Year 2014

VCE Mathematical Methods CAS

Trial Examination 2

Suggested Solutions

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SECTION 1

ANSWERS

1	A	B	C	D	**E**
2	A	**B**	C	D	E
3	A	**B**	C	D	E
4	A	B	C	**D**	E
5	A	B	C	**D**	E
6	A	B	C	**D**	E
7	A	B	**C**	D	E
8	A	B	**C**	D	E
9	**A**	B	C	D	E
10	**A**	B	C	D	E
11	A	**B**	C	D	E
12	**A**	B	C	D	E
13	A	B	**C**	D	E
14	A	B	C	D	**E**
15	A	B	**C**	D	E
16	A	B	C	D	**E**
17	A	B	C	**D**	E
18	**A**	B	C	D	E
19	A	**B**	C	D	E
20	A	B	C	D	**E**
21	A	B	C	D	**E**
22	A	B	**C**	D	E

SECTION 1

Question 1 **Answer E**

$f:[0,4b)\to R\ ,\ f(x)=-x^2+2bx+3b^2$

$f(x)=-(x+b)(x-3b)$

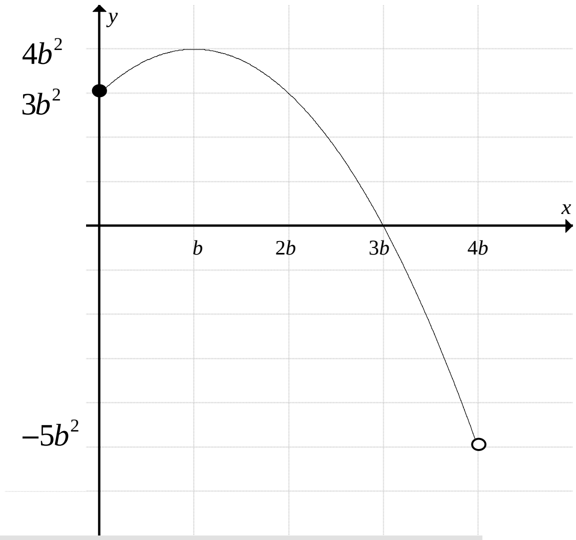
$f(0)=3b^2$

$f(4b)=-5b^2$

$f'(x)=-2x+2b$

$f'(x)=0\ \Rightarrow\ x=b \qquad f(b)=4b^2$

The range is $\left(-5b^2,4b^2\right]$



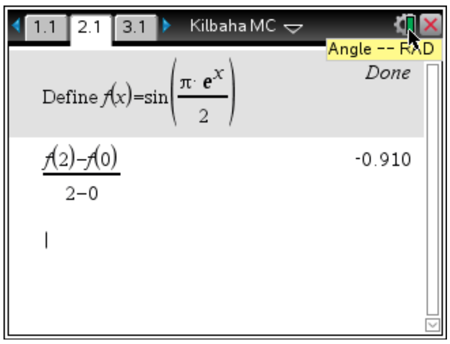
Define $f(x)=-x^2+2\cdot b\cdot x+3\cdot b^2$	*Done*
$f(0)$	$3\cdot b^2$
$f(4\cdot b)$	$-5\cdot b^2$
factor$(f(x))$	$-(x+b)\cdot(x-3\cdot b)$
solve$(f(x)=0,x)$	$x=3\cdot b$ or $x=-b$
$\frac{d}{dx}(f(x))$	$2\cdot b-2\cdot x$
solve$\left(\frac{d}{dx}(f(x))=0,x\right)$	$x=b$
$f(b)$	$4\cdot b^2$

Question 2 **Answer B**

$f(x)=\sin\left(\frac{\pi e^x}{2}\right)$

average rate of change over $[0,2]$

$\frac{f(2)-f(0)}{2-0}=-0.910$



Question 3 **Answer B**

The period $T=\frac{\pi}{\frac{b}{2}}=p \quad \Rightarrow b=\frac{2\pi}{p}$

Question 4 **Answer D**

$f(x) = g(x)e^{h(x)}$ using the product rule

$f'(x) = g'(x)e^{h(x)} + g(x)h'(x)e^{h(x)}$

$f'(2) = g'(2)e^{h(2)} + g(2)h'(2)e^{h(2)}$

Now $g(2) = 4$, $g'(2) = 3$, $h(2) = 1$ and $h'(2) = 2$

$f'(2) = 3 \times e^1 + 4 \times 2e^1 = 11e$

Question 5 **Answer D**

$f(x) = \cos(x) \quad f'(x) = -\sin(x)$

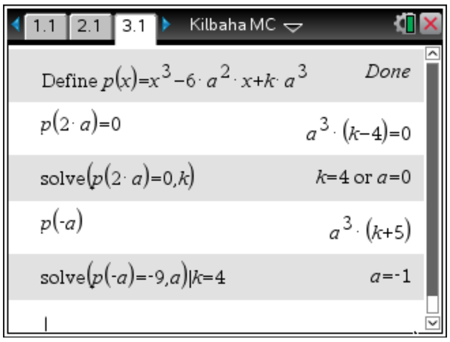
$x = 45^0 = \frac{\pi}{4} \quad h = -1.5^0 = -\frac{3}{2} \times \frac{\pi}{180} = -\frac{\pi}{120}$

$f\left(\frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} \qquad f'\left(\frac{\pi}{4}\right) = -\sin\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

$f(x+h) \approx f(x) + hf'(x)$

$\cos\left(43^0 30'\right) = \frac{\sqrt{2}}{2} + \frac{\pi}{120} \times \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}\left(1 + \frac{\pi}{120}\right) \approx 0.726$

