# Year 2013 VCE Mathematical Methods Trial Examination 1



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# Victorian Certificate of Education 2013

### STUDENT NUMBER

					Lette	r
Figures						
Words						

## **MATHEMATICAL METHOD CAS**

### **Trial Written Examination 1**

Reading time: 15 minutes Total writing time: 1 hour

### QUESTION AND ANSWER BOOK

### **Structure of book**

Number of	Number of questions	Number of
questions	to be answered	marks
11	11	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

### Materials supplied

- Question and answer book of 12 pages with a detachable sheet of miscellaneous formulas at the end of this booklet.
- Working space is provided throughout the booklet.

### **Instructions**

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

### **Instructions**

Answer all questions in the spaces provided.

In all questions where a numerical answer is required an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown. Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1	2 + 2 = 4 marks		
<b>a.</b> If $f(x) = \log_e(\cos(3x))$ , find $f'(\frac{\pi}{18})$ .			
<b>b.</b> If $y = \frac{\sin(4x)}{2x^2}$ and $\frac{dy}{dx} = \frac{g(x)}{x^3}$ , find the function $g(x)$ .			
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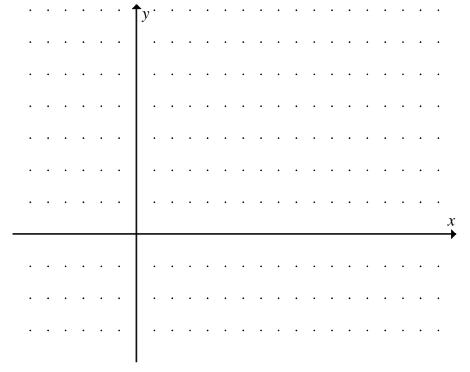
Que	estion 2	3 marks
Cons	sider the linear simultaneous equations	
kx -	-4y=2	
3x-	-(k+4)y = k+1	
whe	re $k$ is a real constant.	
i.	Find the value(s) of $k$ , for which there is a unique solution.	
ii.	Find the value(s) of $k$ , for which there is no solution.	
Que	estion 3	3 marks
A ce	ertain curve has a gradient equal to $\frac{2}{\sqrt{4x+9}}$ . Find the equation of the curve	which
pass	es through the origin.	

**Question 4** 2 + 3 = 5 marks

a. Let  $f: R \to R$ ,  $f(x) = 1 - 2\cos\left(\frac{\pi x}{6}\right)$ .

Find the general solution for the *x*-coordinates where the graph of y = f(x) crosses the *x*-axis.

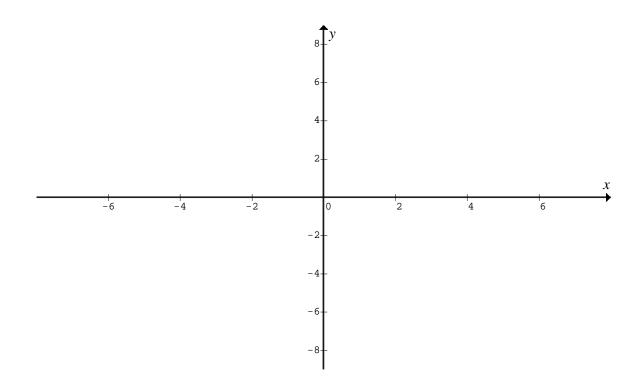
**b.** Sketch the graph of  $g:[0,12] \to R$ ,  $g(x)=1-2\cos\left(\frac{\pi x}{6}\right)$ , stating the amplitude, period and the range. Clearly label the axes scale, intercepts and the coordinates of any turning points and endpoints.



Question 5	3 marks
Solve the equation $\log_x 5 + \log_5 x = \frac{5}{2}$ for x.	
<del>-</del>	
Question 6	3 marks
	3 marks
Question 6 The rule for function $f$ is $f(x) = 3e^{-2x} - 4$ . Find the inverse function $f^{-1}$ .	3 marks
	3 marks

**Question 7** 3 + 2 = 5 marks

a. Sketch the graph of  $y = 3 - \frac{12}{(x+2)^2}$  on the axes below, clearly indicating all axial cuts and the equation of any asymptotes, stating the maximal domain and range.



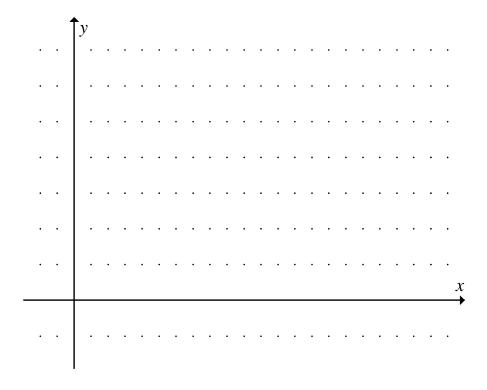
**b.** Describe in words, giving scale factors, the transformations in a suitable order, required to sketch the graph of  $y = 3 - \frac{12}{(x+2)^2}$  from the graph of  $y = \frac{1}{x^2}$ .

**Question 8** 2+1=3 marks

The probability distribution function for the continuous random variable X is given by

$$f(x) = \begin{cases} \frac{1}{16} (4 - |x - 4|) & \text{for } 0 \le x \le 8 \\ 0 & \text{elsewhere} \end{cases}$$

**a.** Sketch the graph of f(x) on the axes below.



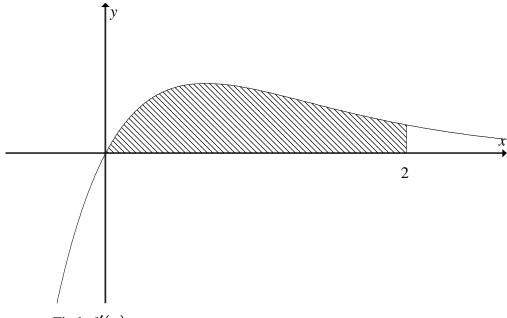
**b.** Find Pr(X < 2 | X < 6)

Question 9 2 marks

Two independent events, A and B, are such that  $\Pr(A) = \frac{1}{3}$  and  $\Pr(B) = \frac{2}{5}$ . If A' and B' denote the complements of A and B respectively, calculate  $\Pr(A' \cup B')$ .

**Question 10** 1 + 2 + 3 = 6 marks

The graph of the function  $f: R \to R$ ,  $f(x) = xe^{-2x}$  is shown below.



**a.** Find f'(x).

\_\_\_\_\_

b.	Find the coordinates of the stationary point on the graph of $y = xe^{-2x}$ .
c.	Use your answer to <b>part a.</b> to find the area of the shaded region.

Question 11 3 marks
A binomial distribution, of the discrete random variable $X$ has $n$ independent trials,
with $p$ as the probability of a success on any trial, where $\Pr(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$ .
It can be shown that $\frac{\Pr(X = k + 1)}{\Pr(X = k)} = \frac{(n - k)p}{(k + 1)(1 - p)}.$
For a certain binomial distribution with 8 independent trials, $Pr(X = 5)$ is the mode.
Find the values of $p_1$ and $p_2$ , where $p_1 .$

# END OF QUESTION AND ANSWER BOOKLET END OF EXAMINATION

# MATHEMATICAL METHODS CAS

# Written examination 1

### **FORMULA SHEET**

### **Directions to students**

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

### **Mathematical Methods CAS Formulas**

### Mensuration

area of a trapezium:  $\frac{1}{2}(a+b)h$  volume of a pyramid:  $\frac{1}{3}Ah$ 

curved surface area of a cylinder:  $2\pi rh$  volume of a sphere:  $\frac{4}{3}\pi r^3$ 

volume of a cylinder:  $\pi r^2 h$  area of triangle:  $\frac{1}{2}bc\sin(A)$ 

volume of a cone:  $\frac{1}{3}\pi r^2 h$ 

### **Calculus**

$$\frac{d}{dx}(x^{n}) = nx^{n-1}$$

$$\int x^{n} dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$$

$$\int \frac{1}{x} dx = \log_{e}|x| + c$$

$$\int \frac{1}{x} dx = \log_{e}|x| + c$$

$$\int \sin(ax) dx = -\frac{1}{a}\cos(ax) + c$$

$$\int \cos(ax) dx = \frac{1}{a}\sin(ax) + c$$

$$\int \cos(ax) dx = \frac{1}{a}\sin(ax) + c$$

$$\int \cos(ax) dx = \frac{1}{a}\sin(ax) + c$$

product rule:  $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$ 

quotient rule:  $\frac{d}{dx} \left( \frac{u}{v} \right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$ 

Chain rule:  $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$ 

approximation:  $f(x+h) \approx f(x) + h f'(x)$ 

### **Probability**

$$\Pr(A) = 1 - \Pr(A')$$

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$

$$\Pr(A/B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$
**Transition Matrices**  $S_n = T^n \times S_0$ 

mean:  $\mu = E(X)$  variance:  $\operatorname{var}(X) = \sigma^2 = E((X - \mu)^2) = E(X^2) - \mu^2$ 

probabi	lity distribution	mean	variance	
discrete	$\Pr(X=x) = p(x)$	$\mu = \sum x  p(x)$	$\sigma^2 = \sum (x - \mu)^2 p(x)$	
continuous	$\Pr(a < X < b) = \int_{a}^{b} f(x) dx$	$\mu = \int_{-\infty}^{\infty} x f(x) dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx$	