QATs
Quality Assessment Tasks

UNIT 3 ■ OUTCOMES 1, 2 & 3

VCE Mathematics Methods (CAS)

SCHOOL-ASSESSED COURSEWORK

Introduction

Outcome 1
Define and explain key concepts as specified in the content from the areas of study, and apply a range of related mathematical routines and procedures.

Outcome 2
Apply mathematical processes in non-routine contexts, and analyse and discuss these applications of mathematics.

Outcome 3
Select and appropriately use technology to develop mathematical ideas, produce results and carry out analysis in situations requiring problem-solving, modeling or investigative techniques or approaches.

Task
Application Task

This task will contribute 40 of the total marks (60) allocated for SAC in Unit 3.This task will be marked out of 100 and then converted to a mark out of 40 for its contribution to school assessed coursework in Unit 3.

The task has been designed to allow achievement up to and including the highest level in the Performance Descriptors and meets a broad range of **key knowledge** and **key skills** related to each outcome.

The marks for each part are indicated in brackets.
This task is to be done in class over a period of approximately two weeks.
You may use a summary book and an approved CAS calculator. WORK IN PROGRESS will be collected at the end of each class session.

T Indicates where use of the technology is specifically required in order to answer the question.
Answer in spaces provided or as indicated.

Your teacher will advise you of any variation to these conditions.

NAME:

Task

Application Task: Inspiring Architecture

You have been commissioned to design a building for a community civic centre. The centrepiece for the construction is an archway and spire. There are seven elements that are to be incorporated into the design, each of which is based on a separate type of function.

The front facade is symmetrical about the centre. This will allow you to determine the equations of the right hand side and then use transformations, in particular reflection, to determine the equation of the left side.

There are three Parts to this Task, and each part has three smaller sections:

Part 1: The Arches
- a) Decorative Arch
- b) Load Bearing Arch
- c) Underside of Beam

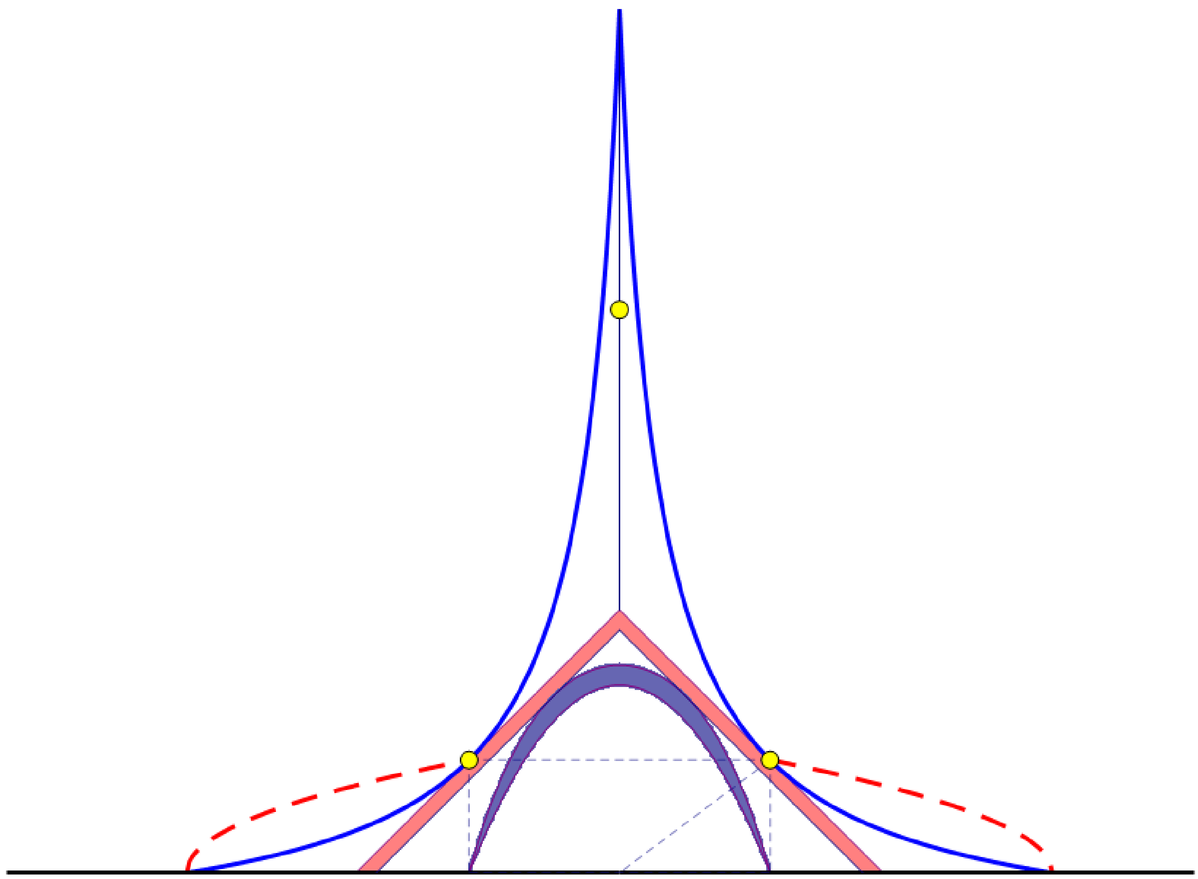
Part 2: The Spire
- a) The Spire itself
- b) Upper side of Beam
- c) Flarings

Part 3: Decorations
- a) Flood Lights
- b) Entrance Way
- c) Scroll Work

For each stage, follow the directions for determining the equation and properties of the given section. A small set of axes is provided for you to sketch that section as a reference. When you have completed a stage, including the reference sketch, add the completed stage to the large set of axes on page 4 of the task.

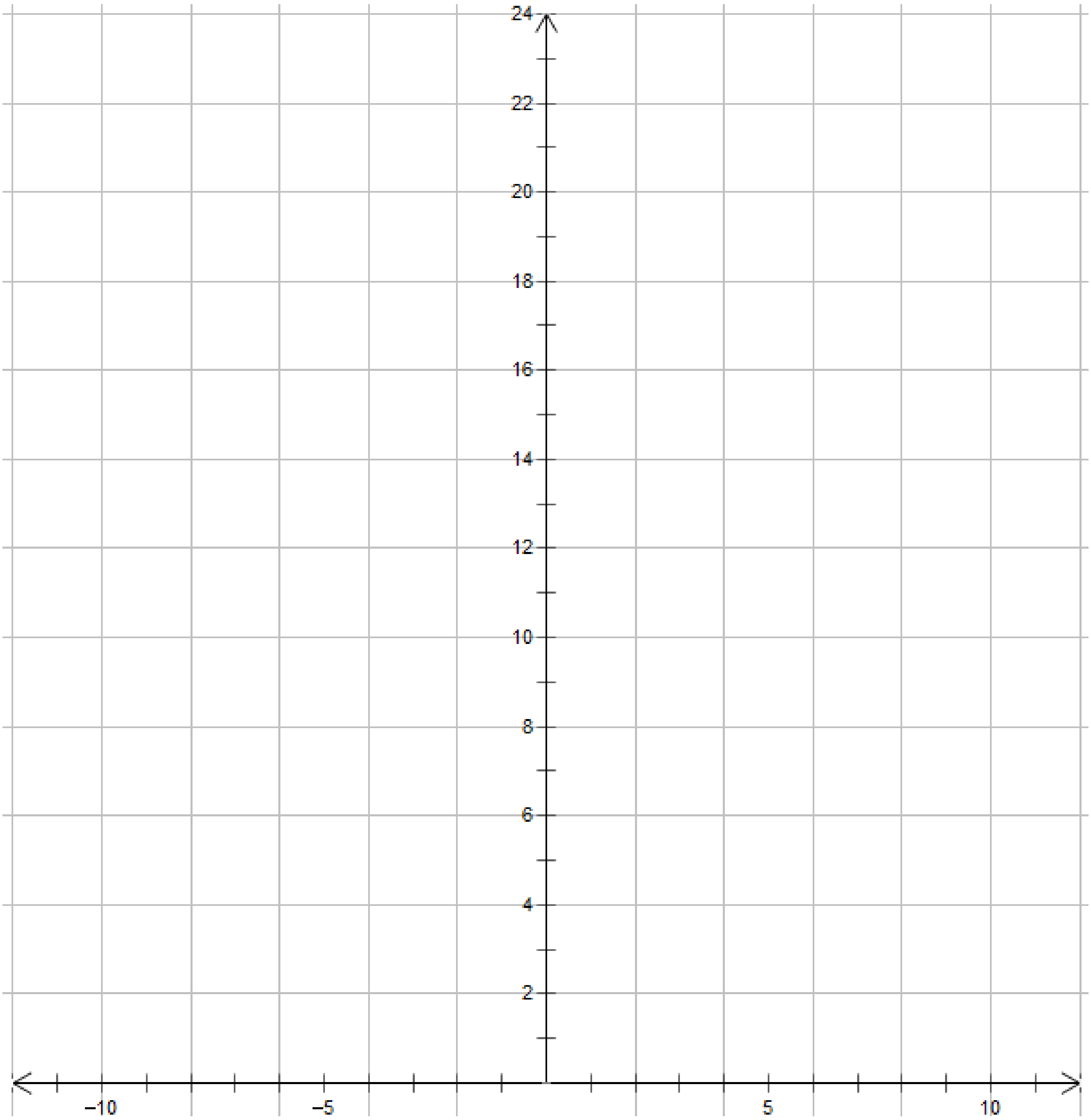
Below is an artist's sketch of what the facade is to look like when it is finished. You can use this to develop a sense of whether you are on the right track or not.

