# Year 2014
# VCE
# Mathematical Methods
# Trial Examination 1



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**Victorian Certificate of Education**
**2014**

**STUDENT NUMBER**

| | | | | | | | | | Letter |
|---|---|---|---|---|---|---|---|---|---|
| Figures | | | | | | | | | |
| Words | | | | | | | | | |

# MATHEMATICAL METHOD CAS

# Trial Written Examination 1

Reading time: 15 minutes
Total writing time: 1 hour

## QUESTION AND ANSWER BOOK

**Structure of book**

| *Number of questions* | *Number of questions to be answered* | *Number of marks* |
|---|---|---|
| 10 | 10 | 40 |

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

**Materials supplied**

- Question and answer book of 14 pages with a detachable sheet of miscellaneous formulas at the end of this booklet.
- Working space is provided throughout the booklet.

**Instructions**

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.**

## Instructions

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

**Question 1** (4 marks)

**a.** If $y=\dfrac{\cos(2x)}{3x}$, find $\dfrac{dy}{dx}$. 2 marks

**b.** Let $f(x)=\tan\left(\sqrt{x}\right)$. Find $f'\left(\dfrac{\pi^2}{16}\right)$. 2 marks

**Question 2** (4 marks)

**a.** Describe in words, giving scale factors, the transformations in a suitable order, required to sketch the graph of $y = 3 - \sqrt{2 - x}$ from the graph of $y = \sqrt{x}$.

2 marks

**b.** The function $f(x) = 3 - \sqrt{2 - x}$ is defined on its maximal domain. Find the inverse function $f^{-1}$.

2 marks

**Question 3** (3 marks)

Consider the linear simultaneous equations

$$3x-(k+2)y=k+1$$
$$kx-5y=4$$

where $k$ is a constant.

**i.** Find the value(s) of $k$, for which there is a unique solution.

**ii.** Find the value(s) of $k$, for which there is an infinite number of solutions.

**Question 4** (2 marks)

Find the general solution to the equation $\sqrt{3}\cos(2x)+\sin(2x)=0$.

**Question 5** (5 marks)

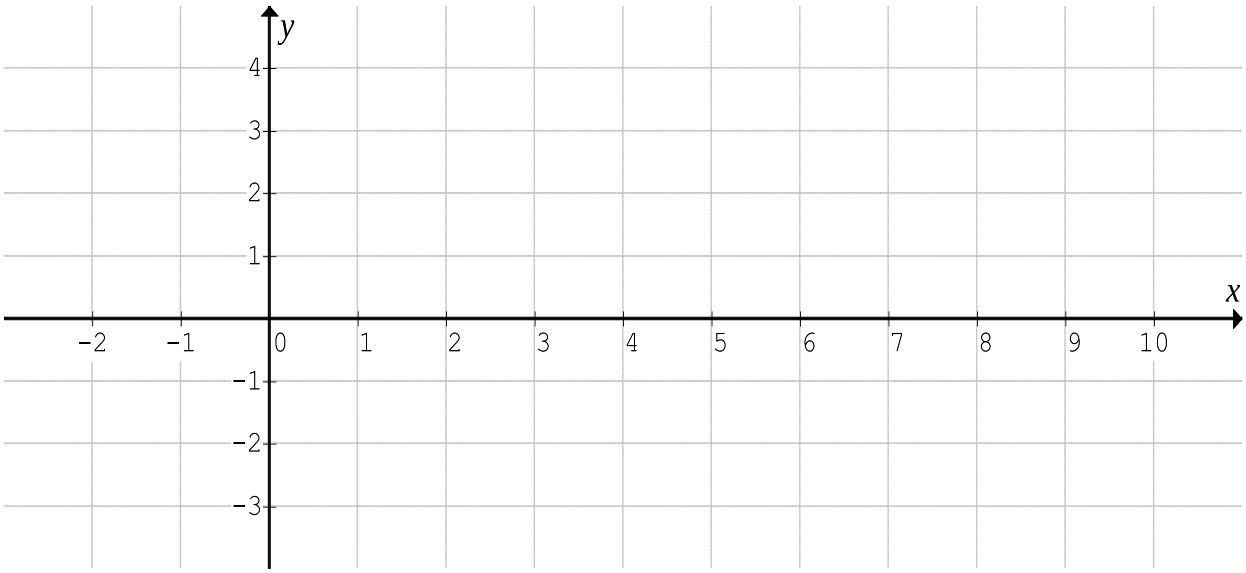
**a.** Solve the equation $2\log_4(x-1)+\log_4(2)-\log_4(x)=\frac{3}{2}$ for *x*. 3 marks

**b.** Solve the equation $8\times 4^{x}+16\times 4^{-x}=129$ for *x*. 2 marks

**Question 6** (6 marks)

Consider the function $f:[0,8]\rightarrow R$, $f(x)=\left|2\sin\left(\frac{\pi x}{4}\right)\right|+1$

**i.** Sketch the graph of the function on the axes below, stating the coordinates of any maximum and minimum points and the endpoints. 2 marks




**ii.** On the axes above, sketch the graph of $y=f'(x)$. 2 marks

**iii.** Determine the average value of the function.

2 marks

**Question 7** (4 marks)

**a.** The probability distribution function for the continuous random variable *X* is given by

$$f(x)=\begin{cases}\dfrac{k}{\sqrt{9-2x}} & \text{for } 0\le x\le 4\\ 0 & \text{otherwise}\end{cases}$$

Find the value of $k$. 2 marks

**b.** If $X$ is a normally distributed random variable with mean 20 and variance 16 and $Z$ is the random variable with the standard normal distribution. If $\Pr(1<Z<2)=p$ and $\Pr(Z>2)=q$, then express $\Pr(X<24 \mid X>12)$ in terms of $p$ and $q$.

