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Student Name……………………………………

MATHEMATICAL METHODS (CAS) UNITS 3 & 4

TRIAL EXAMINATION 1

2014

Reading Time: 15 minutes
Writing time: 1 hour

Instructions to students

This exam consists of 10 questions.
All questions should be answered in the spaces provided.
There is a total of 40 marks available.
The marks allocated to each of the questions are indicated throughout.
Students may **not** bring any calculators or notes into the exam.
Where an exact answer is required a decimal approximation will not be accepted.
Where more than one mark is allocated to a question, appropriate working must be shown.
Diagrams in this trial exam are not drawn to scale.
A formula sheet can be found on page 12 of this exam.

Question 1 (4 marks)

a. If $y = 3x^2e^{4x}$, find $\frac{dy}{dx}$. 2 marks

b. For $g(x) = \frac{\sin(x)}{2x}$, find $g'\left(\frac{\pi}{2}\right)$. 2 marks

Question 2 (2 marks)

Find an antiderivative of $\frac{1}{2x-3}$ with respect to *x*.

Question 3 (2 marks)

Let $f: R \setminus \{0\} \to R,\ f(x) = \dfrac{5}{x}$.

Show that $f(f(x)) = f(f^{-1}(x))$ where f^{-1} is the inverse of *f*.

Question 4 (5 marks)

a. Solve $\log_6(3) - 2\log_6(x) + \log_6(2) = 1$ for *x*. 3 marks

b. Solve $8^{1-2x} = 2^{4+x}$ for *x*. 2 marks

Question 5 (2 marks)

Solve the equation $\sin\left(\frac{x}{3}\right)=\frac{1}{\sqrt{2}}$ for $x \in [0, 6\pi]$.

Question 6 (5 marks)

For the discrete random variable *X*, with probability distribution shown below, $E(X) = 2$.

x	0	1	2	3	4
$\Pr(X = x)$	0.2	0.2	0.1	0.4	0.1

a. Find the median of *X*. 1 mark

b. Find $\Pr(X \le 2 \mid X > 0)$. 2 marks

c. Find the variance of *X*, $\text{Var}(X)$. 2 marks

Question 7 (4 marks)

The continuous random variable *X*, has a probability density function given by

$$f(x)=\begin{cases} a\cos\left(\frac{\pi x}{2}\right) & \text{for } x\in[0,1] \\ 0 & \text{elsewhere} \end{cases}$$

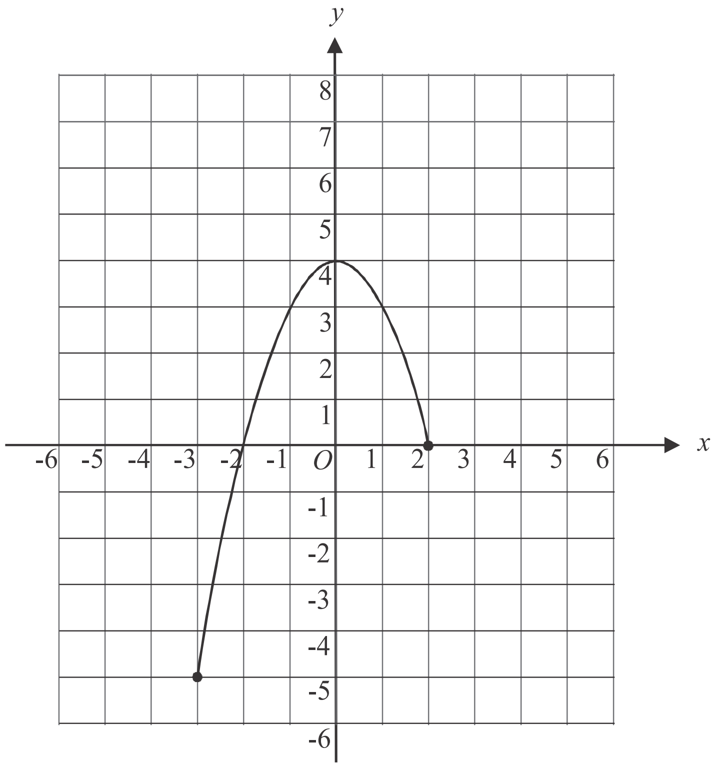
where *a* is a positive constant.

a. Find the value of *a*. 3 marks

b. Find the mode of *X*. 1 mark

Question 8 (6 marks)

The graph of $y = 4 - x^2$, where $x \in [-3,2]$ is shown below.



Let $f : [-3,2] \rightarrow R, f(x) = \left|4 - x^2\right| + 2$.

a. Sketch the graph of $y = f(x)$ on the set of axes above. Indicate clearly the coordinates of the endpoints. 2 marks

b. The graph of f is

- translated 2 units in the negative direction of the y-axis
- translated 2 units in the positive direction of the x-axis
- dilated from the y-axis by a factor of $\frac{1}{2}$

to become the graph of the function h.

