2012 Mathematical Methods (CAS) GA 2: Examination 1

GENERAL COMMENTS

In 2012, 15 823 students sat for Mathematical Methods (CAS) Examination 1.

This examination focused on the topics of calculus (differentiation, anti-differentiation, definite integrals, stationary points, equations of tangents), functions (inverse, sketch graphs, transformations, points of intersection), probability (discrete, continuous, conditional) and logarithms.

Within these topics it was important that students were adept at algebraic manipulation and arithmetic computation (operations with integers, decimals and fractions). Algebraic manipulation was prominent in Questions 3, 7, 8, 9 and 10. Arithmetic proved a problem for some students in Questions 4, 5bii., 7, 8b. and 9b.

Students continue to omit brackets from functions, such as $\sin(x)$ and $\log_e(x)$, and in integral statements containing more than one term, such as $\int (x\cos(x)+\sin(x))dx$. Teachers and students should be aware of problems that occur due to the omission of these brackets, such as writing $\log_e(x-1)$ as $\log_e x-1$ and then using it in further working as $\log_e(x)-1$.

The effective and proper use of brackets by students requires substantial attention. The majority of students who did not gain the first mark on the paper neglected to use brackets for their derivative. A common approach was to write $4\times(x^2-5x)^4\times 2x-5$ instead of $4\times(x^2-5x)^4\times(2x-5)$.

Another problem area is expansions involving negative signs, particularly for the second term in an expression. This affected student answers in Questions 1a., 5bii., 7, 8, 9b. and 10b.

Poor notation continues to be of concern. Some students appeared confused with the difference between function notation and that used for basic algebra. For a 1 to 1 function, f^{-1} represents the inverse of function f. However, the reciprocal of the pronumeral f is $f^{-1}=\frac{1}{f}$. Students should know that $\sin^{-1}(x)$ is the inverse of $\sin(x)$ and not $\frac{1}{\sin(x)}$ and that $\sin^2(x)$ is $(\sin(x))^2$.

When a function has been changed through transformations, then it is no longer the same function. For example, the transformed function in Question 5bii. becomes $-f(x-5)$. Note that f' is a standard notation reserved for the derivative of the function. It would have been preferable for students to use notations such as g, h or even f_1 or f_2. Function notation problems arose in Questions 1b., 3 and 5bii.

A recurring notational problem is the omission of dx in integral statements. Questions 8b. and 9b. required the use of a $\int (\textit{function})\, dx$ statement. Question 2 would have required the use of dx only if the student chose to write the integral statement (which was not necessary to find 'an anti-derivative').

Solving quadratic equations was necessary for Questions 7 and 8b. In both cases, once the quadratic was established, the majority of students completed the question successfully. Students needed to explicitly state the relevant equation and apply the null factor law.

Time can be lost when students continue to engage in a problem beyond the requirement of the question. A number of students chose to find the much more difficult y-coordinate in Questions 6b. and 10ai., despite it not being required. Those who determined the correct y value lost time, whereas students who determined the incorrect y value also risked being penalised for an incorrect solution.

The majority of students were able to complete the paper within the allocated time. Students should make good use of the 15 minutes of reading time. Not only should they read to understand the questions, but they should try to identify those questions that have familiar concepts and routines, and start with those when writing time begins.

SPECIFIC INFORMATION

This report provides sample answers or an indication of what the answers may have included. Unless otherwise stated, these are not intended to be exemplary or complete solutions.

Question 1a.

Marks	0	1	Average
%	38	62	**0.6**

$4(x^2-5x)^3(2x-5)$ or $4x^3(x-5)^3(2x-5)$

The most common errors were to drop the power of 3 or to neglect adding brackets to $2x-5$.

Question 1b.

Marks	0	1	2	Average
%	16	25	59	**1.5**

$$f'(x)=\frac{\sin(x)-x\cos(x)}{\sin^2(x)}$$

$$f'\left(\frac{\pi}{2}\right)=\frac{1-\frac{\pi}{2}\times 0}{1^2}=1$$

This question was generally well done. Some students did not use the formula sheet to their advantage; others did not substitute correctly. Only a few students neglected a substitution.

Question 2

Marks	0	1	2	Average
%	54	19	27	**0.8**

$\dfrac{-1}{4(2x-1)^2}+c$ When c is any real number, c may be omitted.