Task



Task

Fully completed, the combination of graphs from all the section on this set of axes contributes 8 marks to the total grade. 4 marks are awarded to the shapes and positions of the graph elements and another 4 marks for the correct labelling of all important values. **T** (8 marks)



Task

Part 1A: Quadratic (Decorative arch)

The entrance to the spire consists of an archway whose lower component is modelled by a parabola. The parabola has its vertex 5 metres above ground level and touches the ground 4 metres to either side of its axis of symmetry.

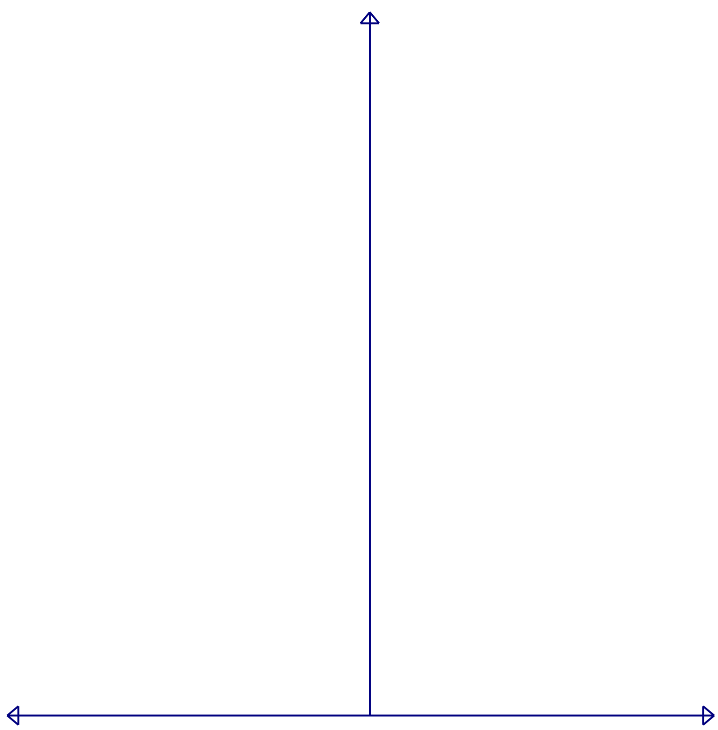
The equation of the parabola is in the form $f(x) = Ax^2 + B$.

a) Write down the coordinates of the vertex of the parabola. (1 mark)

b) Write down the coordinates of both x-intercepts of the parabola. (2 marks)

c) Determine the values of A and B in the equation of the parabola. (2 marks)

d) Sketch the parabola on the axes provided, over a suitable domain. State the domain of this function in set notation. **T** (3 marks)



Task

e) When you are satisfied with this element of the design, add it to the larger sketch on the axes provided on page 4. (Combines to give a total of 8 marks for the entire graph on page 4)

Task

Part 1B: Catenary (Load bearing arch)

The upper component of the archway is to be designed to bear the load of the wall above and around it. For this, the best shape is a catenary. A catenary is the name given to the curve formed by two simple exponential terms added together.

The equation of the upper arch is given as:

$$g(x) = -e^{\frac{x}{2}} - e^{\frac{-x}{2}} + C$$

The x-intercepts of the catenary are (-4, 0) and (4, 0).

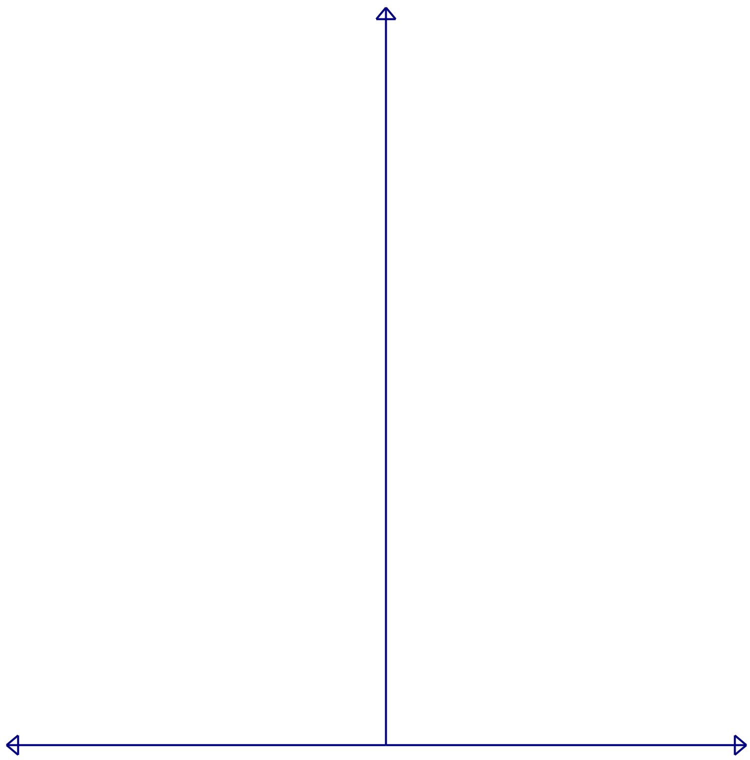
a) Use this information to determine **the exact value** of C. (2 marks)

b) Now write the coordinates of the y-intercept of *g(x)* correct to two decimal places. (2 marks)

c) Now show, using calculus, that the maximum value of the catenary occurs at the y-intercept. (If you use your calculator to verify the derivative, differentiate each term individually rather than entering the entire expression. Some CAS software returns a result that is not part of the Methods course.) (4 marks)

Task

d) Sketch the catenary on the axes provided, over a suitable domain. State the domain of this function in set notation. **T** (3 marks)



e) When you are satisfied with this element of the design, add it to the larger sketch on the axes provided on page 4. (Combines to give a total of 8 marks for the entire graph on page 5)

Task

Part 1C: Lower Tangent (Underside of beam)

Two metal beams are to rest on the load bearing arch. They are inclined at 45° to the ground and meet at a point along the axis of symmetry.

