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Mathematical Methods(CAS)

2013

Trial Examination 2

SECTION 1 Multiple-choice questions

Instructions for Section 1

Answer **all** questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.

Question 1 The asymptote(s) of the graph of $f:[-1,3)\to R, f(x)=\dfrac{3x-7}{x-3}$ is/are

A. $x=3$ only

B. $x=-3$ only

C. $x=3$ and $y=3$

D. $x=-3$ and $y=-3$

E. $x=-3$ and $y=3$

Question 2 $\log_{\frac{1}{e}}\left(\sqrt[3]{e}\times 3\right)$ can be simplified to

A. $3-\log_e 3$

B. $-3-\log_e 3$

C. $\dfrac{1}{3}-\log_e 3$

D. $-\dfrac{1}{3}-\log_e 3$

E. $\dfrac{1}{3}+\log_e 3$

Question 3 The solution to the equation $\sqrt{a-3x}+\log_b(3x)=\log_b a$, where $a,b\in R^+$, is

A. 0

B. 1

C. $\dfrac{1}{3}$

D. $\dfrac{a}{3}$

E. $-\dfrac{3}{a}$

Question 4 Given $f:(a,1]\to R$, $f(x)=x-a$ and $g:(a,1]\to R$, $g(x)=\sqrt{x-a}$,

A. $g\circ f$ is defined for $a\le -10$

B. $g\circ f$ is defined for $a<1$

C. $g\circ f$ is defined for $a=0$ only

D. $g\circ f$ is defined for $a\le 0$

E. $g\circ f$ is defined for $a\le -1$

Question 5 $y=e^{ax}$ and $y=\dfrac{\log_e x}{a}$ intersect at $x=2$ when

A. $a=1$

B. $a=-1$

C. $a=\log_e\sqrt{2}$

D. $a=-\log_e 2$

E. $a=\log_e 2$

Question 6 Given $f(x)=-(x+1)^2(x-5)^2(x-2)^3$ and $g(x)=-f(x-a)$, $g(-x)=-g(x)$ when

A. $a\le -1$

B. $a>-1$

C. $a=-2$

D. $a\ge -2$

E. $a=-5$

Question 7 The point of intersection of the family of lines $4ax+(a+b)y+4b=0$, where $a,b\in R\setminus\{0\}$, is

A. $(-4,1)$

B. $(-1,4)$

C. $(4,-4)$

D. $(1,-4)$

E. $(-1,1)$

Question 8 $y = \sin(x)$ and $y = mx$ intersect at exactly 5 points for a particular ***positive*** value of m. Which one of the following statements is true?

A. $0.1288 < m < 0.1290$

B. $0.1286 < m < 0.1288$

C. $0.1284 < m < 0.1286$

D. $0.1282 < m < 0.1284$

E. $0.1280 < m < 0.1282$

Question 9 The graph of $y = f(1-x)$ is translated in the positive x direction by 1 unit and then reflected in the y-axis. The equation of the resulting graph is

A. $y = f(x)$

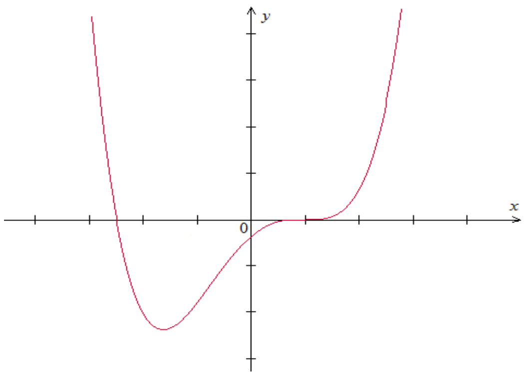
B. $y = -f(x)$

C. $y = f(x-2)$

D. $y = f(x+2)$

E. $y = -f(2-x)$

Question 10 The graph of $y = f(x)$ is shown below.



