Engage
Education
Foundation

Units 3 and 4 Maths Methods (CAS): Exam 1

Technology-free Practice Exam Solutions

Stop!

Don’t look at these solutions until you have attempted the exam.

Any questions?

Check that you’re using the most recent version of these solutions, then email practiceexams@ee.org.au.

Marks allocated are indicated by a number in square brackets, for example, [1] indicates that the line is worth one mark.

Question 1a

$\frac{dy}{dx} = \ln(x) + 1$ [1]

Question 1b

$\frac{dy}{dx} = 2(3x^2 - 2)(x^3 - 2x)$ [1]

At $x = 2$, $\frac{dy}{dx} = 80$ [1]

Question 2

$\int \cos(-2x + 3)dx = -\frac{1}{2}\sin(-2x + 3) + c$ [1]

One such antiderivative is $-\frac{1}{2}\sin(-2x + 3) + 1$ [1]

Question 3a

The inverse of $g(x)$ can be found by swapping x and y:

$x = 2e^{3y} + 1$ [1]

$e^{3y} = \frac{x - 1}{2}$

$y = \frac{1}{3}\ln\frac{x-1}{2}$ [1]

Question 3b

$g\left(g^{-1}(x)\right) = 2e^{3\left(\frac{1}{3}\ln\frac{x-1}{2}\right)} + 1$ [1]

$= 2e^{\ln\frac{x-1}{2}} + 1$

$= x$ [1]

Question 4a

$E(X) = 0.1 + 2(0.25) + 3(0.4) = 1.8$ [1]

Question 4b

$\Pr = 0.25 \times 0.25 \times 0.25 = \frac{1}{64}$ [1]

Question 4c

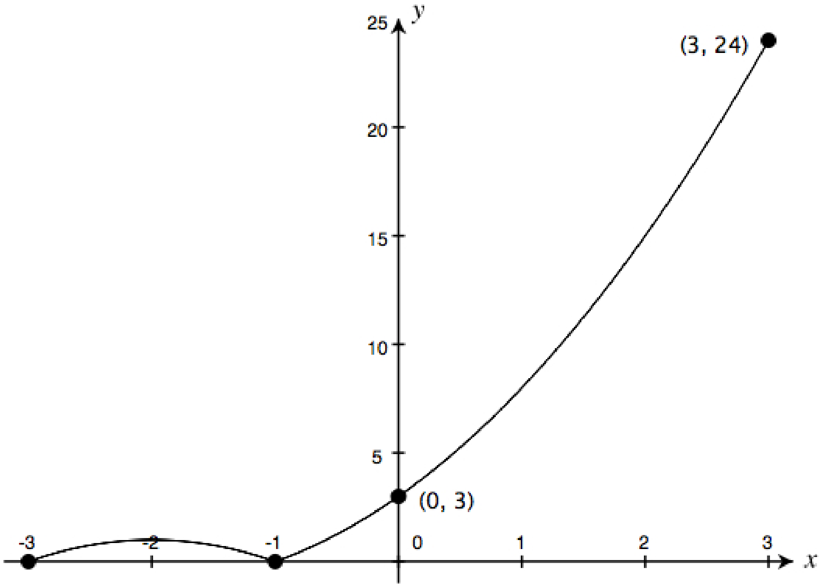
$\Pr(\text{3 calls over 2 day span})$

$= \Pr(\text{1 call then 2 calls}) + \Pr(\text{2 calls then 1 call}) + \Pr(\text{0 calls then 3 calls}) + \Pr(\text{3 calls then 0 calls})$ [1]

$= 0.1 \times 0.25 + 0.25 \times 0.1 + 0.25 \times 0.4 + 0.4 \times 0.25$ [1]

$= 0.25$ [1]

Question 5a



Graph should contain the following:

- End points are at $(-3, 0)$ and $(3, 24)$ [1]
- Axis-intercepts are at $(-3, 0)$, $(-1, 0)$ and $(0, 3)$ [1]
- Correct shape [1]

Question 5b
$(0, -1)$ [1]

Question 5c
The mapping is $(x, y) \rightarrow (-x, y)$ [reflection in y axis] → $(-x, y + 2)$ [translation 2 units up]. This maps to (x', y'). So $x = -x'$ and $y = y' - 2$. So $y' - 2 = |-(-x')^2 - 4(-x') - 3|$.

