VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education 2012

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER

									Letter
Figures									
Words									

MATHEMATICAL METHODS (CAS)
Written examination 1

Wednesday 7 November 2012

Reading time: 9.00 am to 9.15 am (15 minutes)
Writing time: 9.15 am to 10.15 am (1 hour)

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

Materials supplied
- Question and answer book of 10 pages, with a detachable sheet of miscellaneous formulas in the centrefold.
- Working space is provided throughout the book.

Instructions
- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

a. If $y = (x^2 - 5x)^4$, find $\frac{dy}{dx}$.

1 mark

b. If $f(x) = \frac{x}{\sin(x)}$, find $f'\left(\frac{\pi}{2}\right)$.

2 marks

Question 2

Find an anti-derivative of $\frac{1}{(2x-1)^3}$ with respect to x.

2 marks

Question 3

The rule for function h is $h(x) = 2x^3 + 1$. Find the rule for the inverse function h^{-1}.

2 marks

Question 4

On any given day, the number X of telephone calls that Daniel receives is a random variable with probability distribution given by

x	0	1	2	3
$\Pr(X = x)$	0.2	0.2	0.5	0.1

a. Find the mean of X.

2 marks

b. What is the probability that Daniel receives only one telephone call on each of three consecutive days?

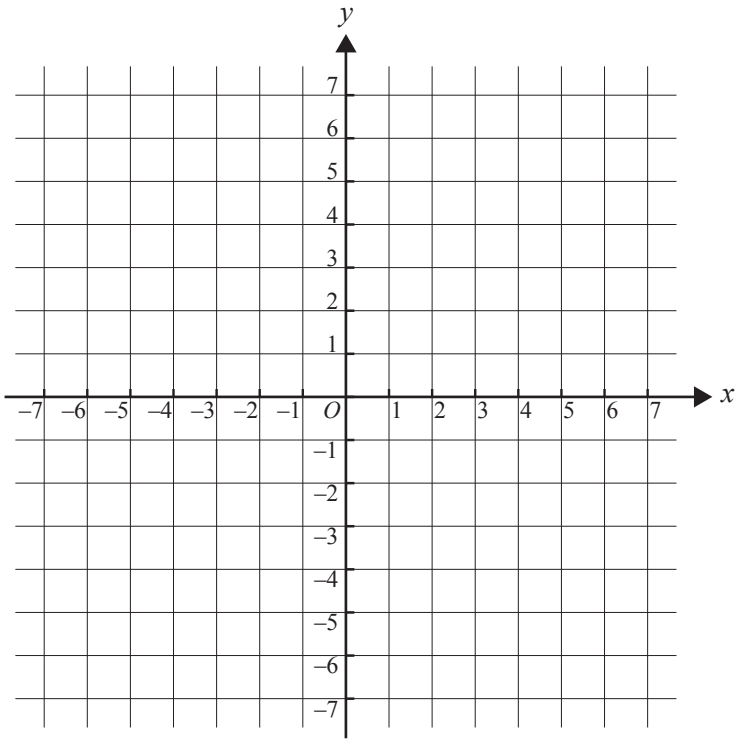
1 mark

c. Daniel receives telephone calls on both Monday and Tuesday.
What is the probability that Daniel receives a total of four calls over these two days?

3 marks

Question 5

a. Sketch the graph of $f: [0, 5] \rightarrow R,\ f(x) = -|x - 3| + 2$. Label the axes intercepts and endpoints with their coordinates.



3 marks

b. **i.** Find the coordinates of the image of the point (3, 2) under a reflection in the x-axis, followed by a translation of 5 units in the positive direction of the x-axis.

ii. Find the equation of the image of the graph of f under a reflection in the x-axis, followed by a translation of 5 units in the positive direction of the x-axis.

1 + 2 = 3 marks

Question 6

The graphs of $y = \cos(x)$ and $y = a\sin(x)$, where a is a real constant, have a point of intersection at $x = \frac{\pi}{3}$.

a. Find the value of a.

2 marks

b. If $x \in [0, 2\pi]$, find the x-coordinate of the other point of intersection of the two graphs.

1 mark

Question 7

Solve the equation $2\log_e(x + 2) - \log_e(x) = \log_e(2x + 1)$, where $x > 0$, for x.

3 marks

Question 8

a. The random variable X is normally distributed with mean 100 and standard deviation 4.
If $\Pr(X < 106) = q$, find $\Pr(94 < X < 100)$ in terms of q.

2 marks

b. The probability density function f of a random variable X is given by

$$f(x) = \begin{cases} \frac{x+1}{12} & 0 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

Find the value of b such that $\Pr(X \leq b) = \frac{5}{8}$.

3 marks

Question 9

a. Let $f: R \rightarrow R,\ f(x) = x \sin(x)$.
Find $f'(x)$.

1 mark

b. Use the result of **part a.** to find the value of $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} x\cos(x)\,dx$ in the form $a\pi + b$.

3 marks

Question 10

Let $f: R \rightarrow R,\ f(x) = e^{-mx} + 3x$, where m is a positive rational number.

a. **i.** Find, in terms of m, the x-coordinate of the stationary point of the graph of $y = f(x)$.

ii. State the values of m such that the x-coordinate of this stationary point is a positive number.

2 + 1 = 3 marks

b. For a particular value of m, the tangent to the graph of $y = f(x)$ at $x = -6$ passes through the origin. Find this value of m.

3 marks

END OF QUESTION AND ANSWER BOOK

MATHEMATICAL METHODS (CAS)

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

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Mathematical Methods (CAS)
Formulas

Mensuration

area of a trapezium: $\frac{1}{2}(a+b)h$

curved surface area of a cylinder: $2\pi rh$

volume of a cylinder: $\pi r^2 h$

volume of a cone: $\frac{1}{3}\pi r^2 h$

volume of a pyramid: $\frac{1}{3}Ah$

volume of a sphere: $\frac{4}{3}\pi r^3$

area of a triangle: $\frac{1}{2}bc\sin A$

Calculus

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

$$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$$

$$\frac{d}{dx}\left(\sin(ax)\right) = a\ \cos(ax)$$

$$\frac{d}{dx}\left(\cos(ax)\right) = -a\ \sin(ax)$$

$$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\ \sec^2(ax)$$

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$$

$$\int \frac{1}{x} dx = \log_e|x| + c$$

$$\int \sin(ax) dx = -\frac{1}{a}\cos(ax) + c$$

$$\int \cos(ax) dx = \frac{1}{a}\sin(ax) + c$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

approximation: $f(x+h) \approx f(x) + hf'(x)$

Probability

$\Pr(A) = 1 - \Pr(A')$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

transition matrices: $S_n = T^n \times S_0$

mean: $\mu = \mathrm{E}(X)$

variance: $\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X-\mu)^2) = \mathrm{E}(X^2) - \mu^2$

Probability distribution		Mean	Variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x\, p(x)$	$\sigma^2 = \sum (x-\mu)^2 p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)dx$	$\mu = \int_{-\infty}^{\infty} x\, f(x)dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx$

END OF FORMULA SHEET