The most common error with this question was students giving a logarithm statement as part of their answer. The degree of the exponent not being –1 was critical to students' success with this question. Neglecting to divide by the coefficient of x was another problem.

Question 3

Marks	0	1	2	Average
%	9	34	56	**1.5**

$h(x)=2x^3+1$, let $y=2x^3+1$

for inverse, swap x and y

$\Rightarrow x=2y^3+1$, make y the subject

$y=\sqrt[3]{\dfrac{x-1}{2}}$, y is the inverse of $h(x)$

$h^{-1}(x)=\sqrt[3]{\dfrac{x-1}{2}}$, $x\in R$

It is important that students do not proceed directly from $y=2x^3+1$ to $x=2y^3+1$. This is not correct working. Students need to indicate that new working is starting. An approach is as shown above. A common error was to write $h^{-1}(x)=x=2y^3+1$.

h(x) is a 1 to 1 function and so there was no domain or range restriction to consider. Use of ± was not appropriate here.

Question 4a.

Marks	0	1	2	Average
%	9	9	82	**1.7**

$$E(X) = 0 + 0.2 + 1 + 0.3$$
$$= 1.5$$

This question was generally well done. Some students either did not answer this question or went about determining the variance, median or some other parameter. Other students knew that they needed to multiply each *x* by *Pr*(*X* = *x*) and add the terms, but either could not do that correctly or divided their result by 4.

Question 4b.

Marks	0	1	Average
%	40	60	**0.6**

0.008

Most students knew that they needed to evaluate 0.2^3 as $2^3 \times 10^{-3}$ or similar, but many students were not able to evaluate correctly.

Question 4c.

Marks	0	1	2	3	Average
%	41	13	43	3	**1.1**

Want $\Pr(\Sigma Calls = 4 \mid Calls \geq 1\ on$ Monday and Tuesday$)$

$\Pr(4Calls \cap Calls \geq 1\ on$ Monday and Tuesday$)$

$= \Pr(M1T3, M2T2, M3T1)$ where: $M1T3 = 1$ call on Monday and 3 calls on Tuesday

$= 0.02 + 0.25 + 0.02 = 0.29$

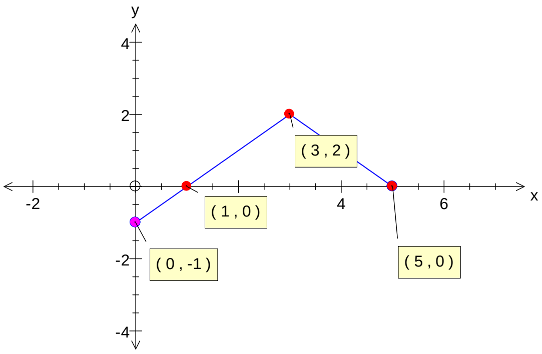
$\Pr(Calls\ both$ Monday and Tuesday$) = 0.8 \times 0.8 = 0.64$

$\Pr(\Sigma Calls = 4 \mid Calls\ both$ Monday and Tuesday$) = \dfrac{29}{64}$

Most students were not aware that the condition of ‘receives telephone calls on both Monday and Tuesday’ would affect the result. A significant number of students incorrectly thought that two calls on Monday and two calls on Tuesday was different from two calls on Tuesday and two calls on Monday. Students who used a key or tree diagram were less likely to make this error.

Question 5a.

Marks	0	1	2	3	Average
%	26	11	17	46	**1.9**



(1, 0) was a very common omission from otherwise correct diagrams.

This question did not require the vertex to be labelled, although many students correctly indicated its location.

Some students indicated the coordinates of an 'included' endpoint in square brackets. Only round brackets are to be used for coordinates.

Question 5bi.

Marks	0	1	Average
%	37	63	0.7

(8, –2)

Variations were numerous depending on how the student moved the point (3, 2). It was apparent that some students were not aware that this point was part of the original function.

Question 5bii.

Marks	0	1	2	Average
%	37	37	26	0.9

$-f(x-5)=|x-8|-2$

$f_1(x)=|x-8|-2$, or similar, was also acceptable as the final answer.

