Year 2014

VCE

Mathematical Methods

Trial Examination 2

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Victorian Certificate of Education
2014

STUDENT NUMBER

									Letter
Figures									
Words									

MATHEMATICAL METHODS CAS

Trial Written Examination 2

Reading time: 15 minutes
Total writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	4	4	58
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator (memory DOES NOT need to be cleared) and, if desired, one scientific calculator. For approved computer-based CAS, their full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer booklet of 32 pages with a detachable sheet of miscellaneous formulas at the end of this booklet.
- Answer sheet for multiple-choice questions.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- Write your **name** and **student number** on your answer sheet for multiple-choice questions, and sign your name in the space provided.
- All written responses must be in English.

At the end of the examination

- Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION 1

Instructions for Section I
Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1 mark, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No mark will be given if more than one answer is completed for any question.

Question 1

Given the function $f:[0,4b)\to R$, $f(x)=-x^2+2bx+3b^2$ where $b\in R^+$, then the range of the function of f is

A. $\left[3b^2,4b^2\right)$

B. $\left(3b^2,4b^2\right]$

C. $\left(3b^2,5b^2\right]$

D. $\left(-5b^2,3b^2\right]$

E. $\left(-5b^2,4b^2\right]$

Question 2

For the function $f(x)=\sin\left(\frac{\pi e^x}{2}\right)$, the average rate of change of $f(x)$ with respect to x over the interval $[0,2]$ is closest to

A. –1.819

B. –0.910

C. 0.079

D. 0.087

E. 0.157

Question 3

The function with the rule $f(x)=\tan\left(\frac{bx}{2}\right)$ where $b \in R^{+}$, has period equal to p, then

A. $b=\frac{4\pi}{p}$

B. $b=\frac{2\pi}{p}$

C. $p=\pi b$

D. $b=\pi p$

E. $b=2\pi p$

Question 4

Given that $f(x)=g(x)e^{h(x)}$ and that

$g(2)=4$, $g'(2)=3$, $h(2)=1$ and $h'(2)=2$, Then $f'(2)$ is equal to

A. $6e$

B. $7e$

C. $8e$

D. $11e$

E. $12e$

Question 5

Using the linear approximation $f(x+h) \approx f(x)+hf'(x)$, with $f(x)=\cos(x)$, the value of $\cos\left(43^{0}30^{'}\right)$ is closest to

A. 0.689

B. 0.713

C. 0.725

D. 0.726

E. 0.727

Question 6

Consider the polynomial $P(x) = x^3 - 6a^2x + ka^3$ where a and k are non-zero real constants.

The polynomial has $x - 2a$ as a factor and when the polynomial is divided by $x + a$ the remainder is equal to -9. Then

A. $k = -4$ and $a = -1$

B. $k = -4$ and $a = 1$

C. $k = 4$ and $a = \sqrt[3]{9}$

D. $k = 4$ and $a = -1$

E. $k = 4$ and $a = 1$

Question 7

If $f(x)$ is a one-one differentiable function, with f^{-1} the inverse function and $f'(x) = \frac{d}{dx}\left(f(x)\right)$, then $\frac{d}{dx}\left[f^{-1}(x)\right]$ is equal to

A. $\dfrac{f'(x)}{f(x)}$

B. $-\dfrac{f'(x)}{\left[f(x)\right]^2}$

C. $\dfrac{1}{f'\left(f^{-1}(x)\right)}$

D. $\dfrac{1}{f^{-1}\left(f'(x)\right)}$

E. $f'\left(f^{-1}(x)\right)$

Question 8

If $y = 3 + \int_4^{2x} e^{t^2}\, dt$ then which of the following is true?