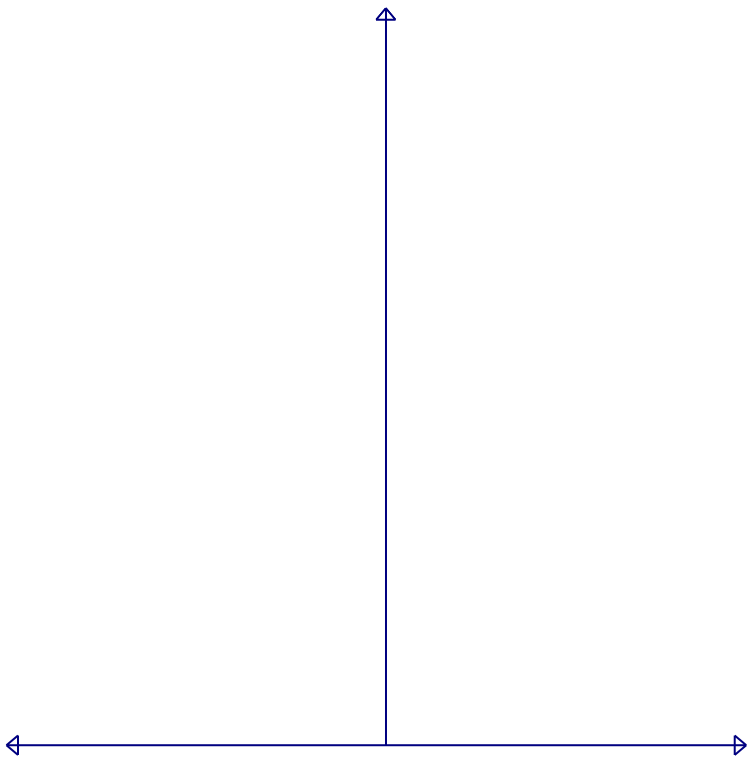
a) Consider the right hand side of the construction. Use algebra to show that the exact value of x at the point on the catenary, $g(x) = -e^{\frac{x}{2}} - e^{\frac{-x}{2}} + C$ where the gradient is -1 is $x = 2\ln(1 + \sqrt{2})$. (5 marks)

b) Now determine the value of y for this point. Write your answer to 2 decimal places. (1 mark)

Task

c) The equation of the tangent to this point is $h(x) = -x + D$. Find the value of D correct to 2 decimal places. (1 mark)

d) Sketch the line segment defined by *h(x)* on the axes provided, over a suitable domain. **T** (2 marks)



Task

e) Use symmetry to determine the hybrid function that fully defines both the left and right sides of the lower sides of the beams. (2 marks)

$$h(x) = \begin{cases} & \{x: \_\_ \leq x \leq \_\_\_\} \\ & \{x: \_\_ \leq x \leq \_\_\_\} \end{cases}$$

f) When you are satisfied with this element of the design, add lower beam sections to the larger sketch on the axes provided on page 4. (Combines to give a total of 8 marks for the entire graph on page 5)

Task

Part 2A: Hyperbola (Spire)

NOTE: The questions in this stage are designed to test your ability to substitute into and rearrange algebraic equations.

The spire itself is to be a height of 23 m and to meet the ground at 11.5 m on either side of the axis of symmetry. The curve also passes through the point (4, 3) on the right hand side.

a) The equation of the spire is that of an hyperbola with the equation:

$$j(x) = \frac{E}{x + F} + G$$

By substituting the coordinates of each of the three points described above, generate 3 equations with 3 unknowns.

i) Use the height to find the first equation. (1 mark)

ii) Use the coordinates of where the spire touches the ground on the right hand side to find the second equation. (1 mark)

Task

iii) Use the point (4, 3) to find the third equation. (1 mark)

b) Rearrange each of the equations to make E the subject.

i) The first equation becomes:

(1 mark)

ii) The second equation becomes:

(1 mark)

Task

iii) The third equation becomes:

(1 mark)

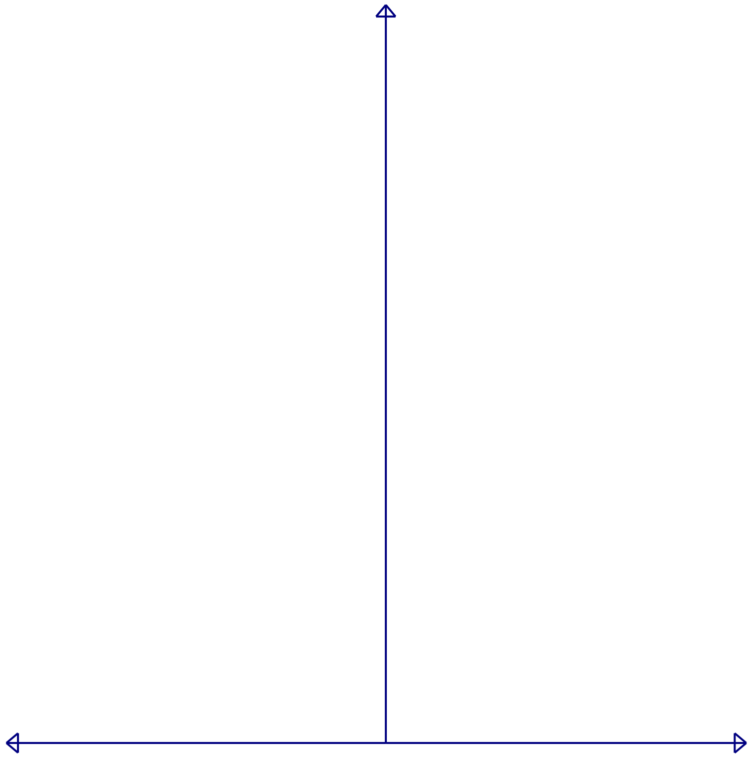
c) Use equations i) and ii) to find G in terms of F only. (2 marks)

Task

d) Now use equations i) and iii) as well as your result from c) to show that F = 1, G = -2 and E = 25.
(5 marks)

Task

e) Sketch the hyperbola on the axes provided, over a suitable domain. **T** (2 marks)



f) Use symmetry to determine the hybrid function that fully defines both the left and right hand sides of the spire. (2 marks)

$$j(x) = \begin{cases} & \{x\colon \_\_ \le x \le \_\_\_\} \\ & \{x\colon \_\_ \le x \le \_\_\_\} \end{cases}$$

g) When you are satisfied with this element of the design, add it to the larger sketch on the axes provided on page 4. (Combines to give a total of 8 marks for the entire graph on page 4)

Task

Part 2B: Upper Tangent (Upperside of beam)

The upper side of the metal beams are tangent to the hyperbola and are parallel to the underside of the beam.

