Student Name:

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MATHEMATICAL METHODS (CAS)

Unit 4
Targeted Evaluation Task for School-assessed Coursework 4

TSSM™
Creating VCE Success

2012 Item Analysis Task on Probability for Outcomes 1, 2 & 3

Recommended writing time*: 120 minutes
Total number of marks available: 60 marks

TASK BOOK

* The recommended writing time is a guide to the time students should take to complete this task. Teachers may wish to alter this time and can do so at their own discretion.

Conditions and restrictions
- Students are permitted to bring into the room for this task: pens, pencils, highlighters, erasers, sharpeners and rulers, bound summary booklet, approved CAS calculator.
- Students are NOT permitted to bring into the room for this task: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question and answer book of 13 pages.

Instructions
- Print your name in the space provided on the top of the front page.
- All written responses must be in English.
- Show appropriate scales on the axes provided when sketching graphs.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the room for this task.

Any question worth more than 1 mark, relevant working must be shown.

The information given below refers to Questions 1 and 2

When Callan plays tennis he wins his first set 65% of the time on average. If he wins the first set his chance of winning the next set is 75% but if he loses the first set his chance of winning the next set is 40%. One day Callan plays two sets of tennis.

Question 1
The probability that he wins one set only is:

A. 0.14
B. 0.1625
C. 0.3025
D. 0.65
E. 0.79

a. Draw up a tree diagram of the situation.

2 marks

b. Show that alternative **C** is the correct answer.

1 mark

c. Explain what error is made if alternatives **A** or **B** are chosen.

1 mark
Total 4 marks

Question 2
If Callan wins one set only the probability that he won the first set is:

A. 0.1625
B. 0.25
C. 0.3025
D. 0.5372
E. 0.9525

a. Find the correct alternative showing your working.

_____

2 marks

b. What misunderstanding of conditional probability would lead to alternative **B** ? Show your working.

_____

_____

_____

1 mark

c. What error would lead to alternative **E**?

_____

_____

1 mark
Total 4 marks

The information given below refers to Questions 3 and 4

x	2	3	4	5
Pr($X = x$)	0.1	0.3	0.4	0.2

Question 3
The mean of the probability distribution shown in the table above is:

A. 0.25
B. 1.0
C. 1.92
D. 3.5
E. 3.7

a. Show that the correct answer is **E**.

_____

_____
1 mark

b. What mistake would most likely lead to alternative **D**?

_____
1 mark

c. If the probabilities remained the same, what change to each of the values of x would lead to an increase of 2 in the mean?

_____
1 mark

d. If Pr($X = 3$) was decreased to 0.2 and Pr($X = 5$) was increased to 0.4, what would the other probabilities have to be changed to, if the mean is to remain at 3.7?

_____

_____

_____

_____

_____

_____

_____
3 marks
Total 6 marks

Question 4
The standard deviation of the distribution shown in the table is:

A. 0.11
B. 0.66
C. 0.81
D. 0.90
E. 10.8

a. Find the correct answer showing your working.

_____

_____

_____

_____

2 marks

b. What error would give
  i.   Alternative C?

_____

  ii.   Alternative E?

_____

$1 + 1 = 2$ marks

c. What changes would need to be made to the given probabilities to make the standard deviation larger?

_____

_____

2 marks
Total 6 marks

Question 5

A probability distribution X is generated from 8 Bernoulli trials and $\Pr(X = 3)$ is found to be 0.2668, where X denotes the number of successes. The values of the mean and standard deviation are:

A. $\mu = 1.6$ and $\sigma = 1.741$
B. $\mu = 2.56$ and $\sigma = 1.319$
C. $\mu = 2.56$ and $\sigma = 1.741$
D. $\mu = 5.44$ and $\sigma = 1.319$
E. $\mu = 5.44$ and $\sigma = 1.741$

a. If p = probability of success in any one trial, use the information given to set up an equation in p only.

_____

_____

_____
1 mark

b. Use your graphics calculator to find the value of p correct to 2 decimal places.

_____
2 marks

c. Hence find the correct solution to the question.

_____

_____
1 mark

d. What error would give:
   i.   Alternative **C**?

_____

   ii.   Alternative **E**?

_____
1 + 1 = 2 marks

e. If the standard deviation was to be 1.2 and the mean greater than that found in part **c.**

    i. Use the quadratic rule to find the value of p (correct to 4 decimal places).

_____

_____

_____

_____

    ii. Use your graphics calculator to find the value of $\Pr(X = 3)$ that would be required in the original question (correct to 4 decimal places).

