

**Year 2014**

**VCE**

**Mathematical Methods**

**Trial Examination 1**



**KILBAHA MULTIMEDIA PUBLISHING**  
**PO BOX 2227**  
**KEW VIC 3101**  
**AUSTRALIA**

**TEL: (03) 9018 5376**  
**FAX: (03) 9817 4334**  
**kilbaha@gmail.com**  
**<http://kilbaha.com.au>**

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**Victorian Certificate of Education  
2014**

**STUDENT NUMBER**

Figures  
Words


Letter

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**MATHEMATICAL METHOD CAS**

**Trial Written Examination 1**

Reading time: 15 minutes

Total writing time: 1 hour

**QUESTION AND ANSWER BOOK**

**Structure of book**

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

**Materials supplied**

- Question and answer book of 14 pages with a detachable sheet of miscellaneous formulas at the end of this booklet.
- Working space is provided throughout the booklet.

**Instructions**

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.**

**Instructions**

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown. Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

**Question 1** (4 marks)

a. If  $y = \frac{\cos(2x)}{3x}$ , find  $\frac{dy}{dx}$ . 2 marks

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b. Let  $f(x) = \tan(\sqrt{x})$ . Find  $f'\left(\frac{\pi^2}{16}\right)$ . 2 marks

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**Question 2** (4 marks)

- a. Describe in words, giving scale factors, the transformations in a suitable order, required to sketch the graph of  $y = 3 - \sqrt{2 - x}$  from the graph of  $y = \sqrt{x}$ . 2 marks

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- b. The function  $f(x) = 3 - \sqrt{2 - x}$  is defined on its maximal domain. Find the inverse function  $f^{-1}$ . 2 marks

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**Question 3** (3 marks)

Consider the linear simultaneous equations

$$3x - (k + 2)y = k + 1$$
$$kx - 5y = 4$$

where  $k$  is a constant.

- i. Find the value(s) of  $k$ , for which there is a unique solution.
- ii. Find the value(s) of  $k$ , for which there is an infinite number of solutions.

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**Question 4** (2 marks)

Find the general solution to the equation  $\sqrt{3} \cos(2x) + \sin(2x) = 0$ .

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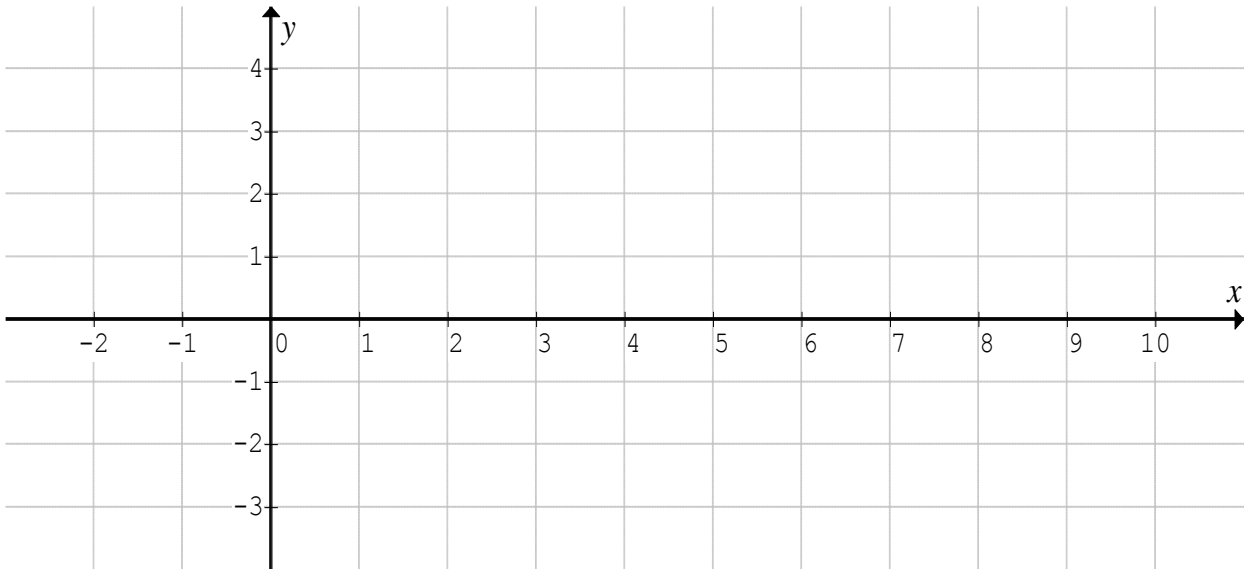
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**Question 6** (6 marks)

Consider the function  $f : [0, 8] \rightarrow \mathbb{R}$ ,  $f(x) = \left| 2 \sin\left(\frac{\pi x}{4}\right) \right| + 1$

- i. Sketch the graph of the function on the axes below, stating the coordinates of any maximum and minimum points and the endpoints. 2 marks



- ii. On the axes above, sketch the graph of  $y = f'(x)$ . 2 marks

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- iii. Determine the average value of the function. 2 marks

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**Question 7** (4 marks)

- a. The probability distribution function for the continuous random variable  $X$  is given by

$$f(x) = \begin{cases} \frac{k}{\sqrt{9-2x}} & \text{for } 0 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

Find the value of  $k$ .