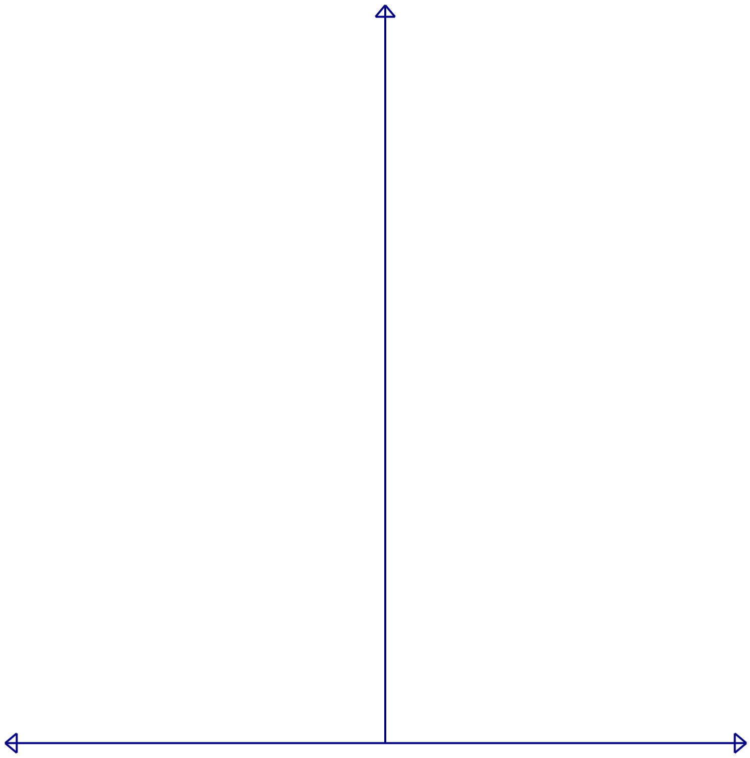
a) Consider the right hand side of the construction. Determine the x-value of the point on the hyperbola where the gradient is -1. (Remember that the equation of the right hand hyperbola is $j(x) = \frac{25}{x+1} - 2$.) (4 marks)

b) Now determine the y-value of this point. (1 mark)

Task

c) The equation of the tangent to this point is $k(x) = -x + H$. Find the value of H. (1 mark)

d) Sketch the line segment defined by *k(x)* on the axes provided, over a suitable domain. **T** (2 marks)



e) Use symmetry to determine the hybrid function that fully defines both the left and right sides of the upper sides of the beams. (2 marks)

$$k(x) = \begin{cases} & \{x:\ \_\_ \le x \le \_\_\_\} \\ & \{x:\ \_\_ \le x \le \_\_\_\} \end{cases}$$

Task

f) When you are satisfied with this element of the design, add upper beam sections to the larger sketch on the axes provided on page 4. (Combines to give a total of 8 marks for the entire graph on page 4)

Task

Part 2C: Square root (Flaring beams)

The outside of the civic centre is to have thin beams flaring out from the point where the spire curve (the hyperbola) meets the tangent and reaching the ground at the point where the spire also touches down.

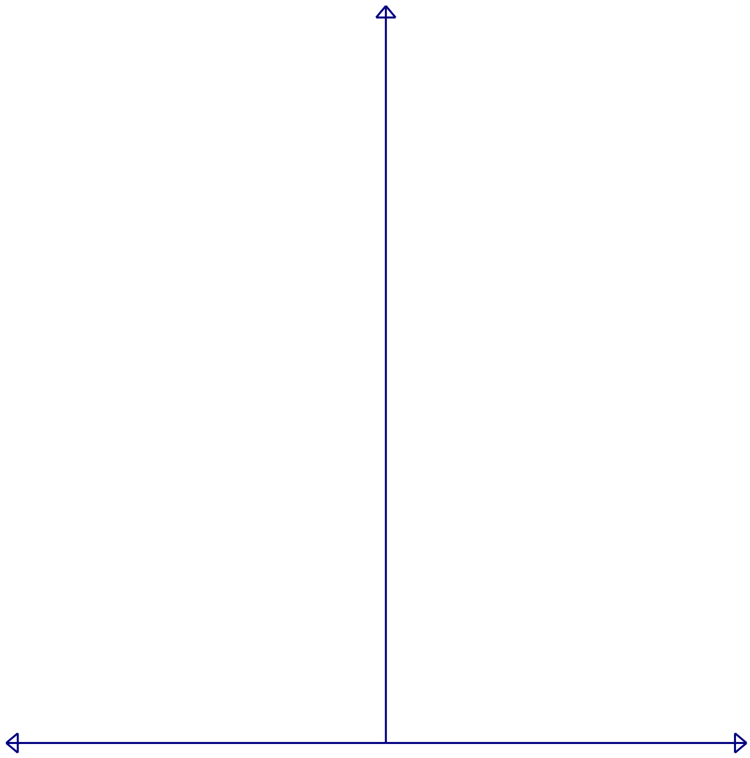
a) Consider the right hand side of the construction. The form of the flaring beam is a square root function of the form $L(x) = J\sqrt{K - x}$.

If the terminating point is at ground level, determine the values of J and K. (2 marks)

b) Show algebraically that $L(x) = \sqrt{\frac{69-6x}{5}}$. (3 marks)

Task

c) Sketch the curve on the axes provided, over a suitable domain. **T**
(2 marks)



d) Use symmetry to determine the hybrid function that fully defines both parts of this flaring section.
(2 marks)

$$L(x) = \begin{cases} \quad & \{x\colon \_\_ \le x \le \_\_\} \\ \quad & \{x\colon \_\_ \le x \le \_\_\} \end{cases}$$

e) When you are satisfied with this element of the design, add it to the larger sketch on the axes provided on page 4. (Combines to give a total of 8 marks for the entire graph on page 4)

Task

Part 3A: Intersection points and midpoint (Floodlight positions)

Three floodlights are to be installed in the positions indicated by the three small circles on the sketch.

a) The two lower floodlights are to be attached to the junction or intersection between the spire (hyperbolas) and the flares (square roots). Write down the coordinates of these two positions. Remember that the right hyperbola was $j(x) = \frac{25}{x+1} - 2$ and the square root equations was $L(x) = \sqrt{\frac{69-6x}{5}}$. (1 mark)

The third floodlight is to be higher than the other two and placed at the midpoint of the vertical line segment from the top of the spire to the top of the tangent beams.

b) Write down the coordinates of the top of the line segment (ie the top of the spire). (1 mark)

c) Write down the coordinates to the top of the upper tangent beam. (1 mark)

Task

d) Determine the length of the vertical line segment. (1 mark)

e) Find the coordinates of the midpoint of this line segment. (2 marks)

f) What is the difference in height between the two lower floodlights and the higher one? (1 mark)

Task

Part 3B: Entrance Way

a) The two upper corners of the rectangular entrance way touch the hyperbola. Hence determine an expression for the height of the rectangle, in terms of x. (1 mark)

b) Show that the Area of this rectangular opening is:

$$A = \frac{50x}{x+1} - 4x$$

Task

c) Use calculus to determine the dimensions of the rectangle that give a maximum area. (5 marks)

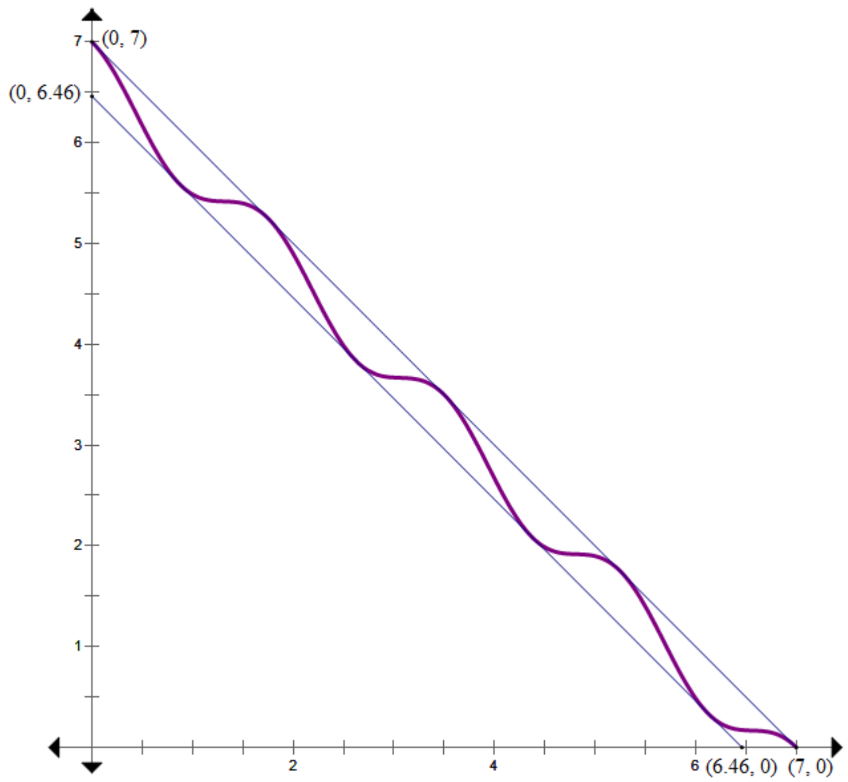
d) Find the maximum possible area for this doorway. Express your answer in exact form. (1 mark)

Task

Part: 3C: Beam Scroll Work

The architect also wants to inscribe a flowing wave onto the beam whose upper and lower sides were worked out in sections 3 and 5. The shape of the wave on the right beam looks as follows and has the equation:

$$M(x) = 0.27\ sin\left(\frac{8\pi}{7}\left(x + \frac{7}{16}\right)\right) - x + 6.73$$



a) What is the linear part of the function and what does it represent graphically?

