2013 Examination Report

VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

2013 Mathematical Methods (CAS) GA 3: Examination 2

GENERAL COMMENTS

In the 2013 Mathematical Methods (CAS) Examination 2, students achieved scores across the whole range of available marks. A number of students sat the computer-based version of this examination. Responses showed that the examination was accessible and that it provided an opportunity for students to demonstrate their knowledge.

In Section 1, the majority of students answered Question 1 correctly, whereas few students answered Questions 16 and 20 correctly. Less than 50% of students obtained the correct answers for Questions 7, 11, 12, 15, 16, 17, 18, 20, 21 and 22.

In Section 2, Question 2 on probability was done well, and many students made good attempts with Questions 1 and 3. Many students found Question 1f., the transformation question, to be challenging. Some were unfamiliar with the notation. Many students were unable to set up the distance formula in Question 3d., and Questions 4c. and 4d. were not done well.

Students need to provide exact answers unless otherwise stated. Approximate answers were often seen in Questions 1c., 1d. and 3a. Some students rounded their answers incorrectly. In Question 2cii., 53 was a common incorrect answer.

Students need to check their responses to make sure they have answered all parts of the question. Many students did not give the minimum distance in Question 4b. Adequate working must be shown for questions worth more than one mark, and students need to be encouraged to attempt to show working even if they are unable to answer a previous part. Students could readily obtain method marks; for example, in Questions 3f. and 3h. The 'show that' questions, Question 1eii. and Question 3ci., were done reasonably well.

Incorrect mathematical notation was often seen, especially in Question 2. This sometimes led to the incorrect expression being entered for computation with technology. Some students did not have their technology set to calculate in mode radians, as was needed for Question 1.

SPECIFIC INFORMATION

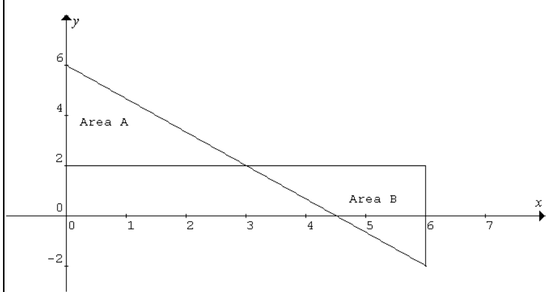
This report provides sample answers or an indication of what answers may have included. Unless otherwise stated, these are not intended to be exemplary or complete responses.

The statistics in this report may be subject to rounding errors resulting in a total less than 100 per cent.

Section 1

The table below indicates the percentage of students who chose each option. The correct answer is indicated by the shading.

Question	% A	% B	% C	% D	% E	% No Answer	Comments
1	2	8	85	2	3	0	
2	74	20	2	2	2	0	
3	4	15	12	10	59	1	
4	3	6	5	2	84	0	
5	75	5	8	4	8	0	
6	2	73	7	11	6	0	
7	37	19	18	14	11	1	The function g must be one to one for the inverse function to exist. The period of g is π. Hence, the domain for g could be $\left[-\frac{\pi}{4}+\frac{\pi}{6}, \frac{\pi}{4}+\frac{\pi}{6}\right]$, $\left[-\frac{\pi}{12}, \frac{5\pi}{12}\right]$. However, the domain for g needed to be in the form $[-a, a]$. Thus, the maximum possible value for a was $\frac{\pi}{12}$.