Question 6 **Answer D**



$P(x) = x^3 - 6a^2 x + ka^3$

$P(2a) = 0$

$P(2a) = a^3(k-4) = 0 \Rightarrow$ since $a \neq 0 \quad k = 4$

$P(-a) = a^3(k+5) = -9$

$P(-a) = 9a^3 = -9 \quad \Rightarrow a = -1$

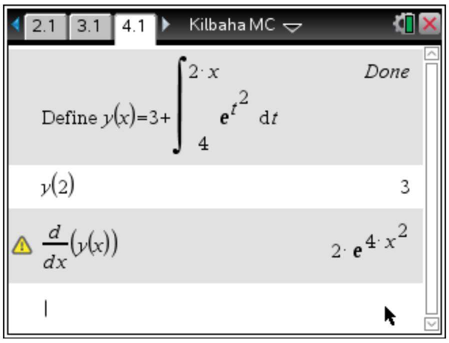
Question 7 **Answer C**

let $y = f^{-1}(x)$

$x = f(y)$ differentiate wrt y $\frac{dx}{dy} = f'(y)$

inverting $\frac{dy}{dx} = \frac{d}{dx}\left[f^{-1}(x)\right] = \frac{1}{f'(y)} = \frac{1}{f'\left(f^{-1}(x)\right)}$

Question 8 **Answer C**



$$y = y(x) = 3 + \int_4^{2x} e^{t^2}\, dt$$

$$y(2) = 3 + \int_4^{4} e^{t^2}\, dt = 3 + 0 = 3$$

$$\frac{dy}{dx} = \frac{d}{dx}(3) + \frac{d}{dx}(2x)\frac{d}{dx}\left[\int e^{4x^2}\, dx\right] = 2e^{4x^2}$$

Question 9 **Answer A**

$$y' = 4 - \sqrt{3 - 2x'}$$

$$4 - y' = \sqrt{3 - 2x'} \qquad y = \sqrt{x}$$

$$\Rightarrow y = 4 - y' \quad \text{and} \quad x = 3 - 2x'$$

$$\Rightarrow \quad y' = 4 - y \quad \text{and} \quad x' = \frac{3}{2} - \frac{x}{2}$$

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -\frac{1}{2} & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} \frac{3}{2} \\ 4 \end{bmatrix}$$

Question 10 **Answer A**

$$\frac{dx}{dt} = 8\cos(2t)$$

$$x = \int 8\cos(2t)\, dt = 4\sin(2t) + c$$

$$\text{when } t = \frac{\pi}{8} \quad x = 2\sqrt{2}$$

$$2\sqrt{2} = 4\sin\left(\frac{\pi}{4}\right) + c \quad \Rightarrow c = 0$$

$$x = 4\sin(2t) \quad \text{when} \quad t = \frac{\pi}{4} \quad x = 4\sin\left(\frac{\pi}{2}\right) = 4$$

$$y = \sqrt{25 - x^2} \qquad \frac{dy}{dx} = \frac{-x}{\sqrt{25 - x^2}}$$

$$\left.\frac{dy}{dx}\right|_{x=4} = \frac{-4}{\sqrt{25 - 16}} = -\frac{4}{3}$$

Question 11 **Answer B**

$g(x)=\int_0^x f(t)\,dt$ now $g(1)=\int_0^1 f(t)\,dt$ is the area bounded by the graph of $f(x)$

and the ordinates $x=0$ and $x=1$. This is the area of the triangle $=\frac{1}{2}\times 1\times {}^{-}2=-1$

The value of the definite integral is negative since the area is below the *x*-axis.

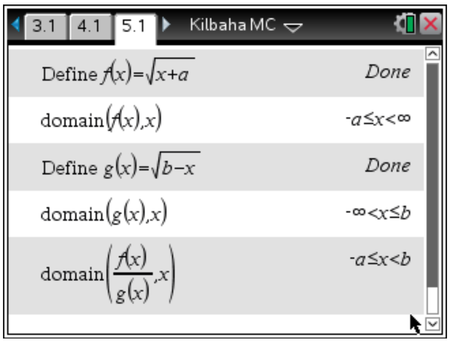
Question 12 **Answer A**

$f(x)=\sqrt{x+a}$ dom $f=[-a,\infty)$

$g(x)=\sqrt{b-x}$ dom $g=(-\infty,b]$

dom $\frac{f}{g}=$ dom $f\ \cap$ dom g and dom $g\neq 0$

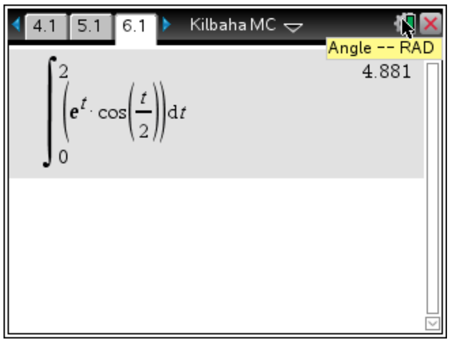
dom $\frac{f}{g}=[-a,b)$



Question 13 **Answer C**

$v(t)=e^t\cos\left(\frac{t}{2}\right)$

$d=\int_0^2 e^t\cos\left(\frac{t}{2}\right)dt=4.881$



Question 14 **Answer E**

$f:(a,\infty)\to R,\ f(x)=x^4-4x^2$

$f(x)=x^4-4x^2$

$=x^2\left(x^2-4\right)$

$=x^2(x+2)(x-2)$

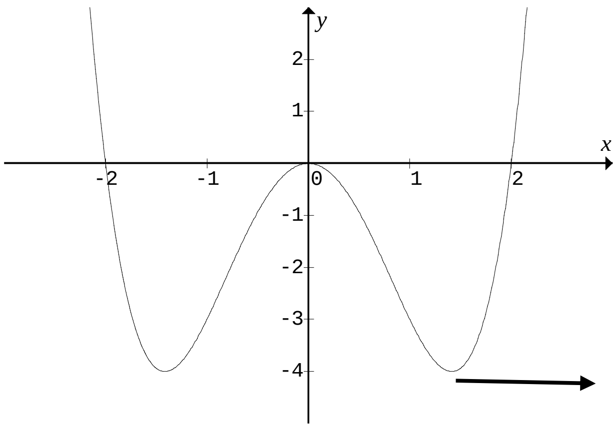
$f'(x)=4x^3-8x$

$=4x\left(x^2-2\right)$

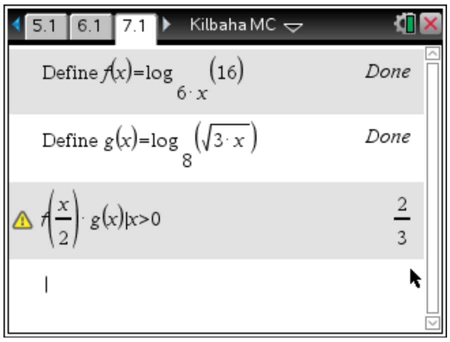
$=4x\left(x+\sqrt{2}\right)\left(x-\sqrt{2}\right)$