(2 marks)

Task

b) Over the domain [0, 7], how many points have a gradient of -1? Explain how you can tell this just from the graph.

(2 marks)

c) Write down the x-values only of the points with a gradient of -1.

(1 mark)

d) Use calculus to find the derivative of $M(x)$.

(2 marks)

e) Show algebraically that the function never has a stationary point.

(3 marks)

Teacher Advice

Key knowledge and key skills

This task covers a broad range of **key knowledge** and **key skills** across Outcomes 1, 2 and 3 in Unit 3 as per the VCAA VCE Study Design 2006-14.

This task covers assessment in:
- Functions and graphs
- Transformations
- Polynomial functions
- Exponential and logarithmic functions
- Circular Functions
- Combinations of the above functions
- Differentiation and its applications

Assessment Planning

This is the Application Task, as suggested to be undertaken during Weeks 12 and 13 in the sample teaching sequence on page 193 of the VCAA Study Design.

This task contributes 40 of the 60 SAC marks in Unit 3.
The coursework scores for this task are:
Outcome 1 15 marks (37.5%)
Outcome 2 20 marks (50%)
Outcome 3 5 marks (12.5%)
TOTAL 40 marks

This weighting can be used in the conversion of their mark out of 110.
For example, a score of 80 results in:

OUTCOME 1	OUTCOME 2	OUTCOME 3
80/100*40*0.375	80/100*40*0.5	80/100*40*0.125
= 12	= 16	= 4

Rounding gives

= 12	= 16	= 4

The above can be established in an Excel file.

Alternatively, the following breakdown of marks could also be used.
Greyed out cells combine to give a total for the final graph which is included at the bottom of the table.

Teacher Advice

Part	Outcome 1	Outcome 2	Outcome 3	Question Total
1 A a	1			1
1 A b	2			2
1 A c	2			2
1 A d	1	1	1	3
1 A e				
1 B a	2			2
1 B b	2			2
1 B c	2	2		4
1 B d	1	1	1	3
1 B e				
1 C a	5			5
1 C b			1	1
1 C c			1	1
1 C d		1	1	2
1 C e		2		2
1 C f				
2 A a i)		1		1
2 A a ii)		1		1
2 A a iii)		1		1
2 A b i)		1		1
2 A b ii)		1		1
2 A b iii)		1		1
2 A c		2		2
2 A d	5			5
2 A e		1	1	2
2 A f		2		2
2 A g				
2 B a	2	2		4
2 B b	1			1
2 B c	1			1
2 B d		1	1	2
2 B e		2		2
2 B f				
2 C a		2		2
2 C b	3			3
2 C c		1	1	2

Teacher Advice

2 C d		1	1	2
2 C e				
3 A a		1		1
3 A b		1		1
3 A c		1		1
3 A d		1		1
3 A e		1	1	2
3 A f		1		1
3 B a		1		1
3 B b	2			2
3 B c	5			5
3 B d		1		1
3 C a		2		2
3 C b		2		2
3 C c		1		1
3 C d		1	1	2
3 C e	1	1	1	3
Final Graph		8		8
Totals	38	50	12	100

This QAT has been designed to meet the highest level in the performance descriptors provided by VCAA for each outcome in unit 3 in the VCAA Mathematics Assessment Handbook.

Solution Pathway

Part 1A: Quadratic (decorative arch)

a) (0, 5) (1 mark)

x-value is on the axis since the structure is symmetric. The y-value is 5 since the ground is taken to be the x-axis.

b) (-4, 0) and (4, 0) (2 marks)

c) Determine the values of A and B in the equation of the parabola.

B = 5 and is the y-intercept (1 mark)

Substitute (4, 0) into *f(x)*

$0 = 16A + 5$

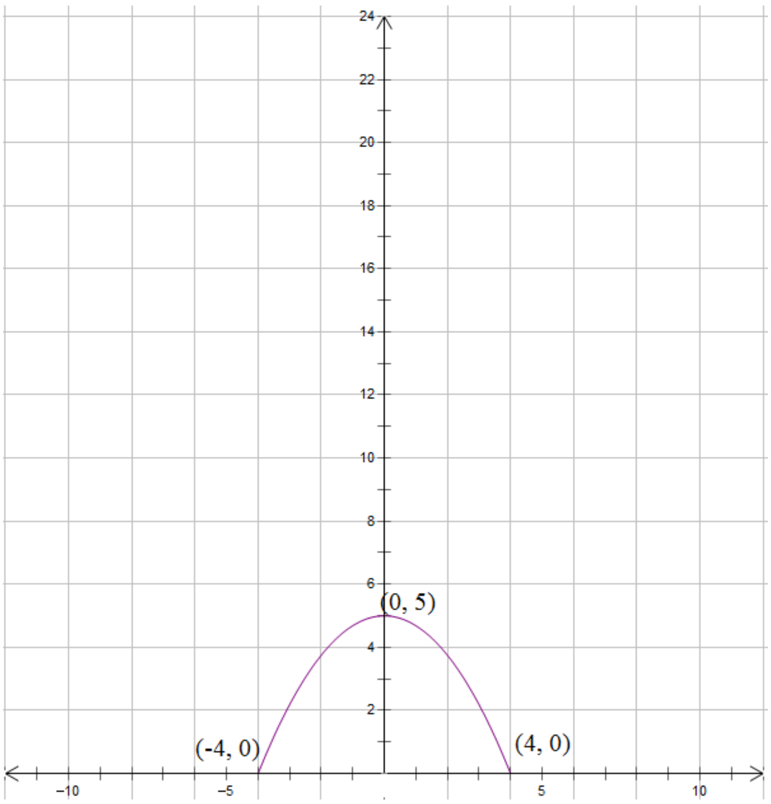
$-5 = 16A$

$A = \frac{-5}{16}$ (1 mark)

d) Sketch on right and domain, [-4, 4] or $\{x: -4 \le x \le 4\}$

(3 marks – shape, domain, points)

e) See end.



Solution Pathway

Part 1B: Catenary (load bearing arch)

a) Use this information to determine **the exact value** of C.