A possible rule of $f(x)$ is

A. $2(x+a)(x+b)^5$

B. $-2(x-a)^3(x+b)$

C. $1-2(x-a)^3(x+b)$

D. $2(x+b)(x^3-a)$

E. $2(x+a)^2(x+b)^3$

Question 11 Given hybrid function $f(x)=\begin{cases} 4a-6x, & x<\dfrac{2a}{3} \\ 6x-4a, & x\ge\dfrac{2a}{3} \end{cases}$ where a is a real constant, $f(x)$ can be expressed as

A. $f(x)=2\sqrt{(2a-3x)^2}$

B. $f(x)=\sqrt{2(3x-2a)^2}$

C. $f(x)=2|3x|-4a$

D. $f(x)=-2|3x-2a|$

E. $f(x)=-2|2a-3x|$

Question 12 The graph of a cubic polynomial function of *x*, where $x\in R$,

A. has an inflection point and *only* one *x*-intercept

B. has an inflection point and *at least* one *x*-intercept

C. has 0, 1, 2 or 3 *x*-intercepts

D. *always* has a stationary inflection point *or* a pair of local maximum and minimum points

E. *always* has a maximum point and a minimum point

Question 13 If $f'(a)=b$ and $g(x)=b-f(-x)$, where *a* and *b* are real constants, then $g'(-a)=$

A. $-a$

B. b

C. -1

D. $\dfrac{1}{b}$

E. $-\dfrac{1}{a}$

Question 14 If $(-2,-1)$ is the only inflection point on the graph of $f(x)$, then

A. $(-2,1)$ is an inflection point on the graph of $f(|x|)$

B. $(2,1)$ is an inflection point on the graph of $f(|x|)$

C. $(2,-1)$ is an inflection point on the graph of $f(|x|)$

D. $(-2,1)$ and $(2,1)$ are inflection points on the graph of $f(|x|)$

E. the graph of $f(|x|)$ does not have an inflection point

Question 15 Given $\int_a^b f(x)dx = \log_e\left|\frac{b}{a}\right|$, which one of the following choices is ***false***?

A. $a=-2$ and $b=-1$

B. $a=-1$ and $b=-2$

C. $a=1$ and $b=2$

D. $a=2$ and $b=1$

E. $a=-1$ and $b=2$

Question 16 Function g is defined as $g(x)=|x-\pi|+\sin x-\frac{\pi}{2}$. The average value of g in the interval $\left[\frac{2\pi}{3},\frac{4\pi}{3}\right]$ is closest to

A. 0

B. -1

C. $-\frac{\pi}{3}$

D. $-\frac{\pi}{2}$

E. $-\pi$

Question 17 One of the factors of $f(\theta)=\cos^2\theta+2\sin^3\theta-1$ is

A. $\sin\theta-1$

B. $\sin\theta+1$

C. $2+\sin\theta$

D. $\cos\theta$

E. $\cos\theta-1$

Question 18 X is a discrete random variable. A possible probability distribution of X is given by

A.

x	-1	-3	1	-2	5
$\Pr(X=x)$	0.130	0.180	0.296	0.311	0.086

B.

x	1	2	3	4	5
$\Pr(X=x)$	0.1	0.2	0.3	0.2	0.1

C.

x	1	2	3	2	1
$\Pr(X=x)$	0	0.1	0.2	0.3	0.4

D.

x	5	4	3	2	1
$\Pr(X=x)$	1.1	0.4	0.1	-0.8	0.2

E.

x	1.1	0.4	0.1	-0.8	0.2
$\Pr(X=x)$	0	0.25	0.25	0.30	0.20

Question 19 A and B are the two states of a two-state Markov chain ***starting with state*** A. If $\Pr(A\mid B)=a$ and $\Pr(B\mid A)=b$, then $\Pr(ABABB)=$

A. $b^2a(1-a)$

B. b^2a^2

C. $b(1-b)a^2$

D. $b(1-b)a^2(1-a)$

E. $b^2(1-b)(1-a)^2$

Question 20 A ***biased*** coin is **tossed 10 times** and the number of tails is recorded. This probability experiment is repeated many times.
In the long-run the probability of getting 5 tails is 0.10 (correct to the 2nd decimal place), and the probability of getting 6 tails is 0.20 (correct to the 2nd decimal place).
The probability of getting tail in a single toss of the biased coin is closest to