2 marks

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- b. If  $X$  is a normally distributed random variable with mean 20 and variance 16 and  $Z$  is the random variable with the standard normal distribution. If  $\Pr(1 < Z < 2) = p$  and  $\Pr(Z > 2) = q$ , then express  $\Pr(X < 24 | X > 12)$  in terms of  $p$  and  $q$ .

2 marks

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**Question 8** (4 marks)

- a. A binomial distribution of the random variable  $X$ , with three independent trials, is such that  $\Pr(X = 1) = \frac{p}{3}$ , where  $p$  is the probability of a success on any trial and  $0 < p < 1$ . Determine the value of  $p$ .

2 marks

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- b. A discrete random variable  $X$  has a probability distribution given by

$X$	0	1	2
$\Pr(X = x)$	$k^2$	$\frac{3k}{2}$	$\frac{k}{2}$

Find  $E(X)$ .

2 marks

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**Question 9** (4 marks)

**a.** Differentiate  $e^{-x}(2\cos(2x) + \sin(2x))$  with respect to  $x$ . 1 mark

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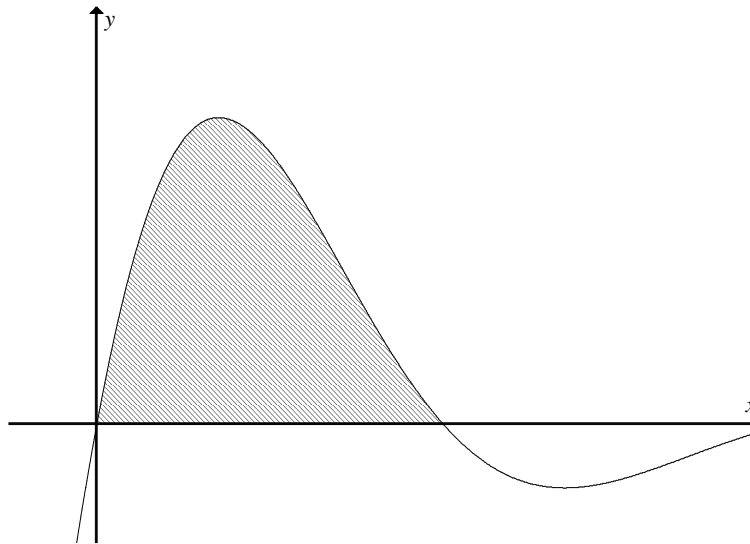


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**b.** The diagram shows part of the graph of  $f : R \rightarrow R$ ,  $f(x) = e^{-x} \sin(2x)$ .  
 Use your answer to **a.** to determine the area of the shaded region. 3 marks




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# MATHEMATICAL METHODS CAS

## Written examination 1

### FORMULA SHEET

#### Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

## Mathematical Methods CAS Formulas

### Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$	volume of a pyramid:	$\frac{1}{3}Ah$
curved surface area of a cylinder:	$2\pi rh$	volume of a sphere:	$\frac{4}{3}\pi r^3$
volume of a cylinder:	$\pi r^2 h$	area of triangle:	$\frac{1}{2}bc \sin(A)$
volume of a cone:	$\frac{1}{3}\pi r^2 h$		

### Calculus

$\frac{d}{dx}(x^n) = nx^{n-1}$ $\frac{d}{dx}(e^{ax}) = ae^{ax}$ $\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$ $\frac{d}{dx}(\sin(ax)) = a \cos(ax)$ $\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$ $\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = a \sec^2(ax)$	$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$ $\int e^{ax} dx = \frac{1}{a} e^{ax} + c$ $\int \frac{1}{x} dx = \log_e x  + c$ $\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$ $\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$
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product rule: 
$$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$$

quotient rule: 
$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

Chain rule: 
$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$$

approximation: 
$$f(x+h) \approx f(x) + h f'(x)$$

### Probability

$$\Pr(A) = 1 - \Pr(A')$$

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$

$$\Pr(A/B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

**Transition Matrices** 
$$S_n = T^n \times S_0$$

mean: 
$$\mu = E(X)$$

variance: 
$$\text{var}(X) = \sigma^2 = E((X - \mu)^2) = E(X^2) - \mu^2$$

probability distribution		mean	variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x p(x)$	$\sigma^2 = \sum (x - \mu)^2 p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x) dx$	$\mu = \int_{-\infty}^{\infty} x f(x) dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx$