Year 2013

VCE

Mathematical Methods

Trial Examination 1

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Victorian Certificate of Education
2013

STUDENT NUMBER

									Letter
Figures									
Words									

MATHEMATICAL METHOD CAS

Trial Written Examination 1

Reading time: 15 minutes
Total writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
11	11	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

Materials supplied

- Question and answer book of 12 pages with a detachable sheet of miscellaneous formulas at the end of this booklet.
- Working space is provided throughout the booklet.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 2 + 2 = 4 marks

a. If $f(x) = \log_e(\cos(3x))$, find $f'\left(\frac{\pi}{18}\right)$.

b. If $y = \frac{\sin(4x)}{2x^2}$ and $\frac{dy}{dx} = \frac{g(x)}{x^3}$, find the function $g(x)$.

Question 2 3 marks

Consider the linear simultaneous equations

$kx - 4y = 2$

$3x - (k+4)y = k+1$

where k is a real constant.

i. Find the value(s) of k, for which there is a unique solution.

ii. Find the value(s) of k, for which there is no solution.

Question 3 3 marks

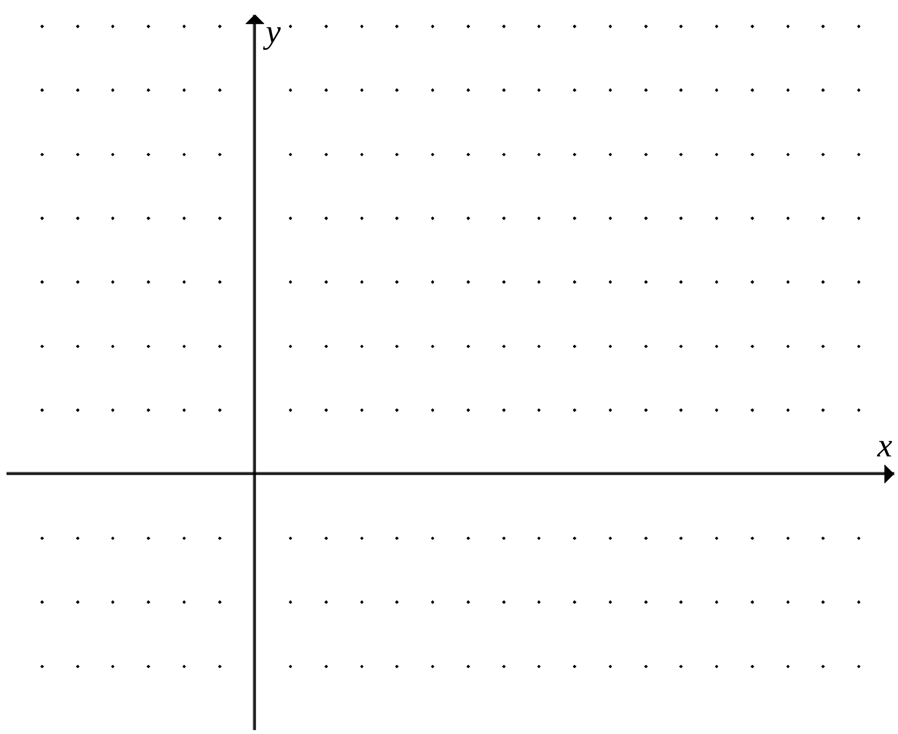
A certain curve has a gradient equal to $\dfrac{2}{\sqrt{4x+9}}$. Find the equation of the curve which passes through the origin.

Question 4 2 + 3 = 5 marks

a. Let $f:R \to R$, $f(x)=1-2\cos\left(\frac{\pi x}{6}\right)$.

Find the general solution for the *x*-coordinates where the graph of $y=f(x)$ crosses the *x*-axis.

b. Sketch the graph of $g:[0,12] \to R$, $g(x)=1-2\cos\left(\frac{\pi x}{6}\right)$, stating the amplitude, period and the range. Clearly label the axes scale, intercepts and the coordinates of any turning points and endpoints.



Question 5 3 marks

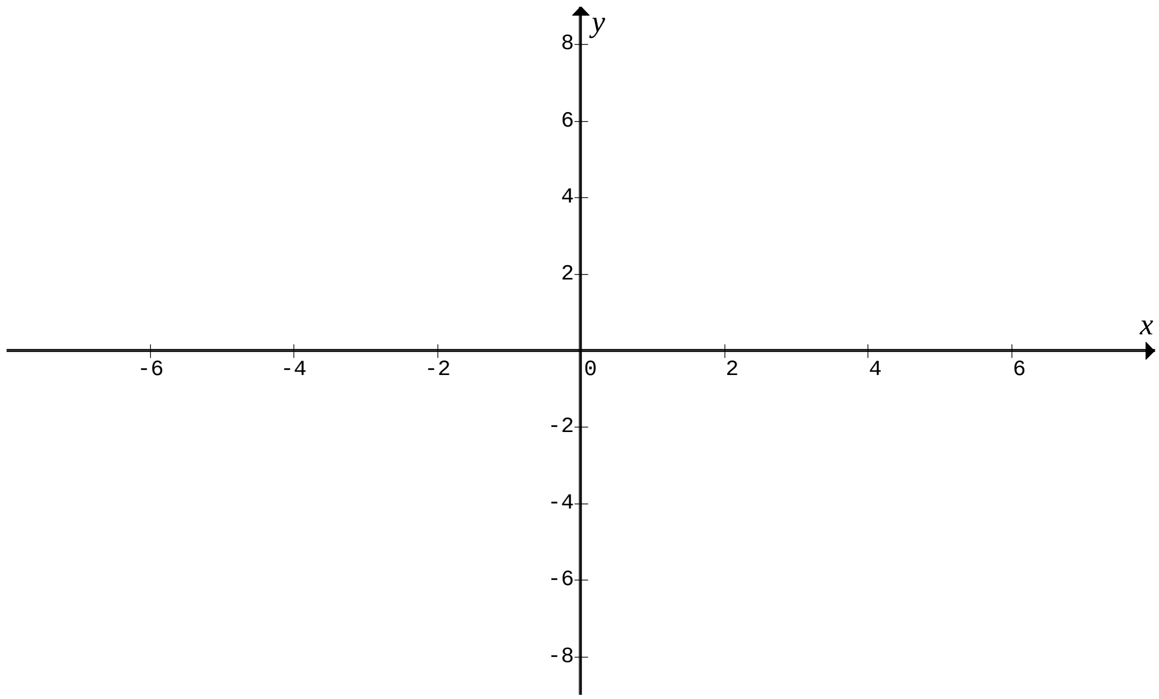
Solve the equation $\log_x 5 + \log_5 x = \frac{5}{2}$ for *x*.

Question 6 3 marks

The rule for function f is $f(x) = 3e^{-2x} - 4$. Find the inverse function f^{-1}.

Question 7 3 + 2 = 5 marks

a. Sketch the graph of $y=3-\frac{12}{(x+2)^2}$ on the axes below, clearly indicating all axial cuts and the equation of any asymptotes, stating the maximal domain and range.



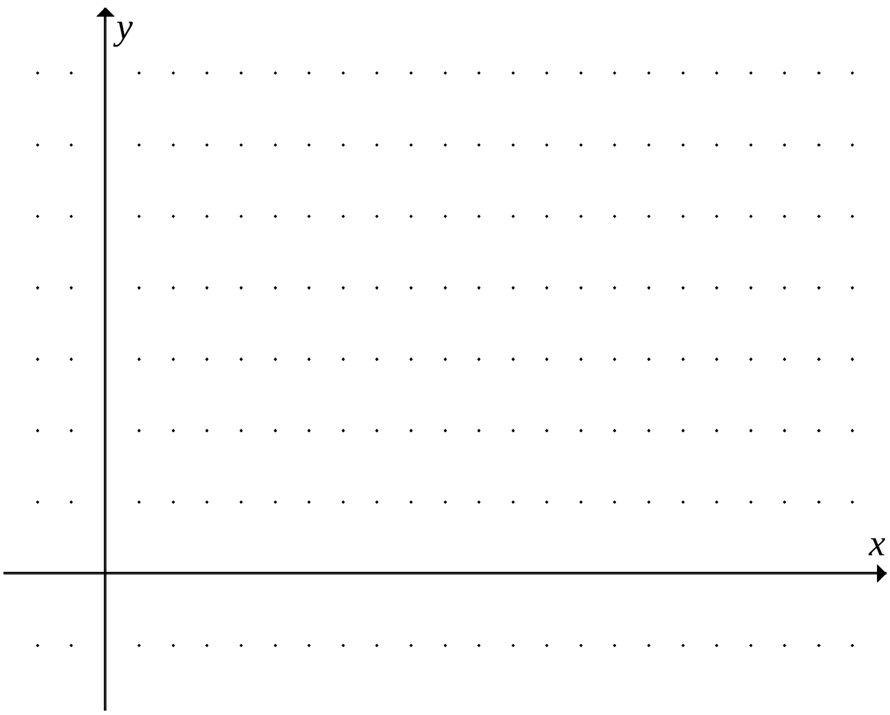
b. Describe in words, giving scale factors, the transformations in a suitable order, required to sketch the graph of $y=3-\frac{12}{(x+2)^2}$ from the graph of $y=\frac{1}{x^2}$.

Question 8 2 + 1 = 3 marks

The probability distribution function for the continuous random variable X is given by

$$f(x)=\begin{cases}\frac{1}{16}\left(4-\left|x-4\right|\right) & \text{for } 0\le x\le 8\\ 0 & \text{elsewhere}\end{cases}$$

a. Sketch the graph of $f(x)$ on the axes below.



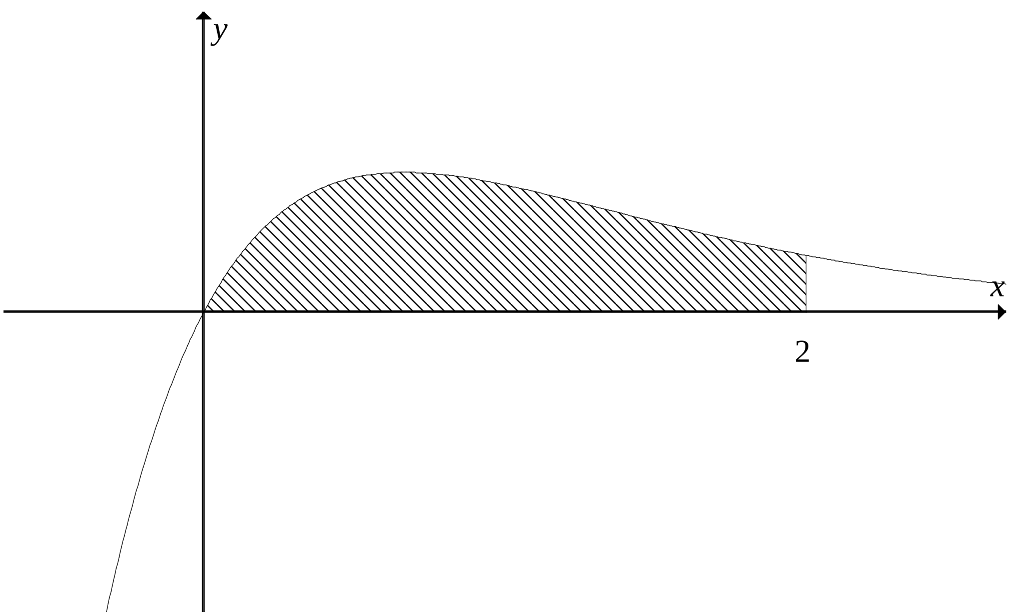
b. Find $\Pr(X<2\mid X<6)$

Question 9 2 marks

Two independent events, *A* and *B*, are such that $\Pr(A) = \frac{1}{3}$ and $\Pr(B) = \frac{2}{5}$. If A' and B' denote the complements of *A* and *B* respectively, calculate $\Pr(A' \cup B')$.

Question 10 1 + 2 + 3 = 6 marks

The graph of the function $f: R \to R$, $f(x) = xe^{-2x}$ is shown below.



a. Find $f'(x)$.

b. Find the coordinates of the stationary point on the graph of $y = xe^{-2x}$.

c. Use your answer to **part a.** to find the area of the shaded region.

Question 11 3 marks

A binomial distribution, of the discrete random variable X has n independent trials, with p as the probability of a success on any trial, where $\Pr(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$.

It can be shown that $\dfrac{\Pr(X = k+1)}{\Pr(X = k)} = \dfrac{(n-k)p}{(k+1)(1-p)}$.

For a certain binomial distribution with 8 independent trials, $\Pr(X = 5)$ is the mode.

Find the values of p_1 and p_2, where $p_1 < p < p_2$.

END OF QUESTION AND ANSWER BOOKLET

END OF EXAMINATION

MATHEMATICAL METHODS CAS

Written examination 1

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

Mathematical Methods CAS Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$	volume of a pyramid:	$\frac{1}{3}Ah$
curved surface area of a cylinder:	$2\pi rh$	volume of a sphere:	$\frac{4}{3}\pi r^3$
volume of a cylinder:	$\pi r^2 h$	area of triangle:	$\frac{1}{2}bc\sin(A)$
volume of a cone:	$\frac{1}{3}\pi r^2 h$		

Calculus

$$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$$

$$\int x^n dx=\frac{1}{n+1}x^{n+1}+c\ ,\ n\neq -1$$

$$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$$

$$\int e^{ax}dx=\frac{1}{a}e^{ax}+c$$

$$\frac{d}{dx}\left(\log_e(x)\right)=\frac{1}{x}$$

$$\int \frac{1}{x}dx=\log_e|x|+c$$

$$\frac{d}{dx}\left(\sin(ax)\right)=a\cos(ax)$$

$$\int \sin(ax)\,dx=-\frac{1}{a}\cos(ax)+c$$

$$\frac{d}{dx}\left(\cos(ax)\right)=-a\sin(ax)$$

$$\int \cos(ax)\,dx=\frac{1}{a}\sin(ax)+c$$

$$\frac{d}{dx}\left(\tan(ax)\right)=\frac{a}{\cos^2(ax)}=a\sec^2(ax)$$

product rule: $\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$

Chain rule: $\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$

approximation: $f(x+h)\approx f(x)+hf'(x)$

Probability

$\Pr(A)=1-\Pr(A')$

$\Pr(A\cup B)=\Pr(A)+\Pr(B)-\Pr(A\cap B)$

$\Pr(A/B)=\frac{\Pr(A\cap B)}{\Pr(B)}$

Transition Matrices $S_n=T^n\times S_0$

mean: $\mu=E(X)$

variance: $\operatorname{var}(X)=\sigma^2=E\left((X-\mu)^2\right)=E\left(X^2\right)-\mu^2$

probability distribution		mean	variance
discrete	$\Pr(X=x)=p(x)$	$\mu=\sum x\,p(x)$	$\sigma^2=\sum(x-\mu)^2\,p(x)$
continuous	$\Pr(a<X<b)=\int_a^b f(x)\,dx$	$\mu=\int_{-\infty}^{\infty}x\,f(x)\,dx$	$\sigma^2=\int_{-\infty}^{\infty}(x-\mu)^2\,f(x)\,dx$