A. $\frac{dy}{dx} = e^{x^2}$ and $y(0) = 3$

B. $\frac{dy}{dx} = 2xe^{4x^2}$ and $y(0) = 3$

C. $\frac{dy}{dx} = 2e^{4x^2}$ and $y(2) = 3$

D. $\frac{dy}{dx} = 2xe^{4x^2}$ and $y(2) = 3$

E. $\frac{dy}{dx} = e^{4x^2}$ and $y(2) = 3$

Question 9

The transformation $T: R^2 \to R^2$, which maps the curve with equation $y = \sqrt{x}$ to the curve with equation $y = 4 - \sqrt{3 - 2x}$, has the rule

A. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -\frac{1}{2} & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} \frac{3}{2} \\ 4 \end{bmatrix}$

B. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 3 \\ 4 \end{bmatrix}$

C. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -\frac{3}{2} \\ 4 \end{bmatrix}$

D. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 3 \\ 4 \end{bmatrix}$

E. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -3 \\ -4 \end{bmatrix}$

Question 10

If $y=\sqrt{25-x^2}$ and *x* is a function of *t* such that $\frac{dx}{dt}=8\cos(2t)$ and when $t=\frac{\pi}{8}$, $x=2\sqrt{2}$.

The value of $\frac{dy}{dx}$ when $t=\frac{\pi}{4}$ is

A. $-\frac{4}{3}$

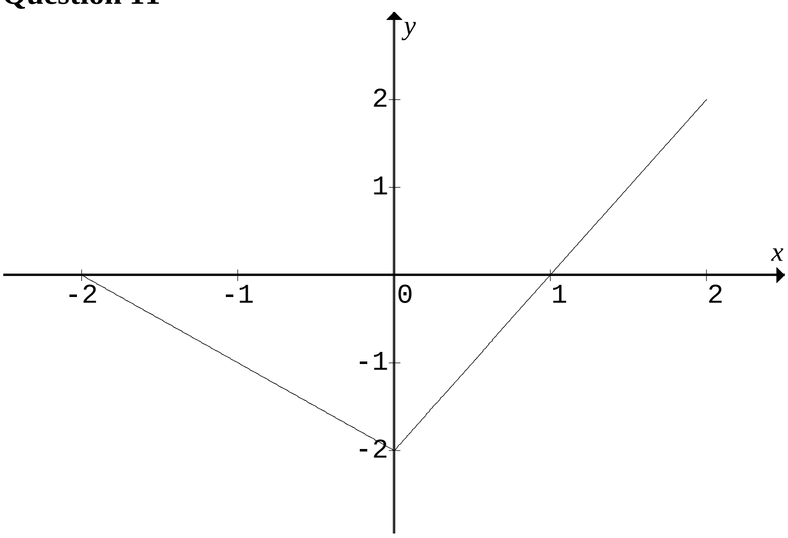
B. $-4\sqrt{2}$

C. $-\frac{8}{3}$

D. $-\frac{3}{4}$

E. $-\frac{2}{3}$

Question 11



The graph of the function f is shown above and consists of two straight line segments.

If $g(x)=\int_0^x f(t)\,dt$, then $g(1)$ is equal to

A. -2

B. -1

C. 0

D. 1

E. 2

Question 12

The functions $f(x)=\sqrt{x+a}$ and $g(x)=\sqrt{b-x}$ are defined on their maximal domains, where $b>a>0$. The maximal domain of the function $\frac{f}{g}$ is

A. $[-a,b)$

B. $[a,-b)$

C. $[-a,b]$

D. $[a,-b]$

E. $[a,b]$

Question 13

A particle moves in a straight line and has a velocity given by $v(t)=e^{t}\cos\left(\frac{t}{2}\right)$ metres per second, where t is the time in seconds and $t\geq 0$. Over the first two seconds, the distance travelled in metres by the particle is closest to

A. 0.883

B. 2.440

C. 4.881

D. 6.389

E. 7.387

Question 14

The function $f:(a,\infty)\to R$, $f(x)=x^{4}-4x^{2}$ has an inverse function.
The value of a could be equal to

A. -2

B. $-\sqrt{2}$

C. 0

D. 1

E. $\sqrt{2}$

Question 15

If $f(x)=\log_{6x}(16)$ and $g(x)=\log_{8}\left(\sqrt{3x}\right)$, $x>0$. Which of the following equations is true for all positive real values of *x*?