_____

<div align="right">

$2 + 1 = 3$ marks

Total 9 marks

</div>

Question 6

Claire, a market researcher, is surveying attitudes towards a certain product. Previous research indicates that 60% of people like the product. Claire is in the process of interviewing a sample of 20 people. The probability that at least 11 and not more than 15 people will like the product is closest to:

A. 0.2447
B. 0.2578
C. 0.7044
D. 0.9491
E. 1.1937

a. Find the correct alternative showing your working.

_____

_____

_____

_____

<div align="right">

2 marks

</div>

b. What error would lead to alternative **E** and why can this alternative be rejected before doing any calculation?

_____

_____

<div align="right">

2 marks

Total 4 marks

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The following information refers to Questions 7 and 8.

In a country town 500 people go to see a film each week at one of the two cinemas, Ausfilm or Boyts. In the past year it has been found that 70% of those who go to the Ausfilm cinema in any one week will go there again the following week and 40% of those who go to the Boyts cinema will go to the Ausfilm cinema the following week.
The Ausfilm cinema has been closed for a few weeks for repairs so everyone has been going to the Boyts cinema in that time. However Ausfilm has just reopened.

Question 7
The number of people going to the Ausfilm cinema in the **second** week after it has reopened will be:

A. 195
B. 200
C. 210
D. 240
E. 260

a. Write down the initial state matrix and the transition matrix for this situation.

1 mark

b. Hence find the correct alternative using your graphics calculator. Show the matrix equation you used.

_____
2 marks

c. What errors would lead to :
  i.   Alternative **D** ?

_____
1 mark

  ii.   Alternative **C**?

_____

_____

_____
2 marks
Total 6 marks

Question 8
In the long term, the number of people going to the Ausfilm cinema each week would be:

A. 214
B. 231
C. 260
D. 285
E. 286

a. Without using a graphics calculator, find the correct alternative. Describe your method for finding an answer.

_____

_____

_____

_____

_____

_____

3 marks

b. Give a reason for obtaining alternative **D**.

_____

_____

1 mark

c. If a calculator method involving matrices was used to find the answer, give another reason for obtaining alternative **D**.

_____

_____

_____

1 mark
Total 5 marks

Question 9

X is a random variable with a probability density function given by:

$$f(x) = \begin{cases} \dfrac{x}{2} - \dfrac{1}{2} & 1 \le x \le 3 \\ 0 & \text{elsewhere} \end{cases}$$

The median of X is:

A. $1 - \sqrt{2}$

B. $\dfrac{\sqrt{2}}{2}$

C. $\sqrt{2}$

D. 2

E. $1 + \sqrt{2}$

a. Using calculus find the correct answer for this question.

_____

_____

_____

_____

2 marks

b. What misunderstanding of the median would lead to alternative **D**?

_____

_____

1 mark

c. Using areas explain why alternative **A** can be a correct mathematical solution to this question even though it is not correct within the context of the actual function given.

_____

_____

_____

_____

2 marks
Total 5 marks

Question 10

X is a random variable with a probability density function given by:

$$f(x) = \begin{cases} \dfrac{4x\sin^2 x}{\pi^2} , & 0 \le x \le \pi \\ 0 , & \text{elsewhere} \end{cases}$$

The mean of X is:

A. 0.5295
B. 1.5708
C. 1.7761
D. 1.7927
E. 1.8366

a. Find the correct answer to this question. Describe how you obtained your answer.

_____

_____

_____

_____

2 marks

b. Show that alternative **A** is the standard deviation of X.

_____

_____

_____

_____

2 marks

c. What feature of the graph of the probability density function is at $x = 1.8366$. What misunderstanding of the mean may lead to alternative **E** being chosen as the correct answer?

_____

_____

_____

2 marks
Total 6 marks

Question 11

Spark-O batteries have a mean life of 30 months. If the life of the batteries is normally distributed and the probability that a Spark-O battery lasts longer than 36 months is 0.05 the standard deviation of the life of the batteries is closest to:

A. 1.65 months
B. 3 months
C. 3.65 months
D. 6.32 months
E. 9.87 months

a. Find the correct alternative.

_____

_____

_____

_____

3 marks

b. What is a possible reason for alternative **B** being chosen?

_____

_____

_____

1 mark

c. What change would have to be made to the probability of a battery lasting longer than 36 months for alternative **D** to be the correct answer?

_____

_____

_____

1 mark

Total 5 marks

END OF TASK BOOK