A. 0.3

B. 0.4

C. 0.5

D. 0.6

E. 0.7

Question 21 $\Pr(A) - \Pr(A \mid B')\Pr(B') =$

A. $\Pr(B)$

B. $\Pr(B')$

C. $\Pr(A')$

D. $\Pr(A' \mid B)\Pr(B)$

E. $\Pr(B) - \Pr(B \mid A')\Pr(A')$

Question 22 The probability distribution of continuous random variable X is symmetric and centred at $X = 1.3$.
Given $\Pr(X < 3) = 0.85$, the value of $\Pr(-0.4 < X < 3)$ is closest to

A. 0.60

B. 0.65

C. 0.70

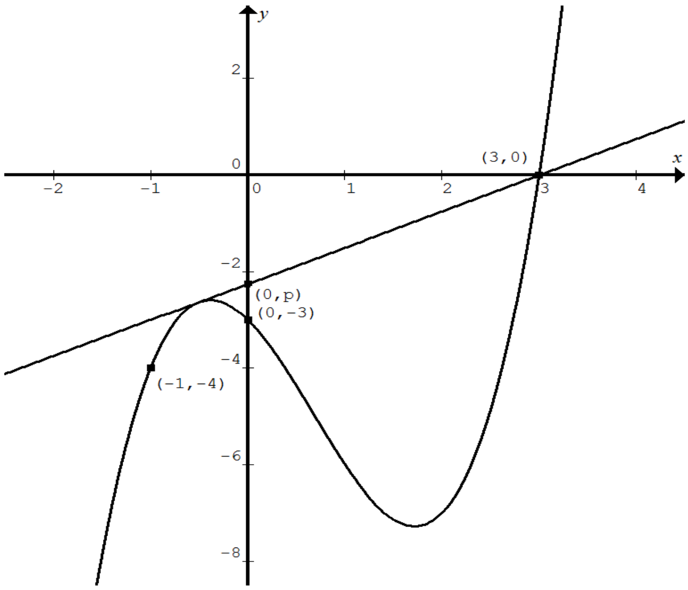
D. 0.75

E. 0.80

Instructions for Section 2

Answer **all** questions.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.

Question 1 The following diagram shows the graphs of a linear function and a cubic function.



a. The y-intercept of the graph of the linear function is $(0, p)$. Find the equation of the linear function in the form $y = A(x - B)$ in terms of p. 2 marks

b. The equation of the cubic function is $y = (x-3)(x^2 + bx + c)$ where $b, c \in R$. Find the values of b and c.
2 marks

c i. ***By using the fact*** that the two graphs intersect at *exactly* two points, find the value of p. 3 marks

c ii. ***Hence*** find the coordinates of the point where the graph of the linear function is a tangent to the graph of the cubic function. 2 marks

d. Find the exact coordinates of another point on the graph of the cubic function where a tangent line is parallel to the graph of the linear function. 2 marks

After the following sequence of transformations: reflection in the x-axis, dilation from the y-axis and dilation from the x-axis, the linear function and the cubic function become $f(x) = -3x + 18$ and $g(x) = -x^3 + 4x^2 + 8x + 24$ respectively.

ei. Find the x-coordinates of the intersections of the graphs of $y = f(x)$ and $y = g(x)$. 1 mark

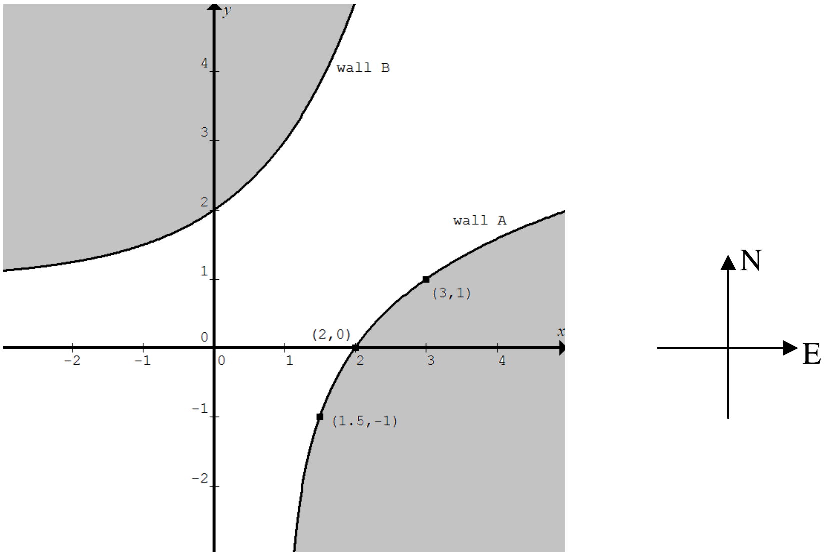
eii. State the factor of dilation from the y-axis. 1 mark