Question	% A	% B	% C	% D	% E	% No Answer	Comments
8	3	12	71	7	6	0	
9	14	9	4	14	58	0	
10	16	13	51	10	9	1	If A and B are independent events $\Pr(A)\times\Pr(B)=\Pr(A\cap B)=p$, $\left(p+p-\frac{1}{8}\right)\times\left(p+\frac{3p}{5}\right)=p,\ p=\frac{3}{8}$.
11	20	47	12	10	10	1	$\frac{dy}{dx}=ae^{ax}$. At $x=c$, the gradient of the tangent is ae^{ac}. As the tangent passes through (0, 0) and $\left(c,e^{ac}\right)$ its gradient can also be written in the form $\frac{e^{ac}}{c}$. Hence, $ae^{ac}=\frac{e^{ac}}{c}$ and $c=\frac{1}{a}$.
12	9	16	35	16	22	1	$\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}=-4\sin(x)\times 3e^{2t}$, $x=\frac{3}{2}e^{2t}+c$, $\frac{3}{2}=\frac{3}{2}e^{0}+c$, c = 0. When $x=\frac{\pi}{2}$, $t=\frac{1}{2}\log_e\left(\frac{\pi}{3}\right)$, $-4\sin(x)\times 3e^{2t}=-4\sin\left(\frac{\pi}{2}\right)\times 3e^{\log_e\left(\frac{\pi}{3}\right)}=-4\pi$.
13	16	8	63	7	5	1	
14	6	6	18	64	6	1	
15	33	20	25	8	12	1	In option C, a possible equation for the line was $y=-\frac{4}{3}x+6$. The average value $=\frac{1}{6}\int_0^6\left(-\frac{4}{3}x+6\right)dx=2$. If the line y = 2 is drawn over the interval $[0,6]$, the area of the two triangles shown are equal. Area A equals Area B. 
16	25	4	22	27	21	0	$f(5)=2$, $f(1)=0$ and $f^{-1}(x)=x^2+1$. $\int_0^2\left(x^2+1\right)dx-2\times 1=\int_0^2\left(x^2\right)dx$
17	8	19	16	49	7	1	$\Pr\left(A\mid B\right)=\frac{\Pr(A\cap B)}{\Pr(B)}=p$, $\Pr\left(A\cap B\right)=p^3$ $\Pr\left(B\mid A\right)=\frac{\Pr(A\cap B)}{\Pr(A)}=\frac{p^3}{p^{\frac{1}{3}}}=p^{\frac{8}{3}}$

Question	% A	% B	% C	% D	% E	% No Answer	Comments
18	12	35	22	17	13	2	$2g(8x) = 2\log_2(8x)$ $= \log_2\left(2^6 x^2\right)$ $= 6 + \log_2\left(x^2\right)$ $= g(x^2) + 6$
19	11	14	17	50	8	1	Solve $f'(x) = 0$, for $0 \le x \le 3\pi$. $x = \frac{5\pi}{6}$ or $x = \frac{11\pi}{6}$ or $x = \frac{17\pi}{6}$ The x-coordinate of T_3 is $\frac{17\pi}{6}$.
20	25	9	53	9	3	1	$x' = x + 5$, $x = x' - 5$ and $y' = -y$, $y = -y'$ $f(x) = -(x+5)^2$, $-y' = -\left(\left(x'-5\right)+5\right)^2$, $y' = \left(x'\right)^2$ or, alternatively, the inverse transformation maps $y = x^2$ onto f.
21	20	25	14	29	11	1	$f'(x) = 3ax^2 - 2bx + c$, $4b^2 - 12ac < 0$, $c > \frac{b^2}{3a}$
22	8	18	18	47	9	1	$T \sim N\left(120, \sigma^2\right)$, $\Pr\left(T \le 90\right) = \frac{150}{2000}$ $\frac{90-120}{\sigma} = -1.4395...$, $\sigma = 20.84... \approx 21$ days

Section 2

Question 1

1a.

Marks	0	1	2	Average
%	15	26	58	**1.5**

Maximum $27\ ^\circ\text{C}$, when $t = 0$ or $t = 16$ h

Some students gave the maximum temperature but not the values of t. Some students did not answer both parts of the question.

1b.

Marks	0	1	Average
%	12	88	**0.9**

Period = $\frac{2\pi}{\frac{\pi}{8}} = \frac{8}{\pi} \times 2\pi = 16$ h

Most students answered this question well.

1c.

Marks	0	1	2	Average
%	12	24	64	**1.5**

Solve $25 + 2\cos\left(\frac{\pi t}{8}\right) = 26$ for t, $t = \frac{8}{3}$ h

An exact answer was required. Some students rounded their answers.

1d.

Marks	0	1	2	Average
%	37	18	45	1.1

Solve $T(t) \geq 26$ for $0 \leq t \leq 24$, $\frac{8}{3} + \frac{56}{3} - \frac{40}{3} = 8$ h

Some students just gave the values of *t* and did not find the difference between them. Others used approximate values in their calculations.

1ei.

Marks	0	1	Average
%	19	81	0.8

$\frac{dy}{dx} = \cos(x)$, $\cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}$

This question was answered well. Some students left their answer as $\cos\left(\frac{2\pi}{3}\right)$.

1eii.

Marks	0	1	Average
%	48	52	0.5

$y - \frac{\sqrt{3}}{2} = -\frac{1}{2}\left(x - \frac{2\pi}{3}\right)$, $y = -\frac{1}{2}x + \frac{\pi}{3} + \frac{\sqrt{3}}{2}$, $0 = -\frac{1}{2}c + \frac{\pi}{3} + \frac{\sqrt{3}}{2}$, $c = \sqrt{3} + \frac{2\pi}{3}$ or $-\frac{1}{2} = \frac{0 - \frac{\sqrt{3}}{2}}{c - \frac{2\pi}{3}}$, $c = \sqrt{3} + \frac{2\pi}{3}$

This was a 'show that' question. Some students thought *c* was the *y*-intercept.

1fi.

Marks	0	1	2	Average
%	83	7	10	0.3

$\frac{\sqrt{3}}{2}k = 10$, $k = \frac{20}{\sqrt{3}} = \frac{20\sqrt{3}}{3}$, $\sqrt{3}m = 30$, $m = \frac{30}{\sqrt{3}} = 10\sqrt{3}$

This question was not answered well. Some students attempted to use matrices, which was not necessary.

1fii.

Marks	0	1	Average
%	88	12	0.1

$\left(\frac{2\pi}{3} \times m, \frac{\sqrt{3}}{2} \times k\right)$, $\left(\frac{20\sqrt{3}\pi}{3}, 10\right)$

Of those who answered this question, some divided by *m* and *k* instead of multiplying.

Question 2

2ai.

Marks	0	1	2	Average
%	17	13	70	1.6

$X \sim \text{Bi}\left(20, \frac{5}{8}\right)$, $\Pr(X \geq 10) = 0.91529\ldots = 0.9153$, correct to four decimal places

This question was answered well. Some students used $\Pr(X > 10)$. Others rounded their answers incorrectly.

2aii.

Marks	0	1	2	3	Average
%	26	14	21	39	1.8

$\Pr(X > 15 \mid X \geq 10) = \frac{\Pr(X > 15))}{\Pr(X \geq 10)} = \frac{0.079041\ldots}{0.915292\ldots} = 0.086$, correct to three decimal places

Most students were able to recognise that the question involved conditional probability. Some used $\Pr(X \geq 15 \mid X \geq 10)$.

2b.

Marks	0	1	2	Average
%	27	22	51	1.3

$S'SSS' + S'SS'S + S'S'SS$, $\frac{3}{32} + \frac{1}{16} + \frac{3}{16} = \frac{11}{32} = 0.34375$

Many students recognised that there were three cases. An exact answer was required.

2ci.

Marks	0	1	2	Average
%	34	20	46	1.1

$\int_{-\infty}^{\infty} (xg(x))dx = \int_1^3 \left(x \times \frac{(x-3)^3 + 64}{256}\right)dx + \int_3^5 \left(x \times \frac{x+29}{128}\right)dx = 3.045833\ldots = 3.0458$, correct to four decimal places

Some students did not include x in the formula. Others used an incorrect formulation such as

$\int_1^3 \left(\frac{x(x-3)^3 + 64}{256}\right)dx + \int_3^5 \left(x \times \frac{x+29}{128}\right)dx$.

2cii.

Marks	0	1	2	Average
%	45	13	42	1

$\int_4^5 \left(\frac{x+29}{128}\right)dx \times 200 = 52.34\ldots = 52$ members

Some students rounded their answers to 53. Others used $\int_{4.01}^5 \left(\frac{x+29}{128}\right)dx \times 200$. Some used $g(4)$.

Question 3

3a.

Marks	0	1	2	3	Average
%	13	4	17	66	2.4

$f'(x)=0$, $\left(\frac{28}{9},-\frac{50}{243}\right)$

This question was answered well. Some students gave approximate answers when exact values were required.

3b.

Marks	0	1	2	Average
%	13	6	81	1.7

$y=-\frac{1}{8}x+\frac{1}{2}$

This question was answered well.

3ci.

Marks	0	1	Average
%	42	58	0.6

Solve $f(x)=y$ for x, $\frac{3x^3}{64}-\frac{7x^2}{32}+\frac{1}{2}=-\frac{1}{8}x+\frac{1}{2}$, $3x^3-14x^2+8x=0$, $x(3x-2)(x-4)=0$, $x=\frac{2}{3}$ as $0<x<4$ or

Solve $\frac{3x^3}{64}-\frac{7x^2}{32}+\frac{1}{2}=-\frac{1}{8}x+\frac{1}{2}$ for x, $x=0$ or $x=\frac{2}{3}$ or $x=4$, $x=\frac{2}{3}$ as $0<x<4$

This was a 'show that' question and adequate working needed to be shown.

3cii.

Marks	0	1	2	Average
%	31	19	50	1.2

$f\left(\frac{2}{3}\right)=\frac{5}{12}$, $\mathrm{d}AD=\sqrt{\left(\frac{2}{3}-0\right)^2+\left(\frac{5}{12}-\frac{1}{2}\right)^2}=\frac{\sqrt{65}}{12}$

Some students found $f\left(\frac{2}{3}\right)=\frac{5}{12}$ and attempted to use the distance formula but did not subtract $\frac{1}{2}$ from $\frac{5}{12}$. Others did not use exact values.

3di.

Marks	0	1	2	Average
%	74	5	21	0.6

$\mathrm{d}(EF)=y-f(x)=-\frac{3x^3}{64}+\frac{7x^2}{32}-\frac{x}{8}$, $\frac{d}{dx}\left(-\frac{3x^3}{64}+\frac{7x^2}{32}-\frac{x}{8}\right)=0$, $x=\frac{2(\sqrt{31}+7)}{9}$

This type of question appeared to be unfamiliar to many students.

3dii.

Marks	0	1	2	Average
%	73	9	17	0.5

Substitute $x = \frac{2(\sqrt{31}+7)}{9}$ into $y - f(x)$, $\mathrm{d}(EF) = 0.33600\ldots = 336$ m, to the nearest metre

Some students attempted to substitute their x value into $y - f(x)$.

3e.

Marks	0	1	Average
%	36	64	0.7

$V(4) = 0$, $k\sqrt{4} - m \times 4^2 = 0$, $k = 8m$

This question was answered well. $k = 2m$ was a common incorrect response.

3f.

Marks	0	1	2	Average
%	40	17	44	1.1

$V(x) = 8m\sqrt{x} - mx^2$, $V'(x) = 0$, $x = 2^{\frac{2}{3}}$

Some students gave the answer without showing any method. Others gave an approximate answer when an exact value was required.

3g.

Marks	0	1	2	Average
%	50	15	35	0.9

$V(x) = 80\sqrt{x} - 10x^2$, $V\left(2^{\frac{2}{3}}\right) = 75.5953\ldots = 75.6$ km/h, correct to one decimal place

Some students left their answer in exact form.

3h.

Marks	0	1	2	Average
%	49	26	25	0.8

Solve $V(2^{\frac{2}{3}}) = 8m\sqrt{2^{\frac{2}{3}}} - m\left(2^{\frac{2}{3}}\right)^2 = 120$ for m, $m = 10 \times 2^{\frac{2}{3}}$

Some students knew to solve $V(x) = 120$ for their value of x. Some calculated $V(120)$. Others gave an approximate answer when an exact value was required.

4ai.

Marks	0	1	2	Average
%	38	12	50	1.1

$m=-1,\ g'(x)=-1,\ g(2)=3,\ y=-x+5$

Some students used $m=1$ instead of $m=-1$. Others used the *y*-intercept of *g*, giving $y=-x+4$ as their final answer.

4aii.

Marks	0	1	2	3	Average
%	45	26	3	26	1.1

$\int_0^5 (-x+5)dx-\int_0^4 (g(x))dx=\frac{11}{6}$ or $\frac{5\times 5}{2}-\int_0^4 (g(x))dx=\frac{11}{6}$ or $\int_0^4 ((-x+5)-g(x))dx+\frac{1\times 1}{2}=\frac{11}{6}$

There were many different approaches to this question. A common incorrect response was $\int_0^5 ((-x+5)-g(x))dx=\frac{35}{12}$.

An exact answer was required.

4b.

Marks	0	1	2	3	Average
%	75	4	6	15	0.6

$d(x)=\sqrt{\left(\frac{16-x^2}{4}\right)^2+x^2}$, $d'(x)=0$, $x=2\sqrt{2}$, $d\left(2\sqrt{2}\right)=2\sqrt{3}$

Some students found $x=2\sqrt{2}$ but did not find the minimum distance. Exact answers were required.

4c.

Marks	0	1	2	Average
%	90	4	6	0.2

Let the coordinates of *P* be $\left(p,\frac{16-p^2}{4}\right)$, $m=g'(p)=-\frac{p}{2}$, solve $\frac{\frac{16-p^2}{4}-k}{p}=-\frac{p}{2}$ for *p*, $p=2\sqrt{k-4}$,

$m=-\sqrt{k-4}$ or $\frac{16-x^2}{4}=mx+k$, $x^2+4mx+4k-16=0$, $m^2-k+4=0$, $m=-\sqrt{k-4}$ as $m<0$ or

$y-\frac{16-p^2}{4}=-\frac{p}{2}(x-p)$, solve $k-\frac{16-p^2}{4}=-\frac{p}{2}(0-p)$ for *p*, $p=2\sqrt{k-4}$, $m=-\sqrt{k-4}$

This question was not answered well. A number of different approaches could have been taken.

4di.

Marks	0	1	2	Average
%	87	9	3	0.2

$y=-\sqrt{k-4}x+k$, *x*-intercept is $\frac{k}{\sqrt{k-4}}$, $A(k)=\frac{k}{2}\times\frac{k}{\sqrt{k-4}}-\int_0^4 (g(x))dx=\frac{k^2}{2\sqrt{k-4}}-\frac{32}{3}$ or

$A(k)=\int_0^{\frac{k}{\sqrt{k-4}}}\left(-\sqrt{k-4}x+k\right)dx-\int_0^4 (g(x))dx=\frac{k^2}{2\sqrt{k-4}}-\frac{32}{3}$

Some students used the equation of their tangent line to attempt to find $A(k)$.

4dii.

Marks	0	1	2	Average
%	94	3	3	0.1

The maximum occurs at the endpoint, $k = 8$, $A = \frac{16}{3}$

Many students did not attempt this question. Most of the students who were successful with Question 4di. were successful with this question.

4diii.

Marks	0	1	2	Average
%	97	1	2	0.1

The minimum occurs at the turning point, $A'(k) = 0$, $k = \frac{16}{3}$, $A\left(\frac{16}{3}\right) = \frac{64\sqrt{3}}{9} - \frac{32}{3}$

Many students did not attempt this question.