The two marks here were linked firstly to the reflection and then the translation. A single incorrect response often gave no indication as to how the student's answer had been achieved. The best responses were those that clearly indicated the process and had a 'new function'. It was common to see *f*(*x*) referred to as the transformed function.

Question 6a.

Marks	0	1	2	Average
%	12	23	65	1.5

$\cos\left(\frac{\pi}{3}\right) = a\sin\left(\frac{\pi}{3}\right) \Rightarrow \frac{1}{2} = \frac{a\sqrt{3}}{2}$ or $\tan(x) = \frac{1}{a} \Rightarrow \tan\left(\frac{\pi}{3}\right) = \sqrt{3} = \frac{1}{a}$

$a = \frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$

This question was generally well answered.

Question 6b.

Marks	0	1	Average
%	48	52	0.5

$\frac{4\pi}{3}$

Many students also identified that the next solution was $\frac{7\pi}{3}$, either by sketch or the link to tan(*x*).

Question 7

Marks	0	1	2	3	Average
%	19	21	8	52	**2**

$\log_e((x+2)^2) = \log_e(2x^2+x)$ or $\log_e\left(\frac{(x+2)^2}{x}\right) = \log_e(2x+1)$

$\Rightarrow x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

$x = 4$ since $x > 0$

Most students could apply at least one of the logarithm rules and many of those were able to successfully eliminate the logarithms and produce an easily factorised quadratic expression.

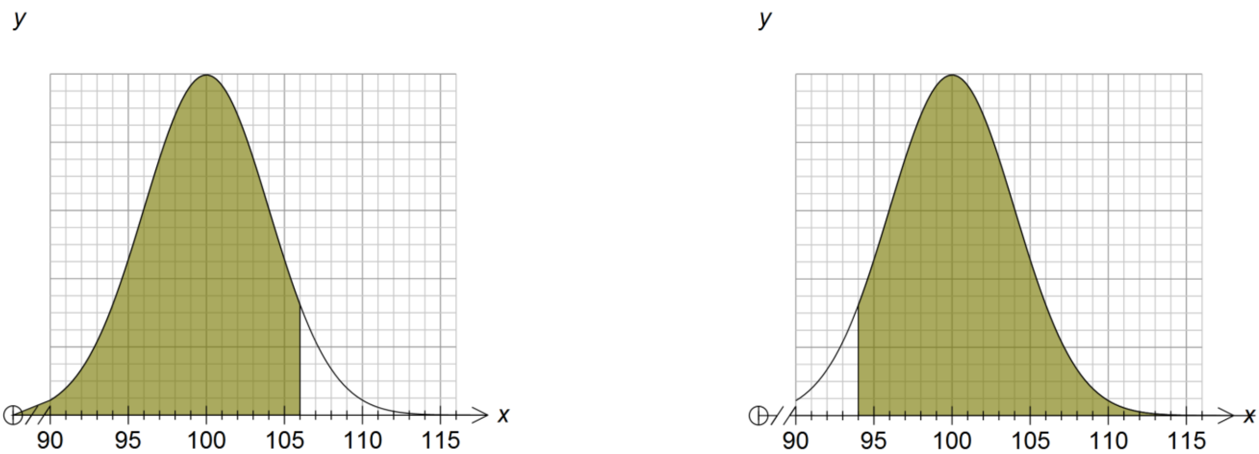
Question 8a.

Marks	0	1	2	Average
%	53	12	34	**0.8**

$\Pr(94 < X < 100) = \Pr(100 < X < 106) = q - 0.5$

OR $\Pr(X > 94) = q$, $\Pr(X < 94) = 1 - q$

$\Pr(94 < X < 100) = 0.5 - (1-q) = q - 0.5$



Pr ($X < 106$) = q is the same area as Pr($X > 94$) and, as Pr($X > 100$) = 0.5.

This question appeared to confuse many students. There were many unsuccessful attempts to allocate an area to 1.5 standard deviations above or below the mean. Those who drew a diagram and realised the symmetry (see graphs), as in the first line of the answer provided here, were most successful.

Question 8b.

Marks	0	1	2	3	Average
%	35	12	9	45	**1.7**

$\int_0^b \left(\frac{x+1}{12}\right)dx = \frac{5}{8}$ or $\frac{1}{12}\int_0^b (x+1)dx = \frac{5}{8}$

$\left[\frac{x^2}{24}+\frac{x}{12}\right]_0^b = \left(\frac{b^2}{24}+\frac{b}{12}\right) = \frac{5}{8}$ or $\frac{1}{12}\left[\frac{(x+1)^2}{2}\right]_0^b = \frac{1}{12}\left[\frac{(b+1)^2}{2}-\frac{(1)^2}{2}\right] = \frac{5}{8}$

$\Rightarrow b^2+2b-15=0$

$(b-3)(b+5)=0$

$b=3$ as a positive area requires $b>0$

Poor anti-differentiation brought on by the presence of the denominator of 12 meant an unhelpful start for many students. Obtaining and solving a quadratic equation was fundamental to further progress. The quadratic formula and null factor law or factorisation can be used only if the quadratic is equated to zero.

A number of students established a quadratic that was not equal to zero and used guesswork to obtain one possible solution for their quadratic.

Alternatively, there was a geometric approach that could have been used. A few students used triangles and rectangles or trapezia to find where the area became $\frac{5}{8}$ square units.

Question 9a.

Marks	0	1	Average
%	21	79	**0.8**

$f'(x)=\sin(x)+x\cos(x)$

This question was generally well answered.

Question 9b.

Marks	0	1	2	3	Average
%	35	12	9	45	**1.3**

$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}(x\cos(x))dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}}(f'(x)-\sin(x))dx$

$=[x\sin(x)+\cos(x)]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$

$=\left(\frac{\pi}{2}+0\right)-\left(\frac{\pi}{6}\times\frac{1}{2}+\frac{\sqrt{3}}{2}\right)$

$=\frac{5\pi}{12}-\frac{\sqrt{3}}{2}$

Those students who obtained the correct answer may not have been aware of the mistakes that they made getting there. It was common to see $\int\sin(x)dx=\cos(x)$ followed by

$(\frac{\pi}{2}+0)-(\frac{\pi}{6}\times\frac{1}{2}-\frac{\sqrt{3}}{2})$

$=\frac{5\pi}{12}-\frac{\sqrt{3}}{2}$

Students needed to provide the answer in the required form of $a\pi + b$.

Question 10ai.

Marks	0	1	2	Average
%	22	22	56	1.4

$f'(x) = -me^{-mx} + 3$

$f'(x) = 0$

$\Rightarrow x = -\frac{1}{m}\log_e\left(\frac{3}{m}\right)$ or $\frac{1}{m}\log_e\left(\frac{m}{3}\right)$

Most students differentiated correctly and equated the derivative to zero in order to find the *x*-coordinate of the stationary point. Some students included an 'absolute value' notation, which was not necessary as $m > 0$.

Question 10aii.

Marks	0	1	Average
%	82	18	0.2

$\frac{1}{m}\log_e\left(\frac{m}{3}\right) > 0 \quad \Rightarrow \frac{m}{3} > 1$ so $m > 3$

Or alternatively,

$-\frac{1}{m}\log_e\left(\frac{3}{m}\right) > 0 \Rightarrow 0 < \frac{3}{m} < 1$ so $m > 3$

Understanding the restrictions within the logarithm caused difficulty for many students.

Question 10b.

Marks	0	1	2	3	Average
%	53	17	9	22	1

$f'(-6) = -me^{6m} + 3,$ and $f(-6) = e^{6m} - 18$

gradient of tangent from $(-6, f(-6))$ through $(0, 0)$ is

$f'(-6) = \frac{f(-6) - 0}{-6 - 0}$

$\Rightarrow f'(-6) \times -6 = f(-6)$

$\Rightarrow 6me^{6m} - 18 = e^{6m} - 18$

$\Rightarrow 6me^{6m} - e^{6m} = 0$

$\Rightarrow e^{6m}(6m - 1) = 0$

so $m = \frac{1}{6}$ as $e^{6m} > 0$

An alternative approach to this problem was to determine the equation of the tangent through (0, 0), then through (–6, *f*(–6)) to solve for *m* (or vice versa).

The most common mistakes were in substitution and algebraic manipulation.

Most students made a start on this question, although many confused m with the common use of m for gradient in $y = mx + c$ and attempted to solve the incorrect equations.