Turning points at $x=0$ and $x=\pm\sqrt{2}$

To restrict the domain to make f a one-one function we require $a=\sqrt{2}$



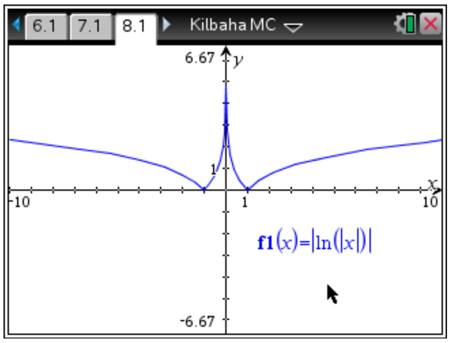
Question 15 **Answer C**



$f(x)=\log_{6x}(16)$, $g(x)=\log_8\left(\sqrt{3x}\right)$, $x>0$

$f\left(\frac{x}{2}\right)g(x)=\log_{3x}(16)\times\log_8\left(\sqrt{3x}\right)=\frac{2}{3}$

Question 16 **Answer E**



$f(x)=\left|\log_e|x|\right|$

$f'(x)>0$ for $x>1$ **A.** is true

$f'(x)<0$ for $x<-1$ **B.** is true

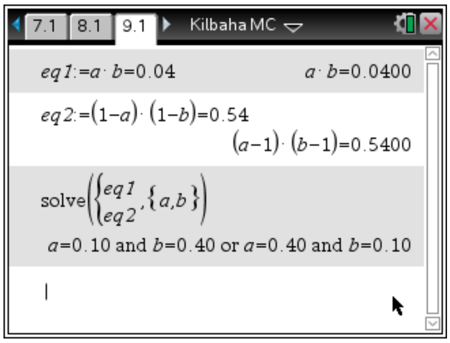
The function is not differentiable

at $x=0$ and at $x=\pm1$. **C.** is true

The range is $[0,\infty)$. **D.** is true

E. is false, the graph is not continuous and is not defined at $x=0$, there is a vertical asymptote at $x=0$.

Question 17 **Answer D**



Let $\Pr(A)=a$ and $\Pr(B)=b$

Since A and B are independent

$\Pr(A\cap B)=\Pr(A).\Pr(B)=ab=0.04$

So A' and B' are also independent

$\Pr(A'\cap B')=\Pr(A').\Pr(B')=(1-a)(1-b)=0.54$

solving $a=0.1$ or $a=0.4$

Question 18 **Answer A**

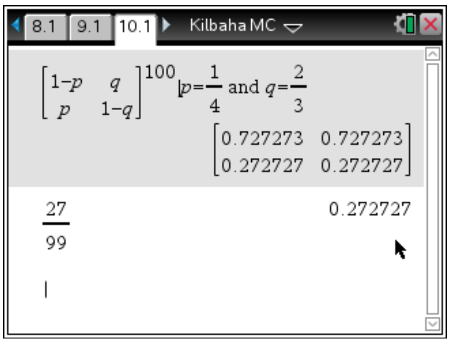
E eggs for breakfast and C cereal for breakfast, long-term cereal is $\frac{27}{99}=0.2727...$

$E\to C=p \quad \Rightarrow \quad E\to E=1-p$

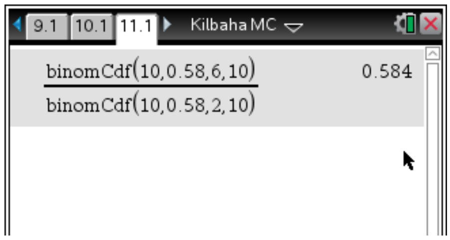
$C\to E=q \quad \Rightarrow \quad C\to C=1-q$

$$\begin{matrix} & E & C \\ E & \multicolumn{2}{c}{} \\ C & & \end{matrix}\begin{bmatrix}1-p & q\\ p & 1-q\end{bmatrix}^{100}\to\begin{bmatrix}\frac{72}{99} & \frac{72}{99}\\ \frac{27}{99} & \frac{27}{99}\end{bmatrix}$$

when $p=\frac{1}{4}$ and $q=\frac{2}{3}$



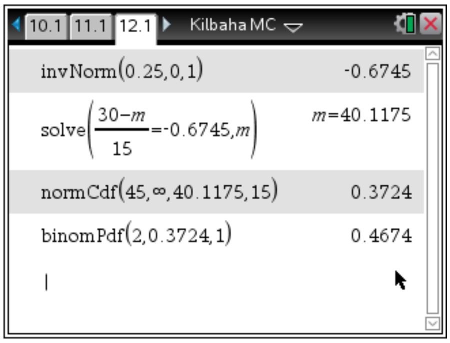
Question 19 **Answer B**



$X \overset{d}{=} Bi\left(n=10,\, p=0.58\right)$

$$\Pr\left(X>5 \mid X\geq 2\right)=\frac{\Pr\left(X\geq 6\right)}{\Pr\left(X\geq 2\right)}=0.584$$

Question 20 **Answer E**



$X \overset{d}{=} N\left(\mu=?,\sigma^2=15^2\right)$

$\Pr\left(X\geq 30\right)=0.75 \quad \Rightarrow \quad \Pr\left(X\leq 30\right)=0.25$

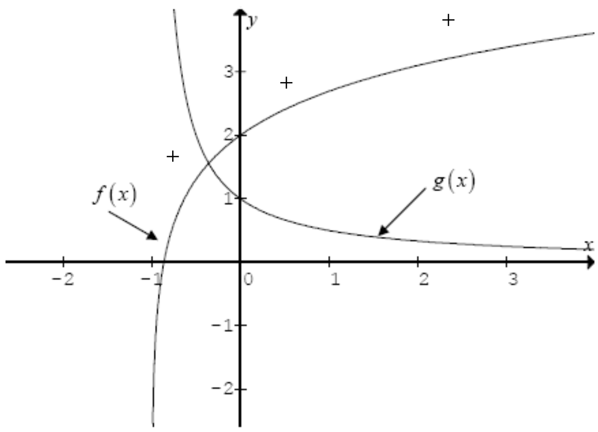
$$-0.675=\frac{30-\mu}{15} \quad \Rightarrow \mu=40.1175$$

$\Pr\left(X\geq 45\right)=0.3724$

$T \overset{d}{=} Bi\left(n=2,\, p=0.3724\right)$

$\Pr\left(T=1\right)=2\times 0.3724\times\left(1-0.3724\right)=0.467$

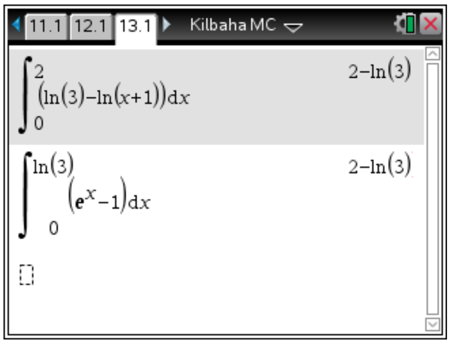
Question 21 **Answer E**



The gradient of the function $f\left(x\right)$ is always positive.
As $x\to -1$ the gradient is large.
At $x=0$ the slope is 45^0, so the gradient is 1.
As $x\to\infty$ the gradient approaches zero,

so $g\left(x\right)=\dfrac{d}{dx}\left[f\left(x\right)\right]$

Question 22 **Answer C**



$y=\log_e\left(x+1\right)$ when $x=2$ $\quad y=\log_e\left(3\right)$

Area with the *x*-axis, $A_x=\int_a^b\left(y_2-y_1\right)dx$

$$A=\int_0^2\left(\log_e\left(3\right)-\log_e\left(x+1\right)\right)dx=2-\log_e\left(3\right)$$

Area with the *y*-axis, $A_y=\int_d^c x\,dy$

$x+1=e^y \quad x=e^y-1$

$$A=\int_0^{\log_e(3)}\left(e^y-1\right)dy=\int_0^{\log_e(3)}\left(e^x-1\right)dx=2-\log_e\left(3\right)$$ by dummy variable property

END OF SECTION 1 SUGGESTED ANSWERS

SECTION 2

Question 1

a. $f(x)=ax^2 \quad f'(x)=2ax$

$f(0)=0$ and $f'(0)=0$ criteria (1) is satisfied

at $A\left(\frac{3}{2},\frac{3}{8}\right)$ $m=\tan\left(45^0\right)=1$

$f\left(\frac{3}{2}\right)=\frac{3}{8} \Rightarrow \frac{9a}{4}=\frac{3}{8} \Rightarrow a=\frac{1}{6}$

$f'\left(\frac{3}{2}\right)=1 \Rightarrow 3a=1 \Rightarrow a=\frac{1}{3}$ M1

contradiction for the value of a, so it is not possible to satisfy criteria (2) A1

Define $f(x)=a\cdot x^2$	*Done*
Define $df(x)=\frac{d}{dx}(f(x))$	*Done*
$f\left(\frac{3}{2}\right)=\frac{3}{8}$	$\frac{9\cdot a}{4}=\frac{3}{8}$
solve$\left(f\left(\frac{3}{2}\right)=\frac{3}{8},a\right)$	$a=\frac{1}{6}$
$df\left(\frac{3}{2}\right)=1$	$3\cdot a=1$
solve$\left(df\left(\frac{3}{2}\right)=1,a\right)$	$a=\frac{1}{3}$

b.i. $g(x)=bx^3+cx^2 \quad g'(x)=3bx^2+2cx$

$g\left(\frac{3}{2}\right)=\frac{3}{8} \Rightarrow \frac{27b}{8}+\frac{9c}{4}=\frac{3}{8}$ A1

$g'\left(\frac{3}{2}\right)=1 \Rightarrow \frac{27b}{4}+3c=1$ A1

solving these simultaneous equations gives

gives $b=\frac{2}{9}$ and $c=-\frac{1}{6}$ A1

Define $g(x)=b\cdot x^3+c\cdot x^2$	*Done*
Define $dg(x)=\frac{d}{dx}(g(x))$	*Done*
$g\left(\frac{3}{2}\right)=\frac{3}{8}$	$\frac{27\cdot b}{8}+\frac{9\cdot c}{4}=\frac{3}{8}$
$dg\left(\frac{3}{2}\right)=1$	$\frac{27\cdot b}{4}+3\cdot c=1$
solve$\left(g\left(\frac{3}{2}\right)=\frac{3}{8}\text{ and }dg\left(\frac{3}{2}\right)=1,\{b,c\}\right)$	$b=\frac{2}{9}$ and $c=\frac{-1}{6}$

ii. $g(x)=\dfrac{2x^3}{9}-\dfrac{x^2}{6}$

$g'(x)=\dfrac{2x^2}{3}-\dfrac{x}{3}=\dfrac{x}{3}(2x-1)$

Now $g'(x)=0 \Rightarrow x=0$ and $x=\dfrac{1}{2}$ A1

so $g(0)=0$ and $g'(0)=0$ criteria (1) is satisfied

however for criteria (3) we require $g'(x)>0$ for $x\in(0,3]$,

there is another turning point at $x=\dfrac{1}{2}$ so criteria (3) is not satisfied. A1