A. $f(x)g\left(\frac{x}{2}\right)=\frac{1}{3}$

B. $f(x)g\left(\frac{x}{2}\right)=\frac{2}{3}$

C. $f\left(\frac{x}{2}\right)g(x)=\frac{2}{3}$

D. $f\left(\frac{x}{2}\right)g(x)=\frac{3}{2}$

E. $f(x)g(x)=\frac{1}{3}$

Question 16

Which of the following is **not** true about the function defined by the rule $f(x)=\left|\log_e|x|\right|$?

A. $f'(x)>0$ for $x>1$.

B. $f'(x)<0$ for $x<-1$.

C. The function is not differentiable at $x=0$ and at $x=\pm1$.

D. The range is $[0,\infty)$.

E. The function is continuous everywhere.

Question 17

For events *A* and *B*, $\Pr(A)=a$, $\Pr(A\cap B)=0.04$ and $\Pr(A'\cap B')=0.54$.

If *A* and *B* are independent, then

A. $a=0.1$ only

B. $a=0.4$ only

C. $a=0.5$ only

D. $a=0.1$ or $a=0.4$

E. $a=0.08$ or $a=0.5$

Question 18

Ashley always has either cereal or eggs for breakfast. If he has eggs one morning the probability he has cereal the next day is p. If he has cereal one morning the probability he has eggs the next day is q. In the long term, the probability Ashley has cereal for breakfast is equal to $\frac{27}{99}$. Then

A. $p = \frac{1}{4}$ and $q = \frac{2}{3}$

B. $p = \frac{1}{4}$ and $q = \frac{1}{3}$

C. $p = \frac{3}{4}$ and $q = \frac{2}{3}$

D. $p = \frac{1}{3}$ and $q = \frac{3}{4}$

E. $p = \frac{2}{3}$ and $q = \frac{3}{4}$

Question 19

It is found that 58% of domestic cats are overweight. From a group of 10 domestic cats, the probability that more than half are overweight, if at least two are overweight is closest to

A. 0.582

B. 0.584

C. 0.592

D. 0.800

E. 0.812

Question 20

The heights of trees in a forest are normally distributed, with a standard deviation of 15 metres. It is found that 75% of the trees have heights greater than 30 metres. A park ranger examines two trees, the probability that one has a height greater than 45 metres is closest to

A. 0.090

B. 0.047

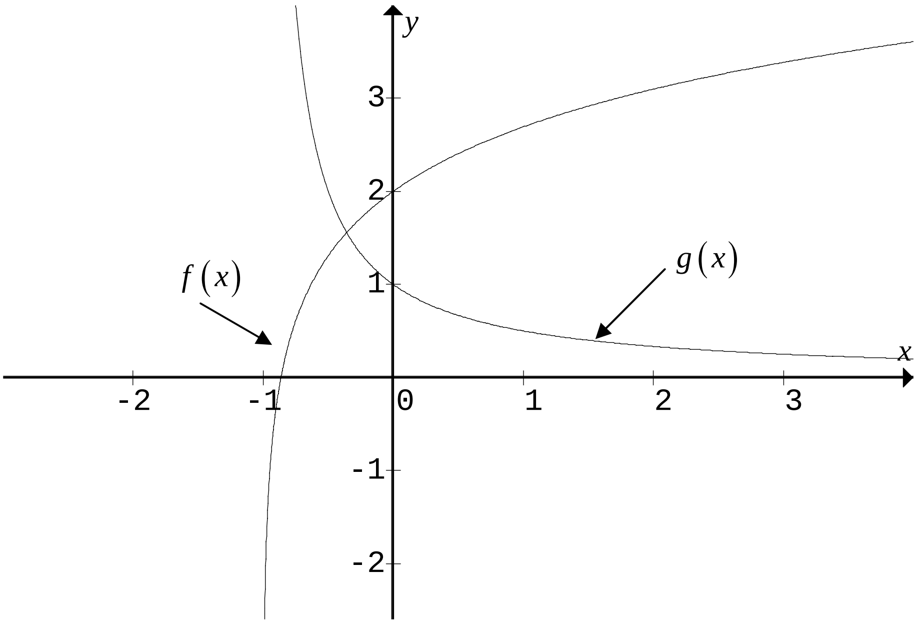
C. 0.275

D. 0.372

E. 0.467

Question 21

The diagram below show the graphs of two functions $f(x)$ and $g(x)$.



