MATHEMATICAL MATHODS (CAS)

Unit 4
Targeted Evaluation Task for School-assessed Coursework 3

TSSM™
Creating VCE Success

2012 Applications Analysis Task on Integration for

Outcomes 1, 2 & 3

SOLUTIONS & RESPONSE GUIDE

The marks given are allocated to the 3 outcomes according to the following:
A – Outcome 1, B – Outcome 2, C – Outcome 3

Question 1

a. $v(t) = \int v'(t)dt$

$v(t) = \int (9 - 3t + 2\sin t)\,dt$

A1

$v(t) = 9t - \frac{3t^2}{2} - 2\cos t + c$

A1

$v(0) = 1 \Rightarrow 0 - 0 - 2\cos 0 + c = 1$

$c = 3$

$v(t) = 9t - \frac{3t^2}{2} - 2\cos t + 3$

B1

b. Graph $v(t)$ on graphics calculator and find *t*-intercept

$t = 6.112$

C1

c. $d(t) = \int v(t)dt$

Therefore displacement over the first 4 seconds is given by:

$\int_0^4 \left(9t - \frac{3t^2}{2} - 2\cos t + 3\right)dt$

A1

$= \left[\frac{9t^2}{2} - \frac{t^3}{2} - 2\sin t + 3t\right]_0^4$

$= (72 - 32 - 2\sin 4 + 12) - (0)$

$= 52 - 2\sin 4$

$= 53.514$

A1, C1

d. The area between the graph of $v(t)$ and the *t*-axis between $t = 0$ and $t = 4$.

B1

Question 2

a. If $y = x^3 \ln x$ then using the product rule

$$\frac{dy}{dx} = 3x^2 \ln x + x^2$$

A1

$$\therefore \int 3x^2 \ln x \, dx + \int x^2 \, dx = x^3 \ln x$$

B1

$$\therefore \int 3x^2 \ln x \, dx + \frac{x^3}{3} = x^3 \ln x$$

$$3\int x^2 \ln x \, dx = x^3 \ln x - \frac{x^3}{3}$$

$$\int x^2 \ln x \, dx = \frac{x^3 \ln x}{3} - \frac{x^3}{9}$$

B2

Now, to find the x-intercept let $f(x) = 0$

$x^2 \ln x = 0$

$x = 0$ or $\ln x = 0$

A1

But $x \neq 0$ as $\ln x$ is undefined at $x = 0$

$\therefore \ln x = 0$ is the only solution

$\Rightarrow x = 1$

B1

Required integral is

$$\int_1^2 x^2 \ln x \, dx$$

$$= \left[\frac{x^3 \ln x}{3} - \frac{x^3}{9}\right]_1^2$$

$$= \left(\frac{8\ln 2}{3} - \frac{8}{9}\right) - \left(0 - \frac{1}{9}\right)$$

$$= \frac{8\ln 2}{3} - \frac{7}{9}$$

A1

b. The integral would have to be evaluated at $x = 0$ and this would mean evaluating ln0 which is undefined. However if the lower limit was evaluated for a very small value of x, say $x = 0.0000001$, a very good approximation of the area would be found.

B2

Question 3

a. $e^{-x}(\ln x + \ln 5)$

C2

b. The x-intercept of $y = e^{-x}\left(\frac{1}{x} - \ln 5x\right)$

B1

c. $x = 0.7537$

C1

d. $\int_{0.5}^{0.7537} e^{-x}\left(\frac{1}{x} - \ln 5x\right) dx - \int_{0.7537}^{1.5} e^{-x}\left(\frac{1}{x} - \ln 5x\right) dx$

B2

e. 0.243 square units

C1

Question 4

a.

x	*f(x)*
1.0	1.3548
1.2	1.4963
1.4	1.7097
1.6	1.9926
1.8	2.3435
2.0	2.7616
2.2	3.2461
2.4	3.7966
2.6	4.4127
2.8	5.0942
3.0	5.8408

C2

b.

i. The lower rectangles in this case will be the left rectangles.
Area of wall $= 0.2[f(1.0) + f(1.2) + f(1.4) + ... + f(2.6) + f(2.8)] = 5.6416 \text{ m}^2$
Volume of wall $= 5.6416 \times 0.25 = 1.410 \text{ m}^3$

A1, C1

ii. The lower rectangles in this case will be the right rectangles.
Area of wall $= 0.2[f(1.2) + f(1.4) + f(1.6) + ... + f(2.8) + f(3.0)] = 6.5388 \text{ m}^2$
Volume of wall $= 6.5388 \times 0.25 = 1.635 \text{ m}^3$

A1, C1

c. Mean of the two values $= \dfrac{1.410 + 1.635}{2} = 1.523 \text{ m}^3$

B1

d. $V = 0.25\int_1^3 \left(\dfrac{1}{3x+1} + e^{-0.1x} + \dfrac{(2x-1)^2}{5} \right) dx$

$V = \dfrac{1}{4}\left[\dfrac{\ln(3x+1)}{3} - 10e^{-0.1x} + \dfrac{(2x-1)^3}{30} \right]_1^3$

A2

$V = \dfrac{1}{4}\left[\left(\dfrac{1}{3}\ln 10 - 10e^{-0.3} + \dfrac{125}{30} \right) - \left(\dfrac{1}{3}\ln 4 - 10e^{-0.1} + \dfrac{1}{30} \right) \right]$

$V = \dfrac{1}{12}\ln\left(\dfrac{5}{2}\right) + \dfrac{5}{2}\left(e^{-0.1} - e^{-0.3}\right) + \dfrac{31}{30}$

A2

Question 5

a. $\sin 2x = -\sqrt{3}\cos 2x$

$\tan 2x = -\sqrt{3}$

$2x = \dfrac{2\pi}{3}$

$x = \dfrac{\pi}{3}$

A1

b. $A = \int_0^{\frac{\pi}{3}} [g(x) - f(x)]\, dx + \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} [f(x) - g(x)]\, dx$

B2

$A = \int_0^{\frac{\pi}{3}} (\sin 2x + \sqrt{3}\cos 2x)\, dx + \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} (-\sqrt{3}\cos 2x - \sin 2x)\, dx$

$A = \left[-\dfrac{1}{2}\cos 2x + \dfrac{\sqrt{3}}{2}\sin 2x \right]_0^{\frac{\pi}{3}} + \left[-\dfrac{\sqrt{3}}{2}\sin 2x + \dfrac{1}{2}\cos 2x \right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$

A1

$A = \left(\dfrac{1}{4} + \dfrac{3}{4} \right) - \left(-\dfrac{1}{2} + 0 \right) + \left(0 - \dfrac{1}{2} \right) - \left(-\dfrac{3}{4} - \dfrac{1}{4} \right)$

$A = 2$

A1

Summary of mark allocation per Outcome

Question	Part	Outcome 1	Outcome 2	Outcome 3
1	a	2	1	
	b			1
	c	2		1
	d		1	
2	a	3	4	
	b		2	
3	a			2
	b		1	
	c			1
	d		2	
	e			1
4	a			2
	b	2		2
	c		1	
	d	4		
5	a	1		
	b	2	2	
Raw Scores		16	14	10
Adjusted Scores		8	7	5