Define $g(x)=b\cdot x^3+c\cdot x^2\mid b=\frac{2}{9}$ and $c=\frac{-1}{6}$	*Done*
Define $dg(x)=\frac{d}{dx}(g(x))$	*Done*
$dg(x)$	$\frac{2\cdot x^2}{3}-\frac{x}{3}$
solve$\left(\frac{2\cdot x^2}{3}-\frac{x}{3}=0,x\right)$	$x=0$ or $x=\frac{1}{2}$

c.i. $h(x)=\dfrac{x^n}{k}$ $h'(x)=\dfrac{nx^{n-1}}{k}$

$h\left(\dfrac{3}{2}\right)=\dfrac{3}{8} \Rightarrow \dfrac{\left(\frac{3}{2}\right)^n}{k}=\dfrac{3}{8}$ $\left(\dfrac{3}{2}\right)^n=\dfrac{3k}{8}$

M1

$h'\left(\dfrac{3}{2}\right)=1 \Rightarrow \dfrac{n\left(\frac{3}{2}\right)^{n-1}}{k}=1$ $\dfrac{2n}{3}\left(\dfrac{3}{2}\right)^n=k$

$\left(\dfrac{3}{2}\right)^n=\dfrac{3k}{8}=\dfrac{3k}{2n} \Rightarrow n=4$

A1

$\dfrac{81}{16}=\dfrac{3k}{8} \Rightarrow k=\dfrac{27}{2}$

ii. $h(x)=\dfrac{2x^4}{27}$

$D=h(3)=\dfrac{2\times 3^4}{27}=6$ metres A1

iii. $h'(x)=\dfrac{8x^3}{27}$

$h'(3)=\dfrac{8\times 3^3}{27}=8=\tan(\theta)$ A1

$\theta=\tan^{-1}(8)$

$\theta=82^0 52'$ A1

iv. $A=\int_0^3 h(x)\,dx=\int_0^3 \dfrac{2x^4}{27}\,dx$ A1

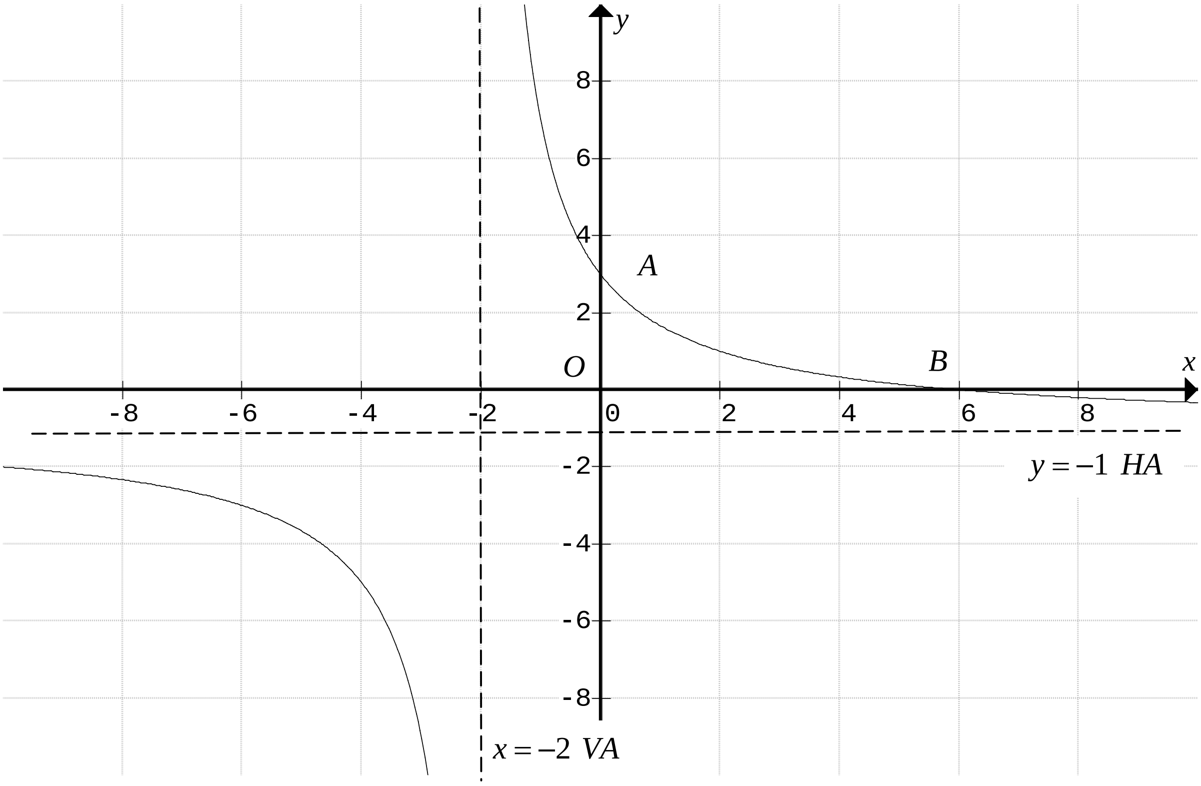
$A=\dfrac{2}{135}\left[x^5\right]_0^3=\dfrac{2}{135}\left[3^5-0\right]=\dfrac{18}{5}$

$A=3\dfrac{3}{5}$ metres^2 A1

Define $h(x)=\dfrac{x^n}{k}$	*Done*
Define $dh(x)=\dfrac{d}{dx}(h(x))$	*Done*
$h\left(\dfrac{3}{2}\right)=\dfrac{3}{8}$	$\dfrac{\left(\frac{3}{2}\right)^n}{k}=\dfrac{3}{8}$
$dh\left(\dfrac{3}{2}\right)=1$	$\dfrac{2\cdot n\cdot\left(\frac{3}{2}\right)^n}{3\cdot k}=1$
solve$\left(h\left(\dfrac{3}{2}\right)=\dfrac{3}{8}\text{ and } dh\left(\dfrac{3}{2}\right)=1,\{n,k\}\right)$	$k=\dfrac{27}{2}$ and $n=4$
Define $h(x)=\dfrac{x^n}{k}\mid k=\dfrac{27}{2}$ and $n=4$	*Done*
$h(3)$	6
$\dfrac{d}{dx}(h(x))\mid x=3$	8
solve$(\tan(\theta)=8,\theta)\mid 0<\theta<90$	$\theta=82.875$
(82.874983651099)▸DMS	82°52'29.9411"
$\int_0^3 h(x)\,dx$	$\dfrac{18}{5}$