Substitute (4, 0) into *g(x)*. (1 mark)

$$0 = -e^{\frac{4}{2}} - e^{\frac{-4}{2}} + C$$

$C = e^{2} + e^{-2}$ (1 mark)

b) Let x = 0

$$y = -e^{0} - e^{0} + e^{2} + e^{-2}$$

$$y = -2 + e^{2} + e^{-2}$$

y = 5.52 Hence the y-intercept is (0, 5.52) (2 marks)

c) $g'(x) = -\frac{1}{2}e^{\frac{x}{2}} + \frac{1}{2}e^{\frac{-x}{2}}$

(1+1 mark)

Let *g'(x)* = 0

$$0 = -\frac{1}{2}e^{\frac{x}{2}} + \frac{1}{2}e^{\frac{-x}{2}}$$

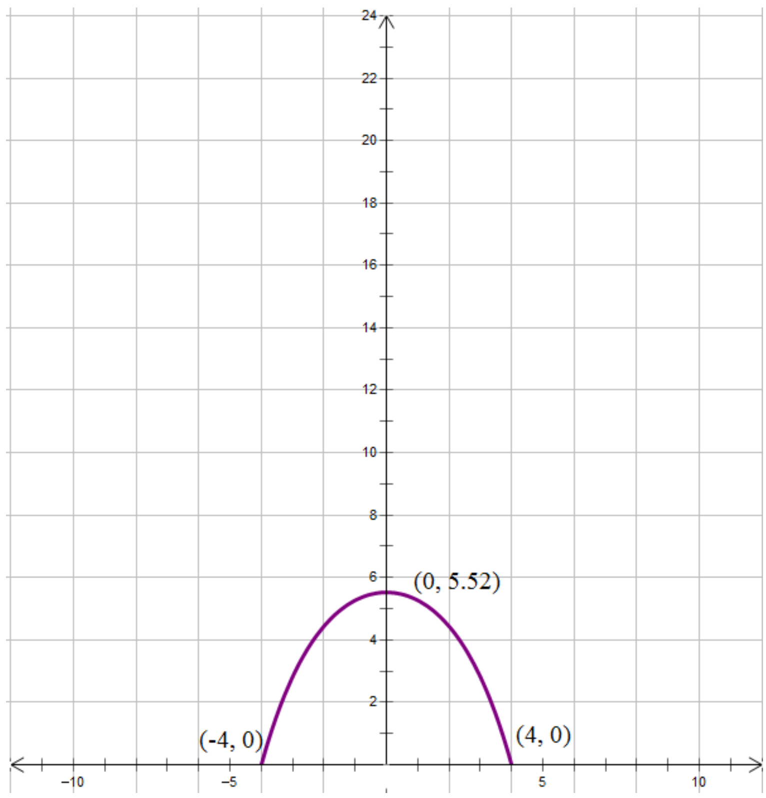
$\frac{1}{2}e^{\frac{x}{2}} = \frac{1}{2}e^{\frac{-x}{2}}$ (1 mark)

Solution Pathway

$e^{\frac{x}{2}} = e^{\frac{-x}{2}}$

$x = -x$, hence x = 0 (1 mark)

d) Sketch at right



(3 marks – shape, domain, points) Domain is [-4, 4] or $\{x: -4 \leq x \leq 4\}$

e) See end.

Solution Pathway

Part 1C: Lower Tangent (Underside of beam)

a) From Section 2) $g'(x) = -\frac{1}{2}e^{\frac{x}{2}} + \frac{1}{2}e^{\frac{-x}{2}}$

Let *g'(x)=-1*

$$-1 = -\frac{1}{2}e^{\frac{x}{2}} + \frac{1}{2}e^{\frac{-x}{2}}$$ (1 mark)

$\frac{1}{2}e^{\frac{x}{2}} - \frac{1}{2}e^{\frac{-x}{2}} - 1 = 0$

$e^{\frac{x}{2}} - e^{\frac{-x}{2}} - 2 = 0$ (Multiply through by 2) (1 mark)

$\left(e^{\frac{x}{2}}\right)^2 - 1 - 2e^{\frac{x}{2}} = 0$ (Multiply by $e^{\frac{x}{2}}$)

$e^{\frac{x}{2}}$ is now solvable using quadratic methods. Let $e^{\frac{x}{2}}$= A (1 mark)

$A^2 - 2A - 1 = 0$

Quadratic Formula: $A = \frac{2\pm\sqrt{2^2+4}}{2}$

$A = 1 \pm \sqrt{2}$

Hence $e^{\frac{x}{2}} = -1 \pm \sqrt{2}$ (1 mark)

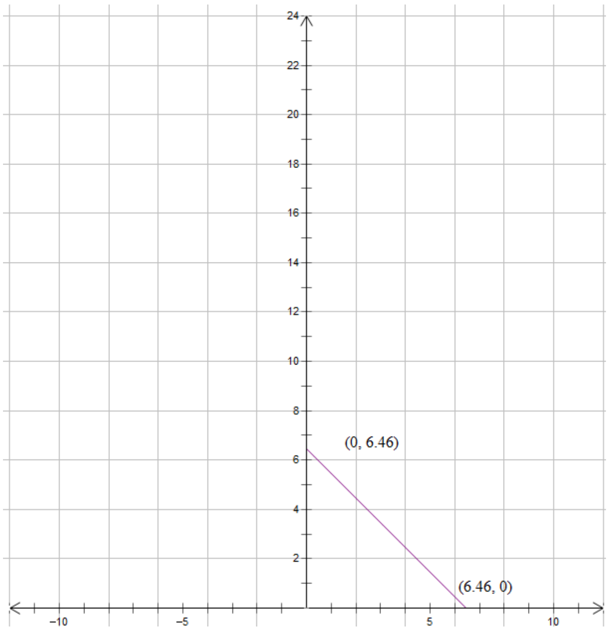
$\frac{x}{2} = \ln(1 \pm \sqrt{2})$

Only $\ln(1 + \sqrt{2})$ exists

Solution Pathway

so $x = 2\ln(-1 + \sqrt{2})$ (1 mark)

b) $g\left(2\ln\left(-1+\sqrt{2}\right)\right) = 4.70$ (1 mark)



c) Substitute (1.76, 4.70) into *h(x)*

$4.70 = -1.76 + D.$

D = 6.46 (1 mark)

d) Sketch on right (2 marks – domain, points)

e) $h(x) = \begin{cases} x + 6.46 & \{x: -6.46 \le 0\} \\ -x + 6.46 & \{x: 0 < \le 6.46\} \end{cases}$ $\begin{matrix} (1\ mark) \\ (1\ mark) \end{matrix}$

f) See end

Solution Pathway

Part 2A: Hyperbola (spire)

The spire itself is to be a height of 23 m and to meet the ground at 11.5 m on either side of the axis of symmetry. The curve also passes through the point (4, 3) on the right hand side.

a)

i) $23 = \frac{E}{F} + G$ ii) $0 = \frac{E}{11.5+F} + G$ iii) $3 = \frac{E}{4+F} + G$

(1 mark) (1 mark) (1 mark)

b)

i) $E = (23 - G)F$ ii) $E = -G(11.5 + F)$ iii) $E = (3 - G)(4 + F)$

(1 mark) (1 mark) (1 mark)

c) $(23 - G)F = -G(11.5 + F)$ (1 mark)

$23F - GF = -11.5G - GF$

$23F = -11.5G$

$G = \frac{23F}{-11.5}$

$G = -2F$ (1 mark)

d) $(23 - G)F = (3 - G)(4 + F)$ (1 mark)

Substitute G = -2F into equation

Solution Pathway

$(23 + 2F)F = (3 + 2F)(4 + F)$

$23F + 2F^2 = 12 + 3F + 8F + 2F^2$ (1 mark)