Which of the following is the relationship between the graphs of f and g ?

A. $g(x)=\dfrac{1}{f(x)}$

B. $g(x)=|f(x)|$

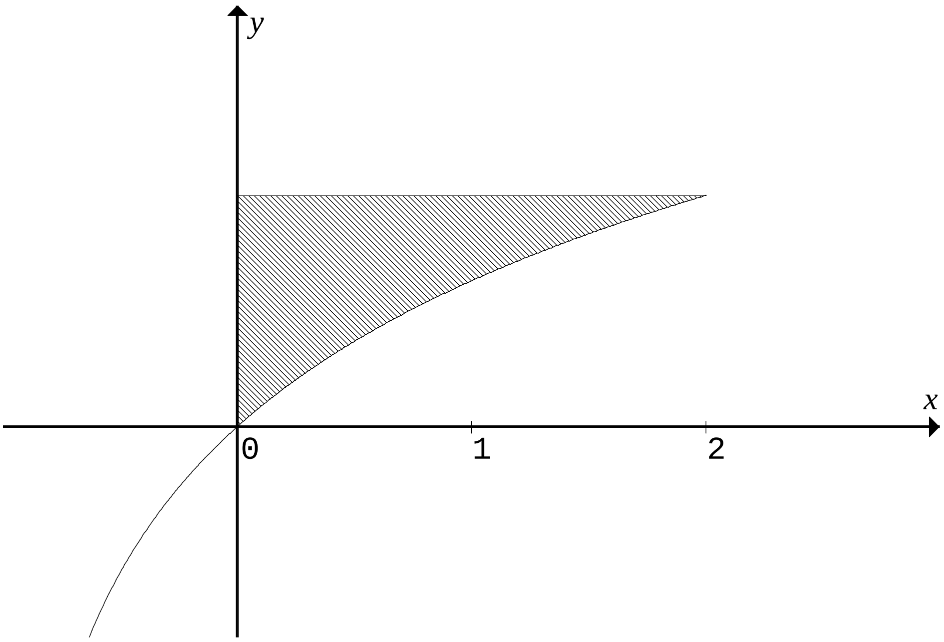
C. $g(x)=f^{-1}(x)$

D. $g(x)=\int f(x)dx$

E. $g(x)=\dfrac{d}{dx}\left[f(x)\right]$

Question 22

Part of the graph of $f:(-1,2]\to R$, $f(x)=\log_e(x+1)$ is shown below.



The shaded area is given by

A. $\int_0^2 \log_e(x+1)\,dx$

B. $\int_0^{\log_e(3)} \left(e^{x+1}\right)dx$

C. $\int_0^{\log_e(3)} \left(e^{x}-1\right)dx$

D. $\log_e(9)-\int_0^{\log_e(3)} \log_e(x+1)\,dx$

E. $\log_e(6)-\int_0^2 \log_e(x+1)\,dx$

END OF SECTION 1

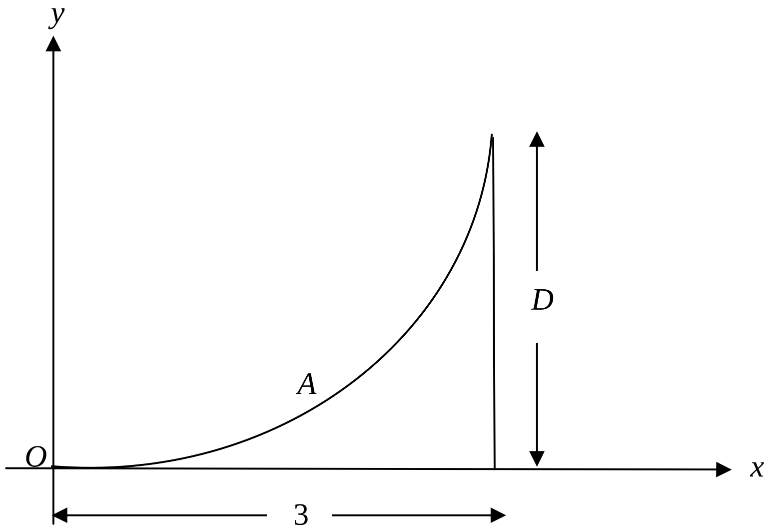
SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (14 marks)