f i. Write down the definite integral for finding the area of the region bounded by the graphs of $y = f(x)$ and $y = g(x)$. 1 mark

f ii. Find the exact area of the region bounded by the graphs of $y = f(x)$ and $y = g(x)$. 1 mark

g. Determine the factor of dilation from the x-axis in the sequence of transformations. 1 mark

Question 2 Part of the floor plan of two structures (shaded) is shown below. All length measures are in metres.



Wall A has the equation $y = a\log_e(x+b)+c$. It passes through points $(3,1)$, $(2,0)$ and $(1.5,-1)$.
Wall B is the reflection of Wall A in the line $y = x$.

a. Write down three simultaneous equations that can be used to find the parameters *a*, *b* and *c* in $y = a\log_e(x+b)+c$. 1 mark

b. Solve the simultaneous equations without using CAS. Show that $a = \dfrac{1}{\log_e 2}$, $b = -1$ and $c = 0$.

3 marks

c. Show that the equation of Wall B is $y = 2^x + 1$. 2 marks

d. Find the exact coordinates of the point at Wall A where the tangent is pointing exactly in the NE direction. 3 marks

e. Find the shortest distance (correct to the nearest 0.01 m) between Wall A and Wall B. 2 marks

f. The horizontal ground between Wall A and Wall B, and bounded by $x = 0$, $x = 3$, $y = 0$ and $y = 3$ is to be covered by a synthetic lawn. Find the area (correct to nearest 0.01 m^2) of synthetic lawn required.
2 marks

Question 3 At $t = 0$ water is pumped into an empty cylindrical tank at a variable rate. The rate is given by $\frac{dV}{dt} = \sin\frac{2t}{\pi} + \cos\frac{2t}{\pi} + \pi$. Length is measured in metres, volume *V* is in m^3 and time *t* is in minutes.

a. Given the radius of the tank is π metres and its height is 4 metres, find $\frac{dh}{dt}$ in terms of t minutes, where h metres is the depth of water in the tank. 2 marks

b. Show that the period of $\frac{dV}{dt}$ is π^2 minutes. 1 mark

c i. Show by calculus the average of $\frac{dV}{dt}$ over *n* periods is π m^3 min^{-1}, where *n* is a whole number. 2 marks

c ii. Hence find the *number of periods* required to fill the tank. 1 mark

d. Find the time (in min, correct to one decimal place) required to fill a second cylindrical tank of the same height but the radius is 3 metres. 2 marks