Question 2

a. $A(0,3)$, $B(6,0)$, $x=-2$ is a vertical asymptote, $y=-1$ is a horizontal asymptote G2



b.

$$f(x)=\frac{8}{x+2}-1$$

$$f'(x)=\frac{-8}{(x+2)^2} \qquad m(AB)=\frac{0-3}{6-0}=-\frac{1}{2}$$ A1

$$f'(k)=\frac{-8}{(k+2)^2}=-\frac{1}{2}$$ M1

$(k+2)^2=16$ since $k>0$ and $f(k)>0 \Rightarrow 0<k<6$

$k=2$ A1

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Define f(x)=8/(x+2)-1                         Done
Define df(x)=d/dx(f(x))                       Done
solve(df(k)=-1/2,k)|0<k<6                     k=2
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c. $s(x)=\sqrt{x^2+\left[f(x)\right]^2}$

$s(x)=\dfrac{\sqrt{x^4+4x^3+5x^2-12x+36}}{|x+2|}$ M1

solving $\dfrac{ds}{dx}=0$ M1

$\Rightarrow x=k=1.188$ A1

Define $s(x)=\sqrt{x^2+(f(x))^2}$ *Done*

$s(x)$ $\dfrac{\sqrt{x^4+4\cdot x^3+5\cdot x^2-12\cdot x+36}}{|x+2|}$

solve$\left(\dfrac{d}{dx}(s(x))=0,x\right)|0<x<6$ $x=1.1881$

d.i. $P(k,f(k))=P\left(k,\dfrac{8}{k+2}-1\right)$ $m_T=f'(k)=\dfrac{-8}{(k+2)^2}$

tangent at P, is $y-\left(\dfrac{8}{k+2}-1\right)=-\dfrac{8}{(k+2)^2}(x-k)$ A1

crosses y-axis at $x=0\Rightarrow y_C=\dfrac{-\left(k^2-12k-12\right)}{(k+2)^2}$ $C\left(0,\dfrac{-\left(k^2-12k-12\right)}{(k+2)^2}\right)$ A1

crosses x-axis at $y=0\Rightarrow x_D=\dfrac{-k^2}{8}+\dfrac{3k}{2}+\dfrac{3}{2}$ $D\left(\dfrac{-k^2}{8}+\dfrac{3k}{2}+\dfrac{3}{2},0\right)$ A1

$y-f(k)=df(k)\cdot(x-k)$ $y-\dfrac{8}{k+2}+1=\dfrac{-8\cdot(x-k)}{(k+2)^2}$

solve$(y-f(k)=df(k)\cdot(x-k),y)|x=0$ $y=\dfrac{-\left(k^2-12\cdot k-12\right)}{(k+2)^2}$

solve$(y-f(k)=df(k)\cdot(x-k),x)|y=0$ $x=\dfrac{-k^2}{8}+\dfrac{3\cdot k}{2}+\dfrac{3}{2}$

ii. $A(k)=\dfrac{1}{2}\left(\dfrac{-\left(k^2-12k-12\right)}{(k+2)^2}\right)\left(\dfrac{-k^2}{8}+\dfrac{3k}{2}+\dfrac{3}{2}\right)$ A1

iii. solving $\frac{dA}{dk}=0$ M1

$\Rightarrow\ k=2$ since $0<k<6$ A1

iv. $m_T = f'(k)=\frac{-8}{(k+2)^2} \Rightarrow m_N=\frac{(k+2)^2}{8}$

normal at P, is $y-\left(\frac{8}{k+2}-1\right)=\frac{(k+2)^2}{8}(x-k)$ M1

this must pass through the origin $(0,0)$ $x=0$ and $y=0$

solving $1-\frac{8}{k+2}=\frac{-k(k+2)^2}{8}$

$k=1.188$ A1

Define $a(k)=\frac{1}{2}\cdot\left(\frac{-k^2}{8}+\frac{3\cdot k}{2}+\frac{3}{2}\right)\cdot\frac{-(k^2-12\cdot k-12)}{(k+2)^2}$ *Done*

solve$\left(\frac{d}{dk}(a(k))=0,k\right)|0<k<6$ $k=2$

solve$\left(y-f(k)=\frac{-1}{df(k)}\cdot(x-k),k\right)|x=0$ and $y=0$ and $0<k<6$ $k=1.1881$

Question 3

a.i. $\frac{1}{50}\int_0^{50} J(t)\,dt$ represents the average amount of orange juice in the cup over the time interval $t\in[0,50]$ A1