At a circus, roller skaters move up and down a ramp. The ramp is designed as a function of x, the horizontal distance of the ramp from the origin O and y represents the height of the ramp above the ground. The lengths are measured in metres. The ramp has a width of 3 metres and a height of D metres. The diagram below show a cross section of the ramp.



The point A, is located horizontally at the midpoint of the ramp and is 0.375 metres above the ground. For the safety of the performers, the function to represent the ramp must meet certain criteria

(1) At the origin, the gradient of the graph is zero.

(2) At the point A, the tangent to the ramp is inclined at 45^0.

(3) The gradient of the ramp is always increasing.

The designers of the ramp try to model the ramp by various functions, $f(x), g(x)$ and $h(x)$.

a. Let $f(x)=ax^2$ where a is a positive constant. Show that this function does satisfy criteria (1), however it is not possible to satisfy criteria (2).

2 marks

b. Let $g(x)=bx^3+cx^2$ where b and c are constants.

i. Find the values of b and c if this function satisfies criteria (2).

3 marks

ii. Explain why this function does not satisfy criteria (3).

2 marks

c. Let $h(x)=\frac{x^n}{k}$ where n is a positive integer and k is a positive constant.

i. If this function satisfies criteria (2) show that $k=\frac{27}{2}$ and determine the value of n.

2 marks

ii. For this function find the value of D.

1 mark

iii. For this function find the angle of inclination, in degrees and minutes, of the tangent at the highest point on the ramp.

2 marks

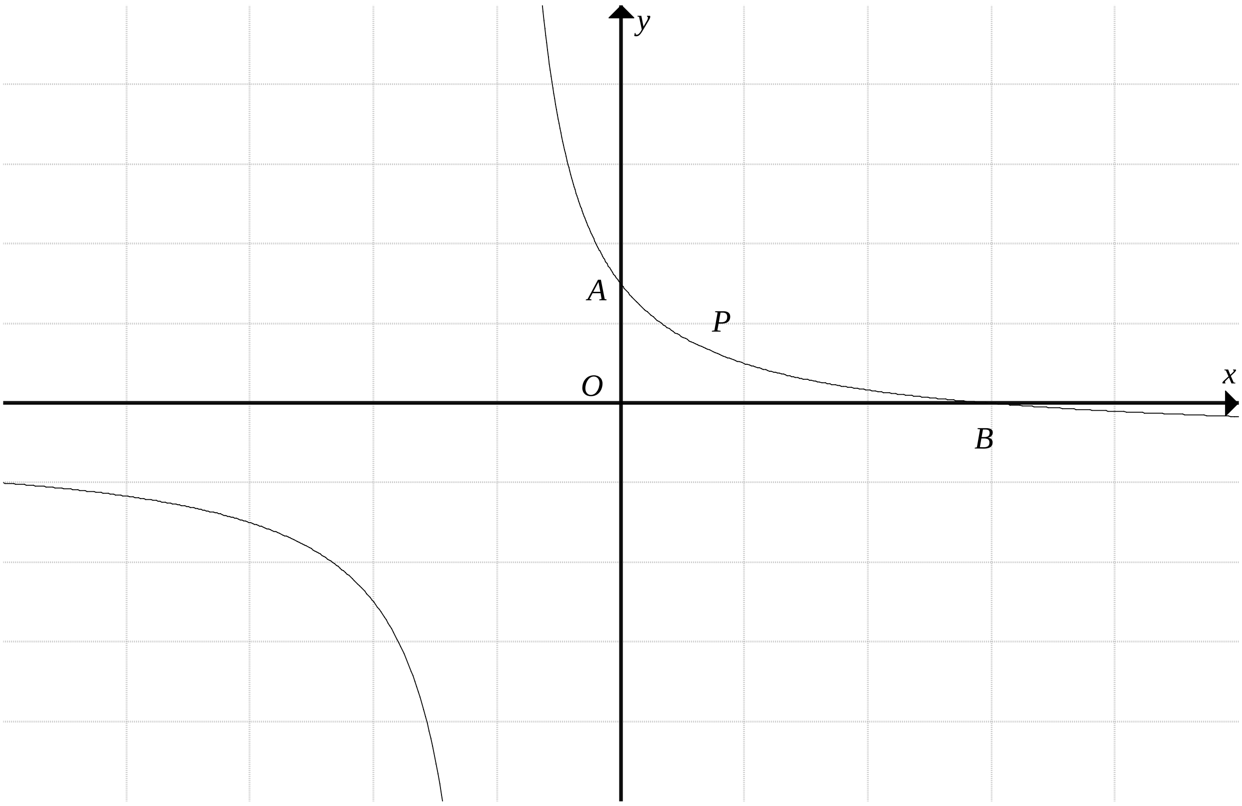
iv. For this function find using calculus, the cross sectional area of the ramp.

2 marks

Question 2 (16 marks)

The diagram below shows part of the graph of the function $f(x)=\dfrac{8}{x+2}-1$.

The curve crosses the *y*-axis at the point *A* and crosses the *x*-axis at the point *B*.



a. Write down the coordinates of the points *A* and *B* and mark in suitable scales on the *x* and *y* axes. Draw and state the equations of the asymptotes of the graph on the diagram above.

2 marks

A point *P* with coordinates $\left(k, f\left(k\right)\right)$, where $k > 0$ and $f\left(k\right) > 0$ lies on the graph of

$y = \dfrac{8}{x+2} - 1$

b. Find the value of k, if the tangent to the curve at the point *P*, is parallel to the line segment joining the points *A* and *B*.

3 marks

c. Find the value of *k*, correct to three decimal places, if the point *P* is closest to the origin.

3 marks

The tangent to the curve at the point P, crosses the y-axis at the point C and the x-axis at the point D.

d.i. Write down in terms of k, the coordinates of the points C and D.

3 marks

ii. Find in terms of k, an expression for the area of the triangle OCD, where O is the origin.

1 mark

iii. Find the value of k for which the area of the triangle OCD is a maximum.

2 marks

e. Find the value of k, correct to three decimal places if the normal to the curve at the point P, passes through the origin.

2 marks

Question 3 (11 marks)

Oranges are squeezed in a hand held juicer. The orange juice drips into a cup. The amount of orange juice in mls in the cup at a time t seconds, is a differentiable function given by $J(t)$.

a. Values of $J(t)$ are given in the table below.

t seconds	0	10	20	30	40	50
$J(t)$ mls	0	70	110	140	155	165

i. Explain the meaning of $\frac{1}{50}\int_0^{50} J(t)\,dt$ in relation to the problem.

1 mark

ii. Using a right rectangle rule with five subintervals of equal length, find an approximation to $\frac{1}{50}\int_0^{50} J(t)\,dt$.

2 marks

iii. Use the data in the table to find a linear approximation to $J'(25)$.

2 marks

b. In fact $J(t) = 180\left(1 - e^{-\frac{t}{20}}\right)$ is a function which models the amount of orange juice in the cup in mls after a time *t* seconds, where $t \in [0, 50]$.

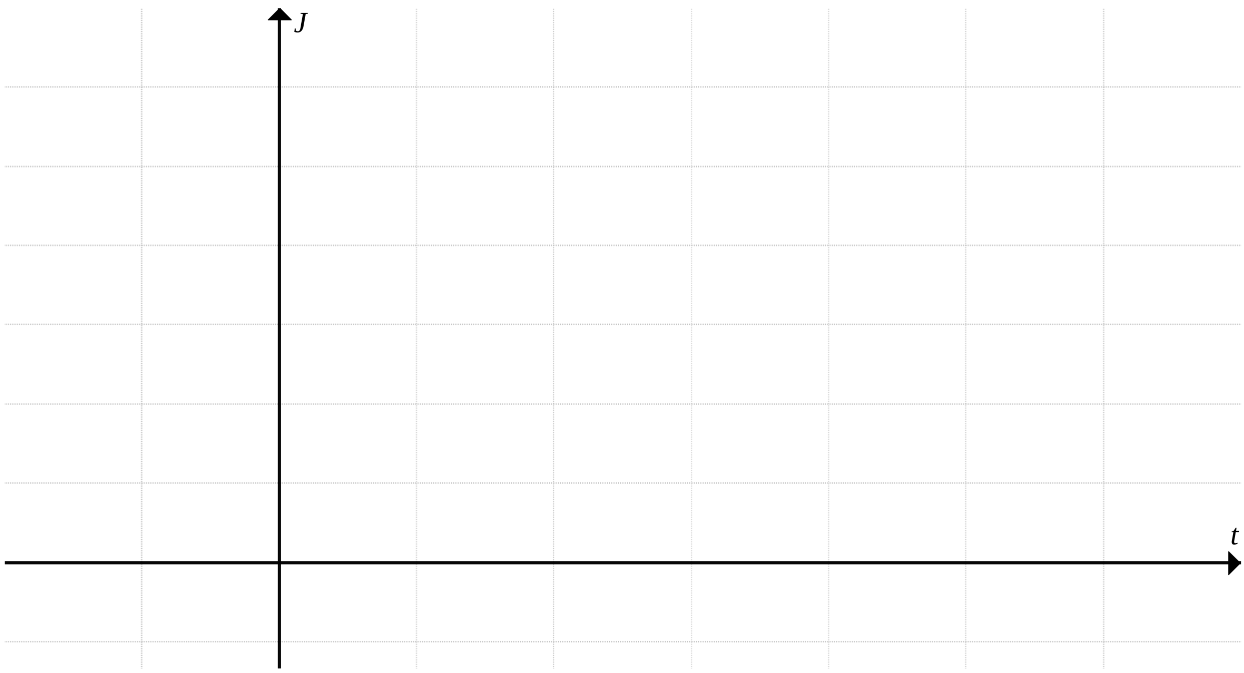
i. Complete the table below giving your answers correct to one decimal place.

1 mark

t seconds	0	10	20	30	40	50
$J(t)$ mls						

ii. Sketch the graph of $J(t)$ on the axes below, clearly labelling the scales, and the coordinates of any stationary points or end-points.

1 mark



iii. Find, using calculus the time rate of change of the amount of orange juice in the cup and evaluate after 25 seconds. Give your answer correct to two decimal places.

2 marks

iv. Find, using calculus the average amount of orange juice in the cup over the time interval $0 \le t \le 50$. Give your answer correct to two decimal places.

2 marks

Question 4 (17 marks)

a. Sharon has either freshly squeezed orange juice or pineapple juice for breakfast every morning. If she has freshly squeezed orange juice for breakfast one morning, the probability she has pineapple juice the next morning is 0.25. If she has pineapple juice one morning, the probability she has freshly squeezed orange juice the next morning is 0.7. On Monday of a certain week Sharon had freshly squeezed orange juice.

i. Find the probability giving your answer correct to three decimal places that she has freshly squeezed orange juice on Thursday.

2 marks

ii. Find the probability giving your answer correct to three decimal places that from Monday to Thursday she has freshly squeezed orange juice at least three times.

