the diagram below.



g. Find the area of the cross-section of the pond. 2 marks

Phil is considering adding a water slide to be built down the other side of the mountain, beginning at point S and entering the pond at point B. Safety regulations require that the average gradient of the slide be no more than one third of the depth of the water at the deepest part of the pond.

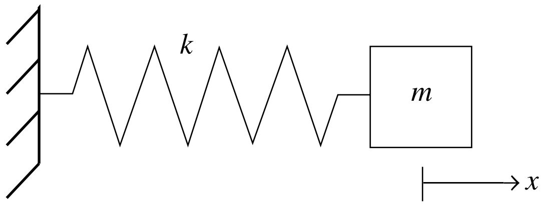
h. Find the coordinates of S, the highest point on the mountain that the water slide could start from and still satisfy the safety regulations. Give your answer to the nearest centimetre. 4 marks

Question 3 (18 marks)

Nathan conducts a physics experiment which involves connecting a mass, m, to a spring. He pulls the mass to the right and releases it. The mass starts to oscillate horizontally on the frictionless surface. Its velocity changes over time according to the function

$$v(t) = -v_m \sin(wt)$$

where v_m is the maximum velocity, t is the time in seconds and $w = \sqrt{\frac{k}{m}}$, k is the spring coefficient. Velocity is positive when the mass is moving to the right and negative when it is moving to the left. The mass oscillates between points A and B. Nathan takes the displacement at the midpoint of the line segment AB as equal to 0, so the displacement is positive from 0 to the right (towards point B) and negative from 0 to the left (towards point A).



a. If the period of the function $v(t)$ is 2 seconds and $m = 0.16$, find the value of k correct to two decimal places. 1 mark

b. At what time after $t = 0$ does velocity becomes equal to 0 for the first time? 1 mark

c. Displacement, $x(t)$, is an antiderivative of velocity.

Find an expression for $x(t)$ if the initial position is 10 cm (when $t = 0$) and $v_m = 10\pi$. 2 marks

d. Acceleration, $a(t)$, is the derivative of velocity.

Find the acceleration at the first instant of time when velocity equals 0. 2 marks

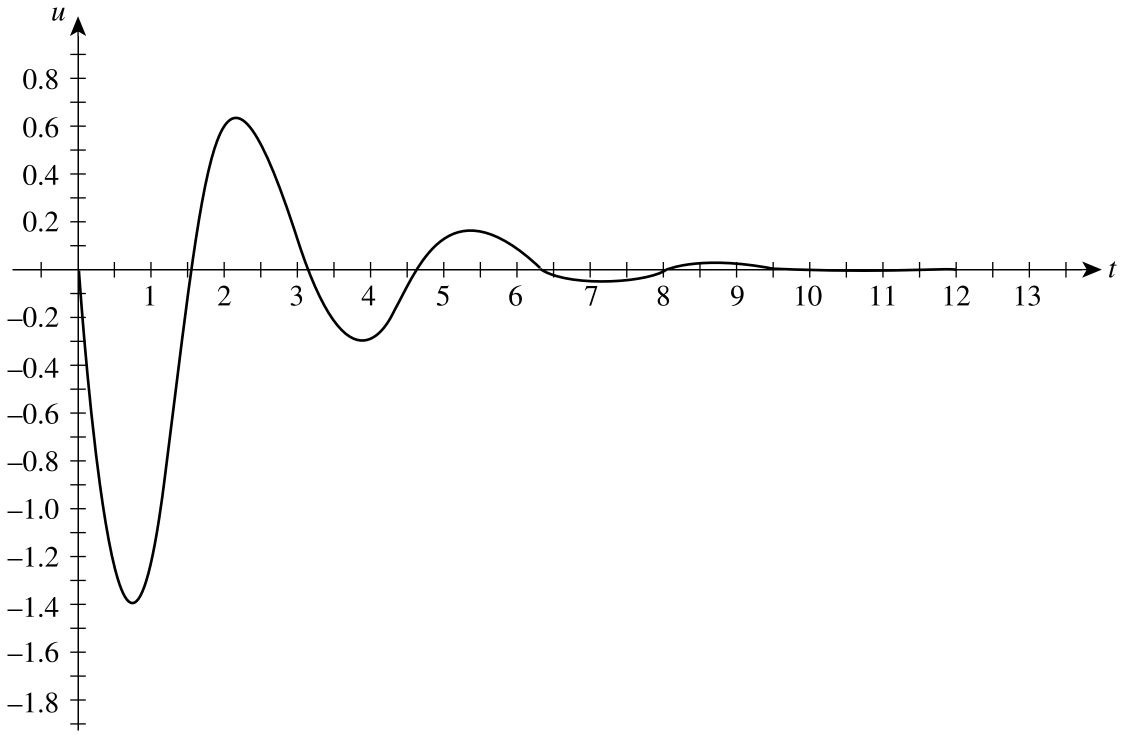
e. The distance travelled by the mass can be found as the area enclosed between the graph $v(t)$ and the time axis.

Find the distance travelled between $t = 1.5$ s and $t = 4.75$ s correct to two decimal places. 3 marks

Vybavi joined Nathan and decided to make some changes to the experiment. First she changed the surface to include friction, and as a result of this the velocity changed according to the function

$$u(t) = -2e^{-rt}\sin(2t)$$

The graph of this function is shown on the diagram below.



f. Find r if the value of the velocity at the third maximum is 0.0266. Give your answer correct to two decimal places. 3 marks

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g. Find at what times, in seconds, the displacement will be greatest (both in negative and positive directions) during the first 5 seconds. 3 marks

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h. Find the displacement of the mass after 10 seconds correct to two decimal places (displacement is the antiderivative of the velocity). 3 marks

Question 4 (12 marks)

A gymnasium owner is considering opening a new gym. He contracted a research company to carry out a survey on the lifestyles of people in two different regions, *X* and *Y*. The research company developed mathematical models from their results.

a. The research company found that the time, in hours, spent each week on sporting activities by people from region *X* is a continuous random variable, *T*, with a probability density function given by

$$f(t) = \begin{cases} \frac{27}{2}t^2 e^{-3t} & t \geq 0 \\ 0 & \text{elsewhere} \end{cases}$$

where *a* is a positive real constant.

i. Find the number of hours a person from region *X* would be expected to spend each week on sporting activities. 2 marks

ii. Find the median time, in hours, that the people from region *X* spent on sporting activities. Give your answer correct to two decimal places. 2 marks

b. It has been found that 30% of the population comes from region X.

If a researcher randomly selected eight people to survey, what is the probability that at least three of the people chosen were from region X? Give your answer correct to four decimal places. 2 marks

The time, in hours, spent on sporting activities each week in region Y is a normally distributed random variable, S. It is known that 55% of the population spend at least one and a half hours on sporting activities and 70% of the population spend less than 2 hours.

c. Find the mean and standard deviation of the time spent each week on sporting activities. Give your answer correct to two decimal places. 3 marks

The gymnasium owner will only offer a gym membership to a person from region Y who spends at least one hour each week on sporting activities.

d. What is the probability that a person selected at random will be offered a gym membership if they spend more than one and a half hours per week in sporting activities? Give your answer correct to four decimal places. 3 marks

END OF QUESTION AND ANSWER BOOKLET