2 marks

**Question 8** (4 marks)

**a.** A binomial distribution of the random variable *X*, with three independent trials, is such that $\Pr(X = 1) = \frac{p}{3}$, where *p* is the probability of a success on any trial and $0 < p < 1$. Determine the value of *p*.

2 marks

**b.** A discrete random variable $X$ has a probability distribution given by

| $X$ | 0 | 1 | 2 |
|---|---|---|---|
| $\Pr(X = x)$ | $k^2$ | $\frac{3k}{2}$ | $\frac{k}{2}$ |

Find $E(X)$.

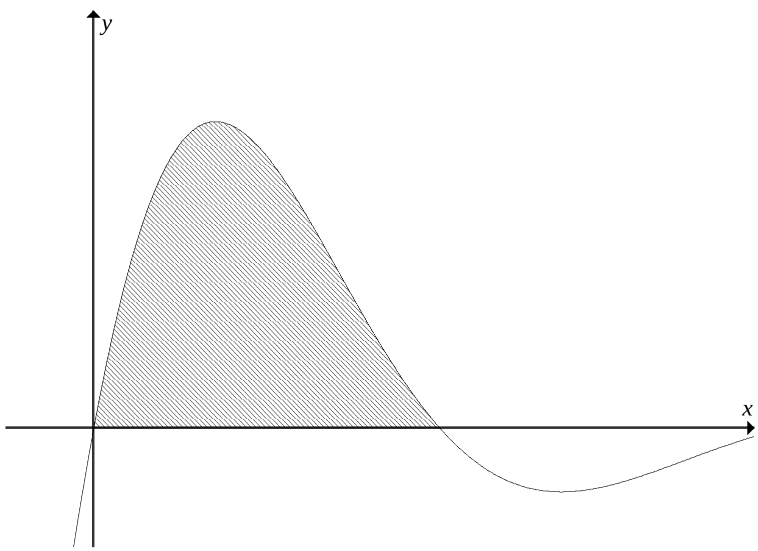
2 marks

**Question 9** (4 marks)

**a.** Differentiate $e^{-x}\left(2\cos(2x)+\sin(2x)\right)$ with respect to *x*. 1 mark

**b.** The diagram shows part of the graph of $f: R \rightarrow R$, $f(x)=e^{-x}\sin(2x)$.
Use your answer to **a.** to determine the area of the shaded region.

3 marks



**Question 10** (4 marks)

Given the function $f:[0,9]\to R$, $f(x)=\pi x+12\cos\left(\frac{\pi x}{6}\right)$, find the maximum and minimum values of the function and the values of *x* for which these occur.

**END OF QUESTION AND ANSWER BOOKLET**

**END OF EXAMINATION**

# MATHEMATICAL METHODS CAS

# Written examination 1

## FORMULA SHEET

### Directions to students

| |
|---|
| Detach this formula sheet during reading time. |
| This formula sheet is provided for your reference. |

# Mathematical Methods CAS Formulas

## Mensuration

| | | | |
|---|---|---|---|
| area of a trapezium: | $\frac{1}{2}(a+b)h$ | volume of a pyramid: | $\frac{1}{3}Ah$ |
| curved surface area of a cylinder: | $2\pi rh$ | volume of a sphere: | $\frac{4}{3}\pi r^3$ |
| volume of a cylinder: | $\pi r^2 h$ | area of triangle: | $\frac{1}{2}bc\sin(A)$ |
| volume of a cone: | $\frac{1}{3}\pi r^2 h$ | | |

## Calculus

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + c\ ,\ n \neq -1$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$$

$$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$$

$$\int \frac{1}{x} dx = \log_e|x| + c$$

$$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$$

$$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + c$$

$$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$$

$$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + c$$

$$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\sec^2(ax)$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

approximation: $f(x+h) \approx f(x) + h\,f'(x)$

## Probability

$\Pr(A) = 1 - \Pr(A')$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A/B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

**Transition Matrices** $S_n = T^n \times S_0$

mean: $\mu = E(X)$

variance: $\operatorname{var}(X) = \sigma^2 = E\left((X-\mu)^2\right) = E\left(X^2\right) - \mu^2$

| probability distribution | | mean | variance |
|---|---|---|---|
| discrete | $\Pr(X = x) = p(x)$ | $\mu = \sum x\,p(x)$ | $\sigma^2 = \sum (x-\mu)^2\, p(x)$ |
| continuous | $\Pr(a < X < b) = \int_a^b f(x)\,dx$ | $\mu = \int_{-\infty}^{\infty} x\,f(x)\,dx$ | $\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx$ |