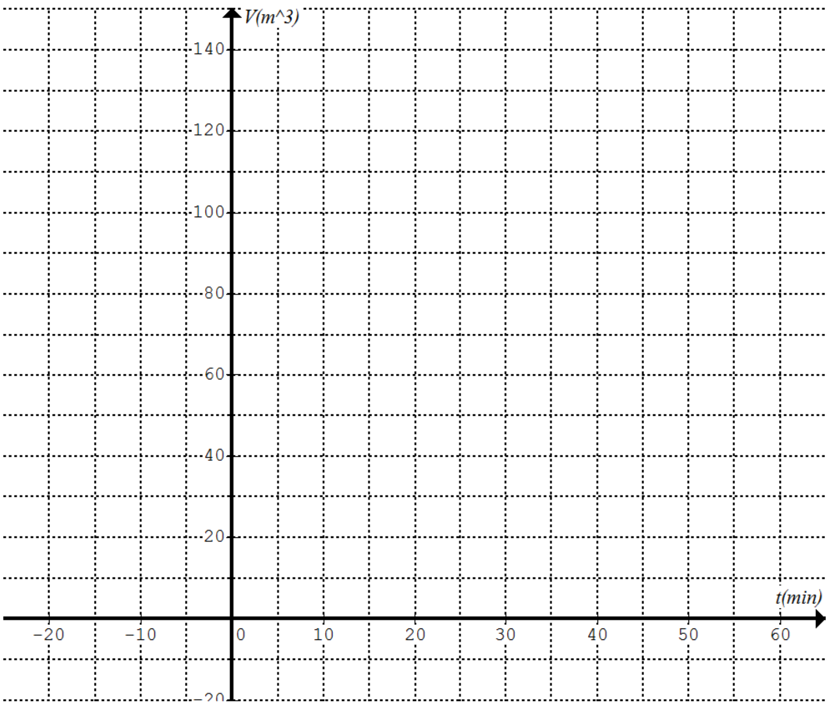
e. Determine the exact time(s) when the depth of water in the second tank ***increases*** at the greatest rate.

3 marks

The pump is turned off when the **first tank** is filled completely and water is allowed to run out of the tank. The volume of water remains in the tank is given by $V(t)=9.164(t+10)^2 e^{-(0.2t+2)}$, and $t=0$ when water starts to run out.

f i. Find the time (in min, correct to one decimal place) when the depth of water is halved. 1 mark

f ii. Sketch the graph of $V(t)=9.164(t+10)^2 e^{-(0.2t+2)}$. Show and label the important features. 2 marks



Question 4 The waiting time in minutes to check out of a supermarket is a random variable and has a normal distribution.
Let A be the waiting time to check out of Allthere Supermarket. It has a mean of 8.53 and a standard deviation of 5.23.
Let B be the waiting time in minutes to check out of Bestbuy Supermarket.

a. Find the probability (correct to 2 decimal places) that an Allthere Supermarket patron has to wait for more than 5 minutes at the checkout. 1 mark

b. Determine the proportion (correct to 2 decimal places) of Allthere Supermarket patrons expected to wait for 7.00 ± 5.23 minutes at the checkout. 1 mark

c i. Find the probability (correct to 2 decimal places) that there are 7 among 10 randomly chosen Allthere Supermarket patrons who have waited for more than 5 minutes at the checkout. 1 mark

c ii. Out of 10 randomly chosen patrons how many (correct to the nearest whole number) were expected to wait more than 5 minutes at the checkout? 1 mark

d. 66.15% of Bestbuy Supermarket patrons have waited for more than 5 minutes at the checkout, and 40.13% have waited for less than 6 minutes. Find the mean and standard deviation (correct to 2 decimal places) of the waiting time at the checkout of Bestbuy Supermarket. 2 marks

e. For how long (in minutes, correct to 2 decimal places) does a Bestbuy Supermarket patron expect to wait at the checkout? 1 mark

f. Given that a Bestbuy Supermarket patron has waited for more than 5 minutes at the checkout, find the probability that the patron has waited for less than 6 minutes. 1 mark

Sofia moves into the neighbourhood and does her grocery shopping at Allthere Supermarket and Bestbuy Supermarket ***only***. If she shops at Allthere the probability that she shops at Allthere again the next time is $\frac{2}{5}$. If she shops at Bestbuy the probability that she shops at Bestbuy again the next time is $\frac{1}{4}$.

g. The probability that Sofia shops at Allthere Supermarket next time is $\frac{3}{4}$. Where did she shop last time?

1 mark

h. Given that she shopped at Bestbuy Supermarket last 5 times, what is the probability that she shops once at each supermarket in her next two shopping trips assuming one supermarket in each trip? 2 marks

i. In the long run what is the ***exact*** probability that Sofia does her grocery shopping at Bestbuy Supermarket? 1 mark

j. If Sofia goes to Bestbuy Supermarket on her 210th shopping trip, what is the expected number of times (correct to 2 decimal places) that she goes to Bestbuy Supermarket in her next ***three*** (i.e. 211th, 212th and 213th) shopping trips? 3 marks

End of exam 2