Image of $y = |-x^2 + 4x - 3| + 2$ [1]

Question 6
$\sin\left(2x - \frac{\pi}{2}\right) = -\frac{\sqrt{2}}{2}$ [1]

$2x - \frac{\pi}{2} = -\frac{\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \frac{13\pi}{4}$ [1]

$x = \frac{\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{15\pi}{8}$ [1]

Question 7

$\ln\left(\frac{(x-3)(2x-1)}{(x+1)^2}\right) = 0$ [1]

$(x-3)(2x-1) = (x+1)^2$

$x^2 - 9x + 2 = 0$ [1]

Reject $x = \frac{9}{2} - \frac{\sqrt{73}}{2}$ since $x > 1$, so:

$x = \frac{9}{2} + \frac{\sqrt{73}}{2}$ [1]

Question 8a

$\Pr(X > 161|X > 150) = \frac{\Pr(X > 161)}{\Pr(X > 150)}$ [1]

Now $P(X > 161) = P(Z > (\frac{(161-150)}{11}) = P(Z > 1) = P(Z < -1)$

and $P(X < 139) = P(Z < \frac{((139-150))}{11} = P(Z < -1) = q.$

So $P(X > 161) = q$. Since $P(X > 150) = P(Z > 0) = 0.5$,

$\frac{\Pr(X > 161)}{\Pr(X > 150)} = \frac{q}{0.5} = 2q$ [1]

Question 8b

$\Pr(X \le k) = \int_2^k \frac{x-2}{8}\, dx$ [1]

$= \frac{1}{8}\left(\frac{k^2}{2} - 2k + 2\right) = \frac{1}{2}$

$k^2 - 4k - 4 = 0$ [1]

Reject $\mathrm{k} = 2 - 2\sqrt{2}$ since $2 < k < 6$

$k = 2 + 2\sqrt{2}$ [1]

Question 9a

$f'(x) = 2x\ln(x) + x$ [1]

Question 9b

$x\ln(x) = \frac{f'(x)-x}{2}$ [1]

$\int_1^e (x\ln x)dx = \int_1^e \frac{f'(x)-x}{2}dx = \int_1^e (\frac{f'(x)}{2} - \frac{x}{2})dx = \left[\frac{f(x)}{2} - \frac{x^2}{4}\right]_1^e = \left[\frac{1}{2}x^2\ln(x) - \frac{x^2}{4}\right]_1^e$ [1]

$= \frac{e^2+1}{4}$ [1]

Question 10
Area below curve: first we must find x-intercepts.

$x^3 - kx = 0$

$x = 0, \pm\sqrt{k}$ [1]

So the area underneath the x-axis for $x > 0$ is:

$-\int_0^{\sqrt{k}} (x^3 - kx)dx = -\left[\frac{x^4}{4} - \frac{kx^2}{2}\right]_0^{\sqrt{k}} = \frac{k^2}{4}$ [1]

Area above the curve: first we must find m, the point of intersection

$f(x) = g(x)$

$x^3 - 2kx = 0$

$x = 0, \pm\sqrt{2k}$ [1]. As $m > 0$, $m = \sqrt{2k}$

To find the area above the x-axis, we find the total area between $g(x)$ and $f(x)$, then subtract the area under the x-axis as found earlier:

$$\begin{aligned}
Area\ between\ g(x)\ and\ f(x) &= \int_0^m \left(kx - (x^3 - kx)\right)dx \\
&= \int_0^m (2kx - x^3)dx \\
&= \int_0^{\sqrt{2k}} (2kx - x^3)dx \\
&= \left[kx^2 - \frac{x^4}{4}\right]_0^{\sqrt{2k}} = k(2k) - \frac{4k^2}{4} = k^2
\end{aligned}$$

Therefore the area bounded by f and g is four times the size of the area bounded by the g and the x-axis.