$23F = 12 + 11F$

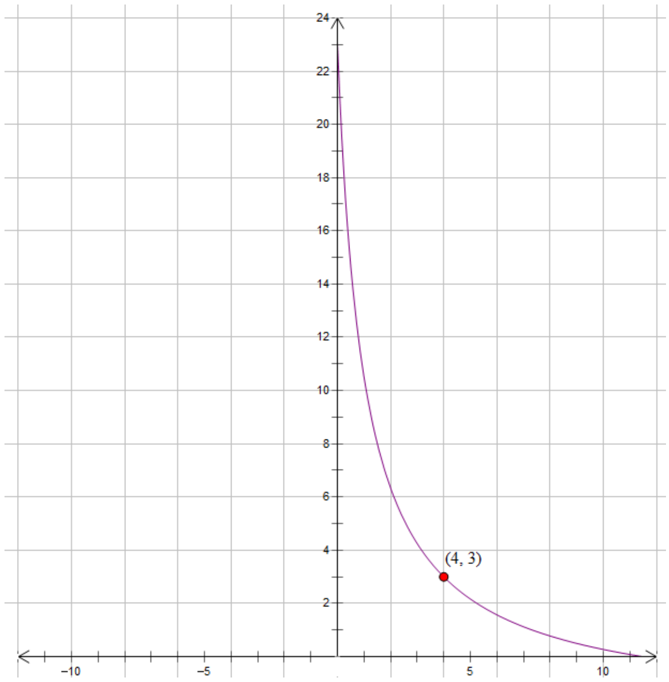
$12F = 12$

Therefore, $F = 1$ (1 mark)

$G = 2(-1) = -2$ (1 mark)

$E = (23 - -2)1$

$= 25$ (1 mark)



e) Sketch on right

(2 marks – shape, domain)

f) $j(x) = \frac{25}{x+1} - 2$ $\{x: 0 \leq x \leq 11.5\}$

(1 mark)

$j(x) = \frac{25}{-x+1} - 2$ $\{x: -11.5 \leq x < 0\}$

(1 mark)

g) See end

Solution Pathway

Part 2B: Upper Tangent (upperside of beam)

The upper side of the metal beams are tangent to the hyperbola and are parallel to the underside of the beam.

a) $j(x) = 25(x + 1)^{-1} - 2$

$j'(x) = -25(x + 1)^{-2}$ (1 mark)

Let $j'(x) = -1$ (1 mark)

$-1 = -25(x + 1)^{-2}$

$(x + 1)^2 = 25$

$x + 1 = 5$ (positive value only since right hand side specified in question) (1 mark)

Therefore $x = 4$ (1 mark)

b) $j(4) = \frac{25}{(4+1)} - 2$

$j(4) = y = 3$ (1 mark)

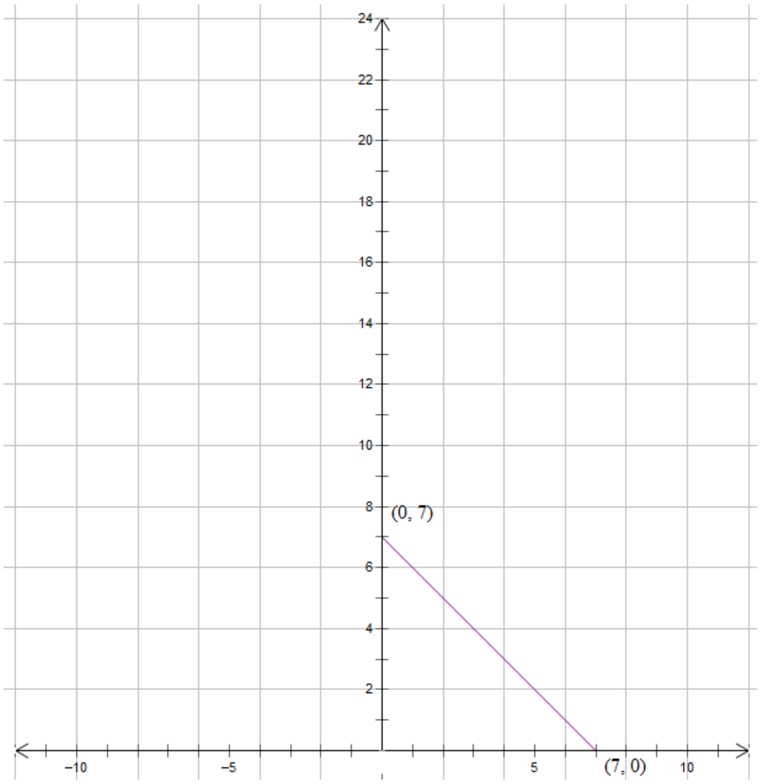
c) Substitute the point (4, 3): $3 = -4 + H$

Therefore, $H = 7$ (1 mark)

Solution Pathway

d) See right

(2 marks- shape, domain)



e) $k(x) = \begin{cases} -x+7 & \{x: -7 \le x \le 0\} \ (1\ mark) \\ x+7 & \{x: 0 < x \le 7\} \ (1\ mark) \end{cases}$

f) See end

Solution Pathway

Part 2C: Square root (flaring beams)

a) If the terminating point is at ground level, determine the values of J and K.

Since the terminating point is on the x-axis, then K = 11.5 (1 mark)

The intersection between the tangent and the hyperbola is at (4, 3), so by substituting:

$$3 = J\sqrt{11.5 - 4}$$

$$J = \frac{\sqrt{30}}{5}\left(= \frac{3}{\sqrt{7.5}} = \frac{3\sqrt{2}}{\sqrt{15}}\right)$$ (1 mark)

b) Show algebraically that $L(x) = \sqrt{\frac{69-6x}{5}}$

$$L(x) = \frac{3\sqrt{2}\,\sqrt{11.5-x}}{\sqrt{15}}$$ (1 mark)

$$L(x) = \frac{3\,\sqrt{23-2x}}{\sqrt{15}}$$

$$L(x) = \frac{3\,\sqrt{23-2x}}{\sqrt{5\times 3}}$$ (1 mark)

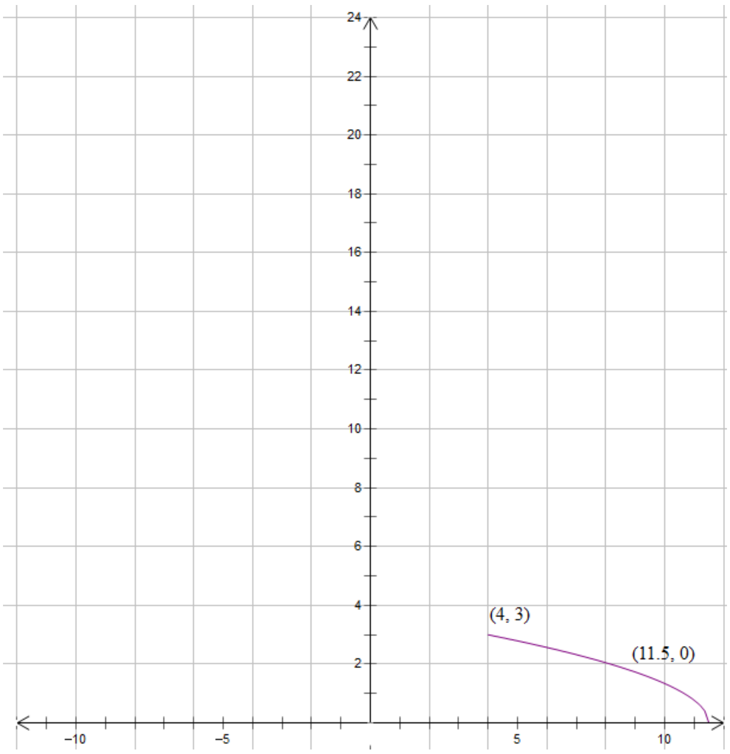
$$L(x) = \frac{\sqrt{3}\,\sqrt{23-2x}}{\sqrt{5}}$$ (1 mark)

$$L(x) = \sqrt{\frac{69-6x}{5}}$$ (as required)

Solution Pathway

c) See right

(2 marks – shape, domain)



d) $L(x) = \begin{cases} \sqrt{\frac{69+6x}{5}} & -11.5 \le x \le -4 \\ \sqrt{\frac{69-6x}{5}} & 4 \le x \le 11.5 \end{cases}$ (1 mark) (1 mark)

e) See end

Solution Pathway

Part 3A: Intersection points and midpoint (Floodlight positions)

a) The right hand point is (4, 3) and the left hand point is (-4, 3). These values were determined in Section 5 and should already be marked clearly on the final version of the sketch. (1 mark)

b) (0, 23) This is from Section 4 and should already be on the final sketch. (1 mark)

c) (0, 7) from Section 5, and already on the final sketch. (1 mark)

d) 23 – 7 = 16 metres (1 mark)

e) y-value = 7 + 8 = 15 (1 mark)

Coordinates are (0, 15) (1 mark)

f) Lower lights are 3 metres above the ground, while the higher light is at a height of 15metres

(1 mark)

The difference in height is 12 metres. (1 mark)

Solution Pathway

Part 3B: Entrance Way

a) The two upper corners of the rectangular entrance way touch the hyperbola. Hence determine an expression for the height of the rectangle, in terms of x.