2 marks

b. The time taken t in minutes for Sharon to finish her breakfast, is found to be a probability density function $T(t)$ defined by

$$T(t)=\begin{cases} a\sin\left(\dfrac{\pi t}{10}\right) & 0 \le t \le 5 \\ b(15-t) & 5 \le t \le 15 \\ 0 & \text{otherwise} \end{cases}$$

i. Show that $a=\dfrac{\pi}{5(\pi+2)}$ and $b=\dfrac{\pi}{50(\pi+2)}$. 2 marks

ii. Sketch the graph of $T(t)$ on the axes below, clearly labelling the scales. 1 mark

iii. Find $\text{Var}(T)$ giving your answer correct to three decimal places.

2 marks

iv. Find the median time correct to three decimal places, taken by Sharon to finish her breakfast.

2 marks

v. Over a period of five days, find the probability that Sharon finishes her breakfast in under three minutes on at least two occasions. Give your answer correct to four decimal places.

2 marks

vi. Find the probability that Sharon finishes her breakfast in under ten minutes if she has taken at least seven minutes to finish her breakfast.

2 marks

c. When Sharon makes toast for breakfast, the time taken for the toast to be ready, is found to be normally distributed with a mean of one minute. 70% of the time, the toaster takes more than 50 seconds for the toast to be ready. Determine the standard deviation of the time for the toast to be ready, giving your answer in seconds correct to one decimal place.

2 marks

END OF EXAMINATION

EXTRA WORKING SPACE

MATHEMATICAL METHODS CAS

Written examination 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

Mathematical Methods and CAS Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$	Volume of a pyramid:	$\frac{1}{3}Ah$
curved surface area of a cylinder:	$2\pi rh$	volume of a sphere:	$\frac{4}{3}\pi r^3$
volume of a cylinder:	$\pi r^2 h$	area of triangle:	$\frac{1}{2}bc\sin(A)$
volume of a cone:	$\frac{1}{3}\pi r^2 h$		

Calculus

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + c\ ,\ n \neq -1$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$$

$$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$$

$$\int \frac{1}{x} dx = \log_e |x| + c$$

$$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$$

$$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + c$$

$$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$$

$$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + c$$

$$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\sec^2(ax)$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

approximation: $f(x+h) \approx f(x) + h\,f'(x)$

Probability

$\Pr(A) = 1 - \Pr(A')$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A/B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

Transition Matrices $S_n = T^n \times S_0$

mean: $\mu = E(X)$

variance: $\mathrm{var}(X) = \sigma^2 = E\left((X-\mu)^2\right) = E(X^2) - \mu^2$

probability distribution		mean	variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x\,p(x)$	$\sigma^2 = \sum (x-\mu)^2\,p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)\,dx$	$\mu = \int_{-\infty}^{\infty} x\,f(x)\,dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2\,f(x)\,dx$

ANSWER SHEET

STUDENT NUMBER

Figures									Letter:
Words									

SIGNATURE ______________________________________

SECTION 1

1	**A**	**B**	**C**	**D**	**E**
2	**A**	**B**	**C**	**D**	**E**
3	**A**	**B**	**C**	**D**	**E**
4	**A**	**B**	**C**	**D**	**E**
5	**A**	**B**	**C**	**D**	**E**
6	**A**	**B**	**C**	**D**	**E**
7	**A**	**B**	**C**	**D**	**E**
8	**A**	**B**	**C**	**D**	**E**
9	**A**	**B**	**C**	**D**	**E**
10	**A**	**B**	**C**	**D**	**E**
11	**A**	**B**	**C**	**D**	**E**
12	**A**	**B**	**C**	**D**	**E**
13	**A**	**B**	**C**	**D**	**E**
14	**A**	**B**	**C**	**D**	**E**
15	**A**	**B**	**C**	**D**	**E**
16	**A**	**B**	**C**	**D**	**E**
17	**A**	**B**	**C**	**D**	**E**
18	**A**	**B**	**C**	**D**	**E**
19	**A**	**B**	**C**	**D**	**E**
20	**A**	**B**	**C**	**D**	**E**
21	**A**	**B**	**C**	**D**	**E**
22	**A**	**B**	**C**	**D**	**E**