The Mathematical Association of Victoria

Trial Exam 2013

MATHEMATICAL METHODS (CAS)

Written Examination 2

STUDENT NAME ____

Reading time: 15 minutes
Writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of examination

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	4	4	58
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator (memory DOES NOT need to be cleared) and, if desired, one scientific calculator. For approved computer based CAS, their full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 21 pages with a detachable sheet of miscellaneous formulas at the back
- Answer sheet for multiple-choice questions.

Instructions

- Detach the formula sheet from the back of this book during reading time.
- Write your **name** in the space provided above on this page.
- All written responses must be in English.

At the end of the examination

- Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION 1

Instructions for Section 1

Answer **all** questions in pencil on the answer sheet provided for multiple – choice questions.
Choose the response that is **correct** for the question.
A correct answer scores1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.

Question 1

The maximal domain and range of $f(x)=\dfrac{2x-1}{3-x}$ are respectively

A. $R\setminus\{-2\}$ and $R\setminus\{3\}$
B. $R\setminus\{-3\}$ and $R\setminus\{2\}$
C. $R\setminus\{2\}$ and $R\setminus\{-3\}$
D. $R\setminus\{3\}$ and $R\setminus\{-2\}$
E. $R\setminus\{2\}$ and $R\setminus\{3\}$

Question 2

The equation(s) of the asymptote(s) of the graph with equation $f(x)=\log_e\left(|x|-1\right)$ is/are

A. $x=-1$ only
B. $x=1$ only
C. $x=-1$ and $x=1$
D. $y=-1$ only
E. $y=1$ only

Question 3

If $f: R^+ \to R, f(x) = \log_e(x)$ and $g: \left(\frac{1}{2}, \infty\right) \to R,\ g(x) = (2x-1)^2$ then the transformations required to obtain the graph of $f\left(g(x)\right)$ from the graph of f are

A. a dilation of a factor of $\frac{1}{2}$ from the y-axis followed by a translation of $\frac{1}{2}$ to the right.

B. a dilation of a factor of 2 from the x-axis, a dilation of a factor of $\frac{1}{2}$ from the y-axis followed by a translation of $\frac{1}{2}$ to the right.

C. a dilation of a factor of 2 from the x-axis, a dilation of a factor of 2 from the y-axis followed by a translation of $\frac{1}{2}$ to the right.

D. a dilation of a factor of 2 from the x-axis, a dilation of a factor of $\frac{1}{2}$ from the y-axis followed by a translation of 1 unit to the right.

E. a dilation of a factor of $\frac{1}{2}$ from the x-axis, a dilation of a factor of 2 from the y-axis followed by a translation of $\frac{1}{2}$ to the left.

Question 4

If the graph of f has a stationary point of inflection on the x-axis at $x = -2$, passes through the origin and is of the form $f(x) = ax^4 + bx^3 - 24x^2 + cx$, where a, b and c are real constants then b equals

A. – 16

B. – 12

C. – 8

D. – 2

E. 12

Question 5

The graph of the image of $y = \tan(x)$ under a translation of $\frac{\pi}{3}$ in the negative direction of the *x* axis followed by a reflection in the *y* axis has the following characteristics

A. an asymptote of $x = -\frac{\pi}{6}$ and a *y* intercept at $\left(0, \sqrt{3}\right)$.

B. an asymptote of $x = \frac{\pi}{6}$ and an *x* intercept at $\left(\frac{\pi}{3}, 0\right)$.

C. an equation of $y = \tan\left(x - \frac{\pi}{3}\right)$.

D. an equation of $y = -\tan\left(x + \frac{\pi}{3}\right)$.

E. a *y* intercept at $\left(0, -\sqrt{3}\right)$ and an *x* intercept at $\left(\frac{\pi}{3}, 0\right)$.

Question 6

The range of the graph of $f(x) = 2\sin(ax - b) - c$, where $\{a, b, c\} \in R$, is

A. $\left[-(a+b),\ (a-b)\right]$

B. $\left[-(a+c),\ (a-c)\right]$

C. $\left[-(2+b),\ (2-b)\right]$

D. $\left[-(2+c),\ (2-c)\right]$

E. $\left[0,\ \frac{2\pi}{3}\right]$

Question 7

Using the Linear Approximation formula $f(x+h) \approx f(x) + hf'(x)$, where $f(x) = \sqrt[3]{x}$, the approximate change in *f* as *x* changes from 8 to 8.1 is

A. 0.1

B. $2\frac{1}{120}$

C. $\frac{1}{120}$

D. $\sqrt[3]{8.1} - 2$

E. 0.008

Question 8

The graph of a cubic function g has turning points at $(a,-3)$ and $(-b,2)$ where a and b are positive real constants. The values of c such that the graph of $f(x)+c=0$ will have exactly one solution are

A. $\{c : c \leq 2\} \cup \{c : c \geq 3\}$
B. $\{c : c < 2\} \cup \{c : c > 3\}$
C. $\{c : c < -3\} \cup \{c : c > -2\}$
D. $\{c : -2 < c < 3\}$
E. $\{c : c < -2\} \cup \{c : c > 3\}$

Question 9

The graph of $g:[-8,8] \to R$, where $g(x) = x^2 - 8|x| + 12$, is **not** differentiable when

A. $x = 0$ only
B. $x = -8,\ x = 0,\ x = 8$
C. $x = -6,\ x = -2,\ x = 2,\ x = 6$
D. $x = -8,\ x = 2,\ x = 6,\ x = 8$
E. $x = 2,\ x = 6$

Question 10

The area under the curve with rule $g(x) = e^{-x}$ between $x = 0$ and $x = 2$ can be approximated using left endpoint rectangles of width 0.5. This area is

A. $0.5\left(1 + \frac{1}{e^{0.5}} + \frac{1}{e} + \frac{1}{e^{1.5}}\right)$

B. $0.5\left(\frac{1}{e^{0.5}} + \frac{1}{e} + \frac{1}{e^{1.5}} + \frac{1}{e^{2}}\right)$

C. $1 + e^{0.5} + e + e^{1.5}$

D. $1 + \frac{1}{e^{0.5}} + \frac{1}{e} + \frac{1}{e^{1.5}}$

E. $1 - \frac{1}{e^{2}}$

Question 11

The average value of $f:[a,b] \to R$, where $f(x) = x - 1$, where a and b are real numbers, will not equal zero over which one of the following intervals?

A. $[-1,3]$

B. $[0,2]$

C. $\left[\frac{1}{2},\frac{3}{2}\right]$

D. $[-2,2]$

E. $[-4,6]$

Question 12

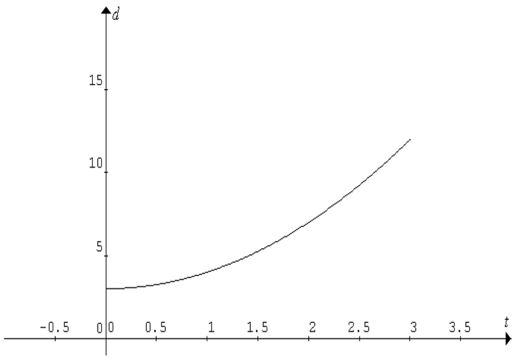
If $\int_1^3 (f(x))dx = 5$, then $2\int_1^3 (f(x) - 1)dx$ equals

A. 3
B. 6
C. 8
D. 9
E. 10

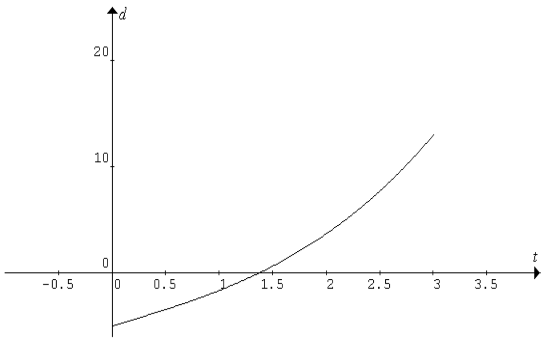
Question 13

The acceleration, a ms^{-2} of a body at time t s is given by the rule $a = t^2 + 3,\ t \geq 0$. If the body has an initial velocity of $-5\ \text{ms}^{-1}$, which one of the following graphs shows its displacement, d m, against time?

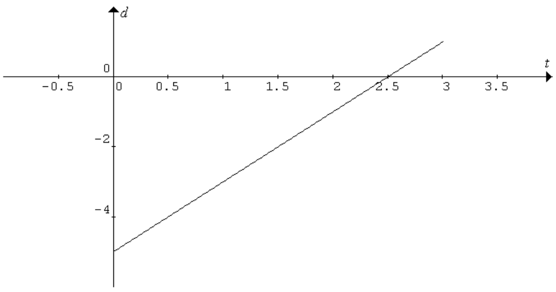
A.



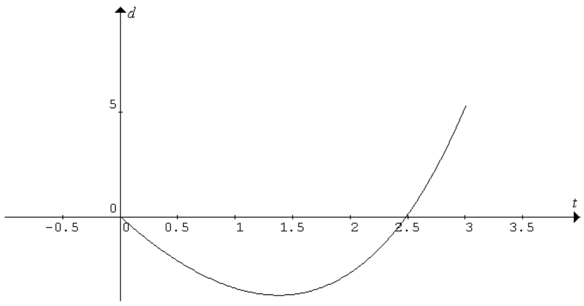
B.



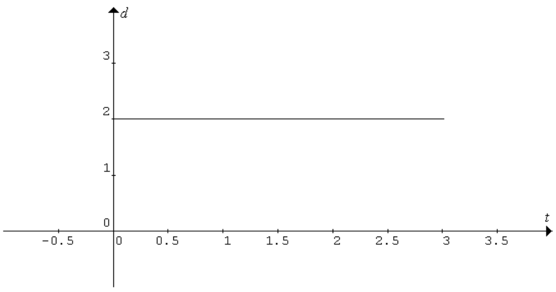
C.



D.



E.



Question 14
For a random variable *X*, $E(X) = a$ and $E(X^2) = 1.44$. The standard deviation of *X* is equal to

A. 1.2
B. $a + 1.2$
C. $1.2 - a$
D. $1.44 - a^2$
E. $\sqrt{1.44 - a^2}$

Question 15
In a certain population the probability a person plays a musical instrument is 0.35. What is the probability correct to four decimal places that, from 60 randomly selected people from this population, less than 25 play a musical instrument?

A. 0.9638
B. 0.0587
C. 0.8874
D. 0.8286
E. 0.1126

Question 16
The probability Collingbush wins a football game given it won the previous game is 0.75. The probability it loses a game given it lost the previous game is 0.55. If the probability that Collingbush wins its first game is 0.8, the probability it wins the third game of the season is

A. 0.6471
B. 0.3529
C. 0.692
D. 0.343
E. 0.657

Question 17
The flowering period for a particular species of plant is between 2 and 9 weeks after the start of Spring. The probability distribution of the flowering period is given by the function

$f(t)=\begin{cases}\dfrac{6}{343}(t-2)(9-t) & \text{for } 2 \le t \le 9 \\ 0 & \text{elsewhere}\end{cases}$. The probability flowers first appear within the first 4 weeks of the flowering period is best described by

A. $\dfrac{6}{343}\displaystyle\int_0^4 \frac{1}{f(t)}dt$

B. $\dfrac{6}{343}\displaystyle\int_4^9 f(t)dt$

C. $-\displaystyle\int_6^2 f(t)dt$

D. $\displaystyle\int_0^4 (f(t))^2 dt$

E. $\dfrac{6}{343}\displaystyle\int_0^4 f(t)dt$

Question 18
The continuous random variable X has probability density function f given by

$f(x)=\begin{cases}|a(x-1)| & 1 \le x \le 3 \\ 0 & \text{elsewhere}\end{cases}$. The value of a is

A. $\dfrac{1}{2}$

B. $\dfrac{2}{5}$

C. $-\dfrac{2}{5}$

D. $\dfrac{1}{4}$

E. 2

Question 19
The variance, correct to four decimal places, of the random variable X which has a probability density function of $f(x)=\begin{cases} 0 & x<1 \cup x>\sqrt{3} \\ x^3-x & 1\le x\le\sqrt{3}\end{cases}$ is

A. 1.5275
B. 2.1542
C. 0.9024
D. 0.1614
E. 0.0261

Question 20
X is a continuous random variable, which is normally distributed with a mean of 13.4 and standard deviation of 3.2. The values a_1 and a_2, which are evenly distributed either side of the mean, are such that $\Pr(a_1 < X < a_2) = 0.85$. The values of a_1 and a_2, correct to two decimal places are respectively

A. 8.79 and 18.01
B. 10.08 and 16.72
C. 9.09 and 22.49
D. -5.39 and 8.01
E. 0.00 and 13.40

Question 21
A discrete random variable X has a probability function $\Pr(X=a)=(1-p)p^a$ where a is a non-negative integer. $\Pr(X>1)$ is equal to

A. $3p-4p^2$
B. $4p^2-3p$
C. $3-4p^2$
D. $(1-2p)^2$
E. $1-3p+4p^2$

Question 22
A right circular cone-shaped tank has a radius of 1000 cm and a height of 3000 cm. Water drains from the tank at a constant rate of 750 cm^3/min. If h cm is the height of water at t minutes, the rate at which the water level is dropping is given by

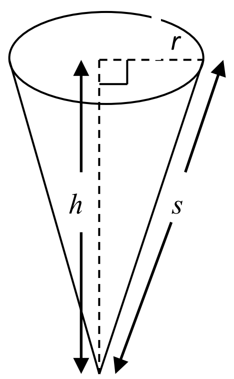
A. $\dfrac{250\pi h^3}{9}$
B. $\dfrac{20250}{\pi h^3}$
C. $\dfrac{6750}{\pi h^2}$
D. $\dfrac{250\pi h^2}{3}$
E. 250

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (14 marks)
Tasmania Jones owns an ice-cream company. He manufactures ice-cream cones as shown in the diagram below.



r is the radius of the cone, s the slant edge and h the height of the cone. All measurements are in centimetres. Tasmania wants the **curved** surface area of each of the cones to be 100 cm^2.
The total surface area of a cone, TSA = $\pi rs + \pi r^2$.

a. Show that $h = \dfrac{\sqrt{10\ 000 - \pi^2 r^4}}{\pi r}$.

3 marks

b. Hence, show that the volume of the cone, V cm^3, in terms of r is given by the rule

$$V = \frac{r\sqrt{10\ 000 - \pi^2 r^4}}{3}.$$

1 mark

c. State the domain of V.

2 marks

d. Find the maximum value for the volume in cm^3 and the radius, in cm, which gives the maximum volume.

2 marks

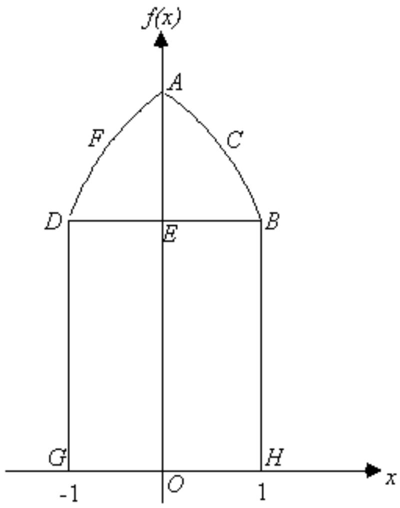
e. Sketch the graph of V against r on the set of axes below. Clearly label the end points and stationary point with their coordinates.

3 marks

f. If Tasmania wants to produce 1000 cones with maximum volume how much wrapping paper will be required to cover the cones. Assume there is no overlay and the top of each cone is covered. Give you answer correct to the nearest square centimetre.

3 marks

Question 2 (13 marks)
The diagram represents the outline of a stained glass window.



The equation $f(x)=\frac{5}{2}\cos\left(\frac{11}{10}\left(x+\frac{2}{5}\right)\right)+5, 0\le x\le 1$ models the curve *ACB*.

All measurements are in metres. **Give answers correct to 3 decimal places for non-integer values.**

a. i. State the coordinates of the points *A* and *B*.

2 marks

ii. The curve *AFD* is a reflection of $f(x)$ in the *y* axis. Write the equation of this second curve in the form $g(x)=\frac{5}{2}\cos(bx+c)+5$.

1 mark

iii. State the domain of $g(x)$.

1 mark

b. **i.** Find the area between by the curve of $f(x)$, the x and y axes and the line $x = 1$.

1 mark

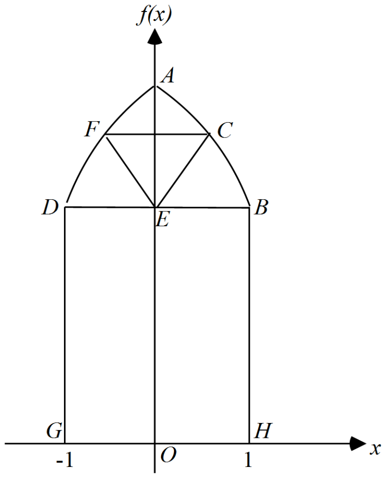
ii. Find the total surface area of the window.

1 mark

iii. Find the area between the curve ACB, the y axis and the line $y = a$, which passes through B.

2 marks

Reinforcements in the form of thin straight line metal rod segments extend from A to O, D to B, C to E, E to F and F to C, where C and F are at the same height above the base of the window. The x-coordinate of C is 0.500.



c. **i.** Find the coordinates of C and F.

2 marks

ii. Hence, find the total length of metal rods used to reinforce the window, correct to nearest cm.

3 marks

Question 3 (15 marks)
A manufacturing process produces aluminium cylinders. The cylinders are considered defective if the diameter is greater than 15.2 mm or less than 14.7 mm. 4% are defective with diameters that are too large and 6% are defective as being too small.

a. i. What percentage of cylinders are non-defective?

1 mark

ii. Show that the mean and standard deviation of the diameters of cylinders are 14.935 mm and 0.151 mm respectively, correct to three decimal places, assuming the diameters are normally distributed. Hence, find the range between which 95% of cylinders lie.

4 marks

iii. Find the probability that a cylinder's diameter lies between 14.8 mm and 15.0 mm. Give your answer correct to two decimal places.

1 mark

Cylinders are packed into boxes of 8. A box of cylinders is considered acceptable if it contains no more than one defective cylinder.

b. **i.** Find the probability, correct to four decimal places, that a box of cylinders, selected at random, is accepted.

2 marks

ii. A box of 8 cylinders gives a profit of \$65 if accepted, otherwise it incurs a loss of \$45. What is the expected profit or loss per box? Give your answer correct to the nearest cent.

2 marks

iii. If three successive boxes are accepted from a large consignment, the entire consignment is passed. What is the probability, correct to four decimal places, the consignment is passed?

1 mark

The company opens a new factory that uses two suppliers of raw materials. Supplier *M* supplies $\frac{5}{8}$ of total raw materials and supplier *N* supplies the remainder. It is estimated that 95% of cylinders manufactured from supplier *M* are non-defective while 88% of cylinders manufactured from supplier *N* are non-defective.

c. i. What percentage of non-defective cylinders are produced from the new factory after two manufacturing runs? Give your answer correct to the nearest percent.

2 marks

ii. In the long run, what overall percentage of cylinders from the new factory are non-defective? Give your answer correct to the nearest percent.

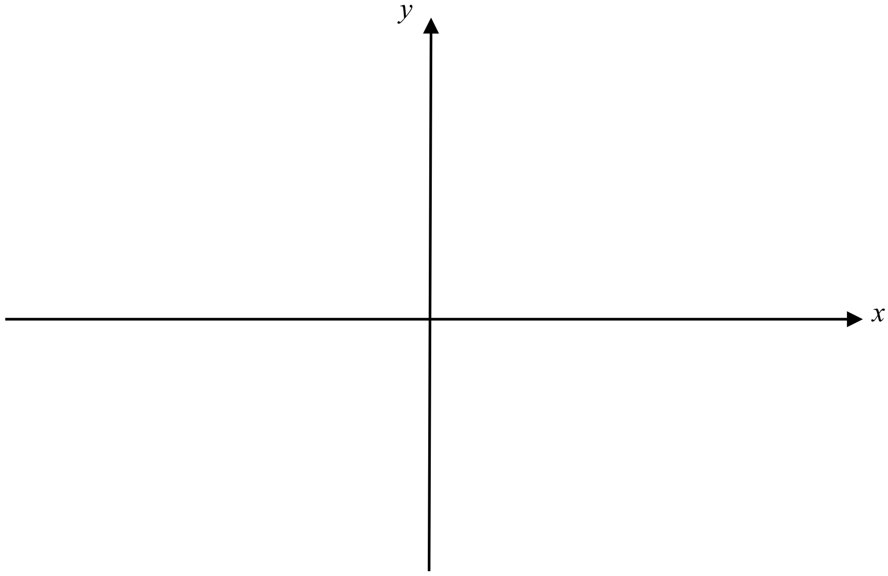
1 mark

iii. Which of the two factories produces less defective cylinders? Explain.

1 mark

Question 4 (16 marks)

a. Sketch the graphs of $f(x)=\left|1-\frac{1}{x}\right|$ and $g(x)=x$ on the set of axes below. Clearly label the asymptotes with their equations and the point of intersection and the axial intercepts with their coordinates.



4 marks

Working Space

b. Write *f* in hybrid form.

$$f(x)=\begin{cases} \_\_\_\_\_\_\_\_\_\_\_\_, & x<0 \ \cup \ \_\_\_\_\_\_\_\_\_\_ \\ \_\_\_\_\_\_\_\_\_\_\_\_, & \_\_\_\_\_\_\_\_\_\_\_\_\_\_ \end{cases}$$

2 marks

c. Find the area bounded by the curves of f and g and the x-axis.

3 marks

Consider $y = x + k$, where k is a real constant.

d. Find the value of k, correct to one decimal place, so that the area bounded by the curve of f, the line $y = x + k$ and the x-axis is the same as that in **part c.**. The area in **part c.** is 0.2902 correct to four decimal places.

4 marks

e. Find the values of k so that $y = x + k$ intersects f three times.

3 marks

END OF QUESTION AND ANSWER BOOK