ii. $\frac{1}{50}\left[10\times(70+110+140+155+165)\right]$ M1

$=128$ mls A1

iii. $J'(25)=\frac{140-110}{30-20}$ M1

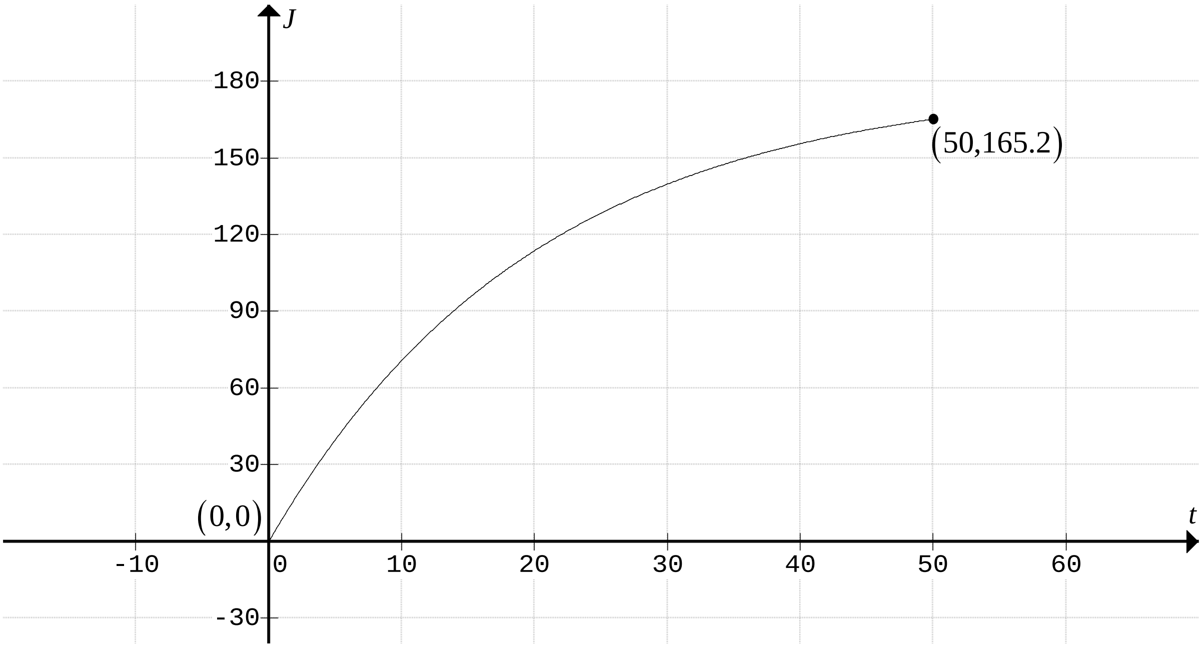
$J'(25)=3$ mls/sec A1

b.i.

t seconds	0	10	20	30	40	50
$J(t)$ mls	0	70.8	113.8	139.8	155.6	165.2

A1

ii. correct graph, scale, for $t \in [0,50]$, end-points $(0,0)$ and $(50,165.2)$
asymptote towards $J = 180$ G1



iii. $J(t) = 180\left(1 - e^{-\frac{t}{20}}\right)$

$J'(t) = 9e^{-\frac{t}{20}}$ A1

$J'(25) = 9e^{-\frac{5}{4}} = 2.58$ mls/sec A1

iv. $\frac{1}{50}\int_0^{50} 180\left(1 - e^{-\frac{t}{20}}\right)dt$

$= \frac{18}{5}\left[t + 20e^{-\frac{t}{20}}\right]_0^{50}$ M1

$= \frac{18}{5}\left[\left(50 + 20e^{-\frac{5}{2}}\right) - (20)\right]$

$= 113.91$ mls A1

Question 4

a.i. Let O be she has freshly squeezed orange juice, and P be she has pineapple juice

$O \to P = 0.25 \quad \Rightarrow O \to O = 0.75$ and $P \to O = 0.7 \quad \Rightarrow P \to P = 0.3$

$$\begin{array}{c} \\ O \\ P \end{array} \begin{array}{c} \begin{array}{cc} O & P \end{array} \\ \begin{bmatrix} 0.75 & 0.7 \\ 0.25 & 0.3 \end{bmatrix}^3 \end{array} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.737 \\ 0.263 \end{bmatrix}$$ M1

orange juice on Thursday 0.737 A1

alternatively

$\Pr(\text{orange juice on Monday and Thursday})$

$= \Pr(OOOO) + \Pr(OOPO) + \Pr(OPOO) + \Pr(OPPO)$

$= 0.75^3 + 0.75 \times 0.25 \times 0.7 + 0.25 \times 0.7 \times 0.75 + 0.25 \times 0.3 \times 0.7$

$= 0.737$

$\begin{bmatrix} 0.75 & 0.7 \\ 0.25 & 0.3 \end{bmatrix}^3 \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\begin{bmatrix} 0.737 \\ 0.263 \end{bmatrix}$

ii. $\Pr(\text{orange juice at least three times})$

$= \Pr(OOOP) + \Pr(OOPO) + \Pr(OPOO) + \Pr(OOOO)$ M1

$= 0.75^2 \times 0.25 + 0.75 \times 0.25 \times 0.7 + 0.25 \times 0.7 \times 0.75 + 0.75^3$

$= 0.825$ A1

b.i. Since the function is continuous at $t = 5$ $a \sin\left(\frac{5\pi}{10}\right) = 10b \quad \Rightarrow a = 10b$

Since the total area under the curve is equal to one.

$\int_0^5 a \sin\left(\frac{\pi t}{10}\right) dt + \int_5^{15} b(15 - t)\, dt = 1$ substituting $a = 10b$ M1

$$b\left[10 \int_0^5 \sin\left(\frac{\pi t}{10}\right) dt + \int_5^{15} (15 - t)\, dt\right] = 1$$

$$b\left(-\left[\frac{100}{\pi} \cos\left(\frac{\pi t}{10}\right)\right]_0^5 + \left[15t - \frac{t^2}{2}\right]_5^{15}\right) = 1$$ M1

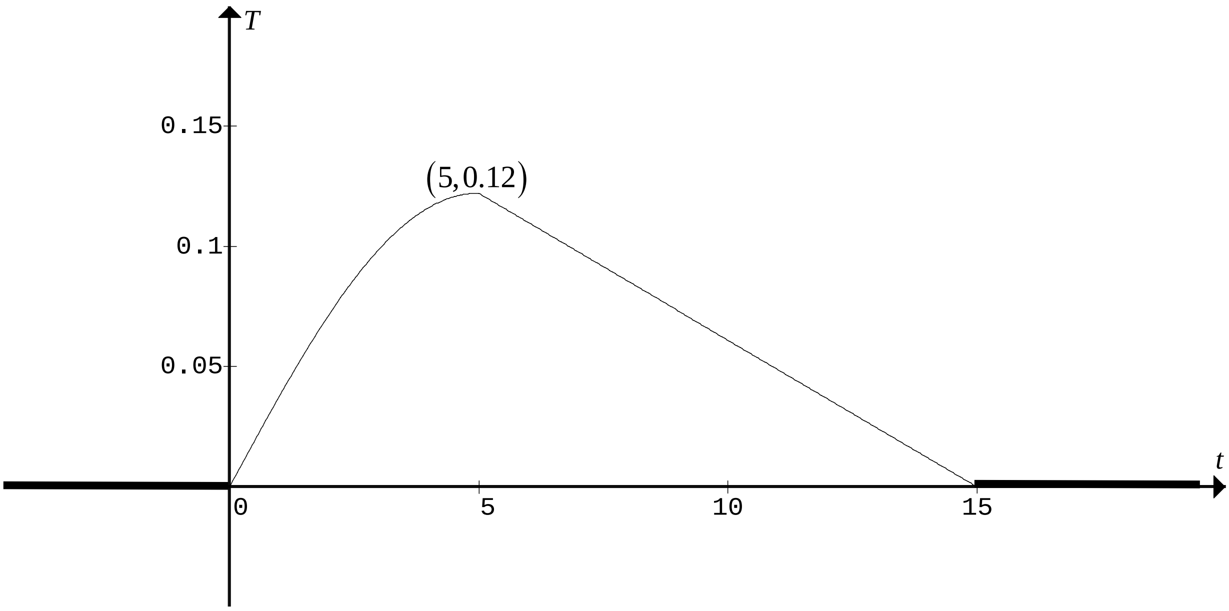
$$b\left(\left[-\frac{100}{\pi} \cos\left(\frac{\pi}{2}\right) + \frac{100}{\pi} \cos(0)\right] + \left[\left(15^2 - \frac{15^2}{2}\right) - \left(75 - \frac{25}{2}\right)\right]\right) = 1$$