For this function h, find

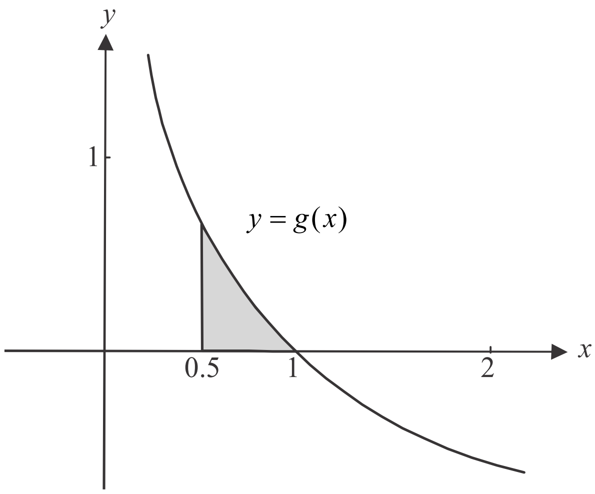
i. the rule. 2 marks

ii. the domain. 1 mark

iii. the range. 1 mark

Question 9 (4 marks)

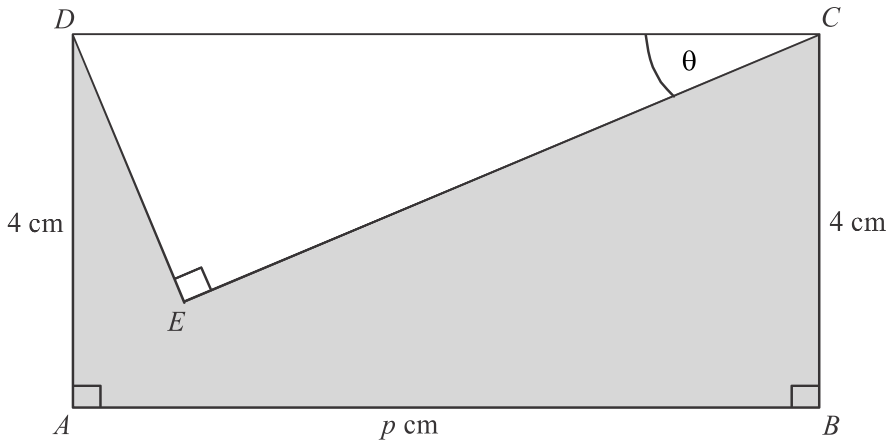
Let $g:(0,\infty)\to R, g(x)=-\log_e(x)$.
The graph of g is shown below.



a. Differentiate $(1-x)\log_e(x)$ with respect to x. 1 mark

b. Hence find the area of the shaded region shown above.
Express your answer in the form $a(\log_e(a)+1)$ where a is a positive, real constant. 3 marks

Question 10 (6 marks)



In the diagram above, *ABCD* is a rectangle and *CDE* is a right-angled triangle where $\angle DCE = \theta$ and $0 < \theta < \frac{\pi}{2}$.
Also, $AD = BC = 4$ cm and $AB = p$ cm where p is a positive constant.

a. Find an expression in terms of p and θ for

i. *DE* 1 mark

ii. *CE* 1 mark

The area of the shaded region in the diagram is $A\text{cm}^2$.

b. Find an expression for A in terms of p and θ. 1 mark

c. Use the derivative $\frac{dA}{d\theta}$ to find the value of θ when A is a minimum. 2 marks

d. If the minimum area of the shaded region is 16cm^2, find the value of p. 1 mark

Mathematical Methods (CAS) Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$	volume of a pyramid:	$\frac{1}{3}Ah$
curved surface area of a cylinder:	$2\pi rh$	volume of a sphere:	$\frac{4}{3}\pi r^3$
volume of a cylinder:	$\pi r^2 h$	area of a triangle:	$\frac{1}{2}bc\sin A$
volume of a cone:	$\frac{1}{3}\pi r^2 h$		

Calculus

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax} + c$$

$$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$$

$$\int \frac{1}{x}\,dx = \log_e|x| + c$$

$$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$$

$$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + c$$

$$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$$

$$\int \cos\,(ax)\,dx = \frac{1}{a}\sin(ax) + c$$

$$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\sec^2(ax)$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

approximation: $f(x+h) \approx f(x) + hf'(x)$

Probability

$\Pr(A) = 1 - \Pr(A')$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

transition matrices: $S_n = T^n \times S_0$

mean: $\mu = \mathrm{E}(X)$

variance: $\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X-\mu)^2) = \mathrm{E}(X^2) - \mu^2$

probability distribution		mean	variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \Sigma\, x\, p(x)$	$\sigma^2 = \Sigma\,(x-\mu)^2\, p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)dx$	$\mu = \int_{-\infty}^{\infty} x\, f(x)dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx$