MATHEMATICAL METHODS

Units 3 & 4 – Written examination 1

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(*TSSM's *2013 trial exam updated for the current study design)

SOLUTIONS

Question 1

a. $f(x) = 1$

$ke^{2x} - 2e^{x} + k = 0$

For two solutions $\Delta > 0$

$4 - 4k^2 > 0 \ \ which\ gives - 1 < k < 1\ ,\ k \neq 0$

M1+A1

2 marks

b. $e^{x} = \frac{2 \pm \sqrt{4-4k^2}}{2k} = \frac{1 \pm \sqrt{1-k^2}}{k}$

$x = \left(\frac{(1 \pm \sqrt{1-k^2})}{k}\right)$

M1+A1

2 marks

Question 2

a. $f'(x) = 2x \times -\frac{1}{2\sqrt{1-x}} + 2\sqrt{1-x} \times 1$

$f'(x) = \frac{-x}{\sqrt{1-x}} + 2\sqrt{1-x}$

$f'(x) = \frac{2-3x}{\sqrt{1-x}}$

M1+A1

2 marks

b. $f'(x) = \frac{-x}{\sqrt{1-x}} + 2\sqrt{1-x}$

$\frac{x}{\sqrt{1-x}} = 2\sqrt{1-x} - f'(x)$

Integrating both sides with respect to x between the limits $x = -2$ and $x = 0$

$\int_{-2}^{0} \frac{x}{\sqrt{1-x}} dx = \int_{-2}^{0} 2\sqrt{1-x} - \left(2x\sqrt{1-x}\right)_{-2}^{0}$

$\int_{-2}^{0} \frac{x}{\sqrt{1-x}} dx = -\frac{4}{3} + 4\sqrt{3} - 4\sqrt{3}$

$Area = \frac{4}{3}\ square\ units$

M2+A1

3 marks

Question 3

a. $g(2x) = 0$

$\sin(2x) + \cos(2x) = 0$

$\tan(2x) = -1$

$2x = n\pi + tan^{-1}(-1), n \in Z$

$2x = n\pi + \frac{3\pi}{4}, n \in Z$

$x = \frac{n\pi}{2} + \frac{3\pi}{8},\ n \in Z$

M1+A1

2 marks

b. $n = 0\ \ gives\ x = \frac{3\pi}{8}$

$n = -1\ \ gives\ x = \frac{-\pi}{8}$

$n = -2\ \ gives\ x = \frac{-5\pi}{8}$

$n = 1\ \ gives\ x = \frac{7\pi}{8}$

$Solutions\ are\ \frac{3\pi}{8}, \frac{-\pi}{8}, \frac{-5\pi}{8}, \frac{7\pi}{8}$

M1+A1

2 marks

Question 4

a. $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

$x' = 2x,\ y' = -3y$

$x = \frac{x'}{2},\ y = \frac{-y'}{3}$

$\frac{-y'}{3} = \frac{1}{2 \times \frac{x'}{2}}$

$g(x) = -\frac{3}{x}$

M2+A1

3 marks

b. $Reflection\ in\ the\ x-axis,$
$dilation\ of\ a\ factor\ of\ 3\ units\ from\ the\ x-axis,$
$dilation\ by\ a\ factor\ of\ 2\ units\ from\ the\ y-axis.$

A2
2 marks

c. $x = -\frac{3}{y}\ \ or\ y = -\frac{3}{x}$
$g^{-1}(x) = -\frac{3}{x}$

M2+A1
3 marks

d. $Domain\ of\ g^{-1}\ is\ R\backslash\{0\}$
$Range\ of\ g^{-1}\ is\ R\backslash\{0\}$

A2
2 marks

Question 5

a. $\int_0^4 0.3p\ dx + \int_4^5 xp\ dx = 1$
$(0.3px)_0^4 + \left(p\frac{x^2}{2}\right)_4^5 = 1$
$1.2p + \frac{9}{2}p = 1\ \ which\ gives\ p = \frac{10}{57}$

M2+A1
3 marks

b. $\int_1^{2.5} 0.3p\ dx = 0.3p(2.5-1)$
$\int_1^{2.5} f(x)dx = \frac{3}{38}$

M1+A1
2 marks

Question 6

a. $\hat{p}$ is the midpoint of the confidence interval
$\hat{p} = \frac{s+r}{2}$

M1+A1
2 marks

b. The margin of error is equal to half the width of the interval
$M = \frac{s-r}{2}$

M1+A1
2 marks

Question 7

$(a, -a^2 - 8a)$

$\frac{dy}{dx} = -2x - 8$

$m_T = -2a - 8$

$Also, m_T = \frac{-a^2-8a-3}{a}$

$\frac{-a^2-8a-3}{a} = -2a - 8 \ which\ gives\ a = \pm\sqrt{3}$

The equations of tangents are:

$y = \left(-8 - 2\sqrt{3}\right)x + 3 \quad and \quad y = \left(-8 + 2\sqrt{3}\right)x + 3$

M2+A2
4 marks

Question 8

a. $\int_0^{\frac{\pi}{8}} sin\left(\frac{4x}{3}\right) dx = \left(-\frac{cos\left(\frac{4x}{3}\right)}{\frac{4}{3}}\right)_0^{\frac{\pi}{8}} = \frac{6-3\sqrt{3}}{8}$

M1+A1
2 marks

b. Average value $= \frac{1}{\frac{\pi}{4}-0} \times \left(-\frac{cos\left(\frac{4x}{3}\right)}{\frac{4}{3}}\right)_0^{\frac{\pi}{4}} = \frac{3}{2\pi}$

M1+A1
2 marks