$$b\left(\frac{100}{\pi} + 50\right) = b\left(\frac{100 + 50\pi}{\pi}\right) = 1$$

$$b = \frac{\pi}{50(\pi + 2)} \quad \text{and} \quad a = \frac{\pi}{5(\pi + 2)}$$

Define $f1(x)=a\cdot\sin\left(\frac{\pi\cdot x}{10}\right)\mid 0\le x\le 5$	*Done*
Define $f2(x)=b\cdot(15-x)\mid 5\le x\le 15$	*Done*
$f1(5)=f2(5)$	$a=10\cdot b$
$\int_0^5 f1(x)\,dx+\int_5^{15} f2(x)\,dx$	$\frac{10\cdot a}{\pi}+50\cdot b$
$\text{solve}\left(\frac{10\cdot a}{\pi}+50\cdot b=1 \text{ and } a=10\cdot b,\{a,b\}\right)$	$a=\frac{\pi}{5\cdot(\pi+2)}$ and $b=\frac{\pi}{50\cdot(\pi+2)}$

ii. Graph passes through $(0,0)$ $(5,a)$ where $a=\frac{\pi}{5(\pi+2)}\approx 0.12$ and is continuous and zero elsewhere. G1



iii. $$E(T)=\frac{\pi}{5(\pi+2)}\left[\int_0^5 t\sin\left(\frac{\pi t}{10}\right)dt+\frac{1}{10}\int_5^{10} t(15-t)\,dt\right]=6.32997$$

$$E\left(T^2\right)=\frac{\pi}{5(\pi+2)}\left[\int_0^5 t^2\sin\left(\frac{\pi t}{10}\right)dt+\frac{1}{10}\int_5^{10} t^2(15-t)\,dt\right]=50.32545$$ M1

$$\text{Var}(T)=E\left(T^2\right)-\left(E(T)\right)^2=50.32545-6.32997^2=10.257$$ A1

Define $f3(x)=\begin{cases} f1(x), 0\le x\le 5 \\ f2(x), 5\le x\le 15 \end{cases} \mid a=\frac{\pi}{5\cdot(\pi+2)}$ and $b=\frac{\pi}{50\cdot(\pi+2)}$	*Done*
$e(x)=\int_0^{15} (x\cdot f3(x))\,dx$	$e(x)=6.330$
$e2(x)=\int_0^{15} (x^2\cdot f3(x))\,dx$	$e2(x)=50.325$
$50.32544781983-(6.32997179986)^2$	10.257

iv. since $\frac{\pi}{5(\pi+2)}\int_0^5 \sin\left(\frac{\pi t}{10}\right)dt=0.389$, the median m, satisfies M1

$\frac{\pi}{50(\pi+2)}\int_5^m (15-t)\,dt=0.5-0.389=0.111$, solving gives

$m=5.954$ minutes A1

$\int_0^5 f1(x)\,dx \mid a=\frac{\pi}{5\cdot(\pi+2)}$	0.389
$0.5-0.38898452964834$	0.111
Define $f2(x)=b\cdot(15-x) \mid b=\frac{\pi}{50\cdot(\pi+2)}$	*Done*
$\text{solve}\left(\int_5^m f2(x)\,dx=0.111,m\right) \mid 5<m<15$	$m=5.954$

v. $\Pr(T\le 3)=\frac{\pi}{5(\pi+2)}\int_0^3 \sin\left(\frac{\pi t}{10}\right)dt=0.160345$ A1

$Y\overset{d}{=}Bi(n=5,p=0.160345)$

$\Pr(Y\ge 2)=0.1841$ A1

Define $f1(x)=a\cdot\sin\left(\frac{\pi\cdot x}{10}\right) \mid a=\frac{\pi}{5\cdot(\pi+2)}$	*Done*
$\int_0^3 f1(x)\,dx$	0.16034516
binomCdf(5,0.16034516,2,5)	0.1841

vi. $\Pr(T \le 10 \mid T \ge 7) = \dfrac{\Pr(7 \le T \le 10)}{\Pr(T \ge 7)}$ M1

$$= \frac{b\int_7^{10}(15-t)\,dt}{b\int_7^{15}(15-t)\,dt}$$

$$= \frac{39}{64}$$ A1

$$\frac{\int_7^{10} f3(x)\,dx}{\int_7^{15} f3(x)\,dx} \qquad \frac{39}{64}$$

c. T is the time in seconds for the toast to be ready, $T \overset{d}{=} N(\mu = 60, \sigma^2 = ?)$

$\Pr(T > 50) = 0.7 \;\Rightarrow \Pr(T < 50) = 0.3$

$\Rightarrow \quad \dfrac{50-60}{\sigma} = -0.5244$ M1

$\sigma = 19.1$ sec A1

invNorm(0.3,0,1)	-0.5244
solve$\left(\frac{-10}{s}=-0.52440051009939, s\right)$	$s=19.0694$

END OF SECTION 2 SUGGESTED ANSWERS