Height = $\frac{25}{x+1} - 2$ (1 mark)

b) Area = Height x width

$A = \left(\frac{25}{x+1} - 2\right)(2x)$ (1 mark)

$A = \frac{50x}{x+1} - 2(2x)$ (1 mark)

$A = \frac{50x}{x+1} - 4x$ (as required)

c) Use Quotient Rule:

$\frac{dA}{dx} = \frac{(x+1)50-50x}{(x+1)^2} - 4$ (1 mark)

$\frac{dA}{dx} = \frac{50x+50-50x}{(x+1)^2} - 4$

$\frac{dA}{dx} = \frac{50}{(x+1)^2} - 4$

Let $\frac{dA}{dx} = 0$ (1 mark)

$0 = \frac{50}{(x+1)^2} - 4$

$4 = \frac{50}{(x+1)^2}$

$(x+1)^2 = 12.5$ (1 mark)

Solution Pathway

$x = \sqrt{12.5} - 1 = \frac{5}{2}\sqrt{2} - 1$ or $x = -\sqrt{12.5} - 1 = -\frac{5}{2}\sqrt{2} - 1$

However, since the context of this question denotes a physical measurement, only the first solution is valid.

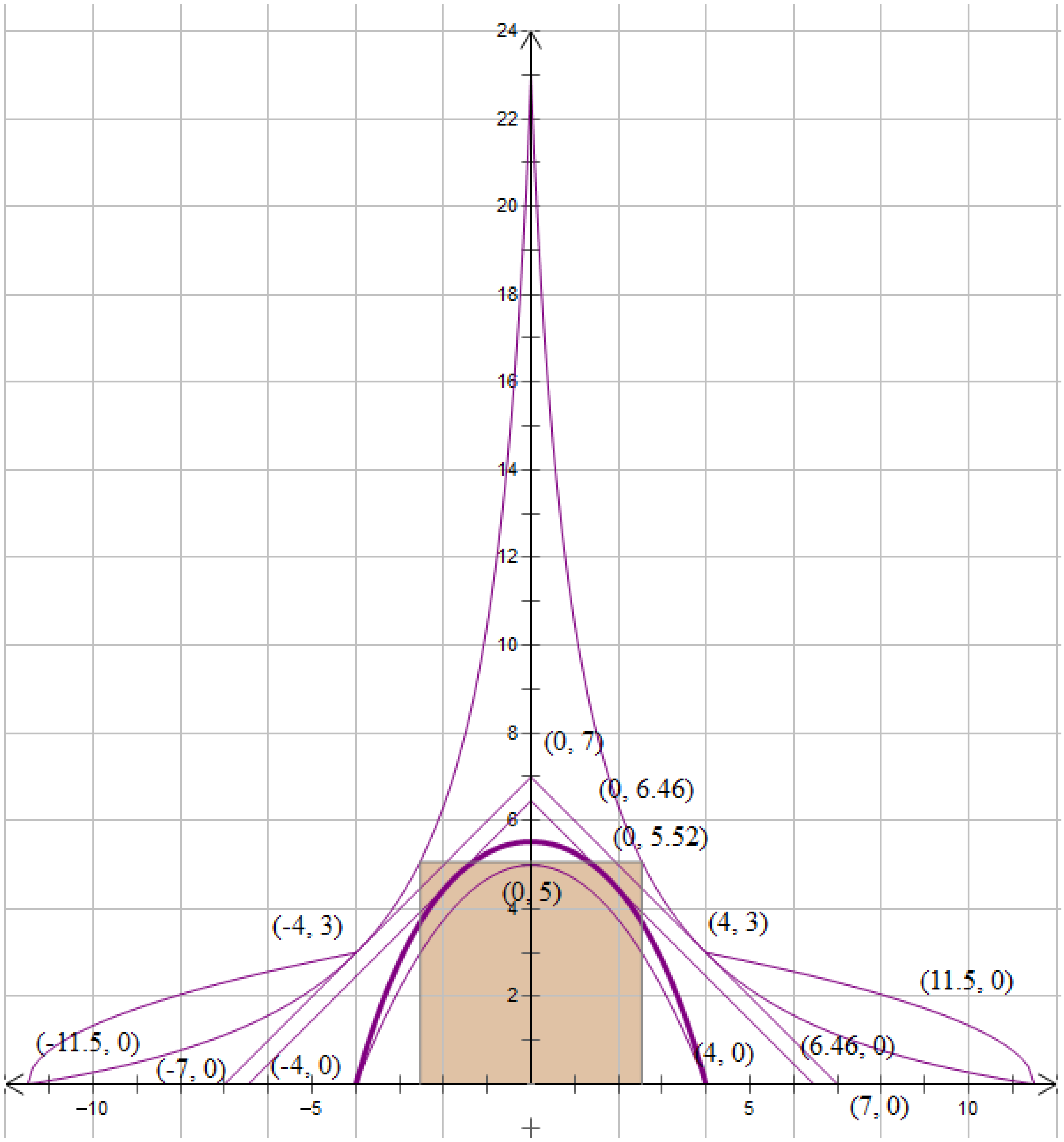
Width = $2(\sqrt{12.5} - 1)$ m = $5\sqrt{2} - 2$ m (1 mark)

Height = $\frac{25}{\sqrt{12.5}} - 2$ m = $5\sqrt{2} - 2$ m (1 mark)

d) Therefore maximum area = $54 - 20\sqrt{2}$ m^2 (1 mark)

(No mark if quoted in decimal form rather than exact. Decimal value is 25.72 to 2 decimal places)

Solution Pathway



4 marks – If all sections present on final graph. Subtract 1 mark for each section missing.

4 marks – All relevant points labelled correctly.

Solution Pathway

Part 3C: Beam Scroll Work

a) The linear part of the function is $-x + 6.73$ and represents the median line of sine curve.

b) There are 9 points with a gradient of -1. They are the points where the curve is tangent to the top and bottom straight lines.

c) $x = 0, \frac{7}{8}, \frac{14}{8}, \frac{21}{8}, \frac{28}{8}, \frac{35}{8}, \frac{42}{8}, \frac{49}{8}, \frac{56}{8}$

d) $M'(x) = -1 + \frac{2.16\pi.\cos\left(\frac{8\pi x}{7} + \frac{\pi}{2}\right)}{7}$

e) Let M'(x) = 0

$0 = -1 + \frac{2.16\pi.\cos\left(\frac{8\pi x}{7} + \frac{\pi}{2}\right)}{7}$ (1 mark)

$7 = 2.16\pi.\cos\left(\frac{8\pi x}{7} + \frac{\pi}{2}\right)$

$\frac{7}{2.16\pi} = \cos\left(\frac{8\pi x}{7} + \frac{\pi}{2}\right)$

$1.03156 = \cos\left(\frac{8\pi x}{7} + \frac{\pi}{2}\right)$ (1 mark)

Since the LHS is greater than 1 there can be no value of cos that gives this. Hence there are no stationary points. (1 mark)