Student Name: ____________________

MATHEMATICAL METHODS (CAS)

Unit 4
Targeted Evaluation Task for School-assessed Coursework 2

2012 Modelling Analysis Task on Trigonometric Functions for Outcomes 1, 2 & 3

Recommended writing time*: 120 minutes
Total number of marks available: 40 marks

TASK BOOK

* The recommended writing time is a guide to the time students should take to complete this task. Teachers may wish to alter this time and can do so at their own discretion.

Conditions and restrictions

- Students are permitted to bring into the room for this task: pens, pencils, highlighters, erasers, sharpeners and rulers, bound summary booklet, approved CAS calculator.
- Students are NOT permitted to bring into the room for this task: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 7 pages.

Instructions

- Print your name in the space provided on the top of the front page.
- All written responses must be in English.
- Show appropriate scales on the axes provided when sketching graphs.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the room for this task.

Any question worth more than 1 mark, relevant working must be shown.

The depth of sea water at any point changes regularly due to the tidal effects of the moon and to a lesser extent the sun. In this task you will use trigonometric functions to analyse these changes and how they can effect ship movements, initially using a simplified model and then a more refined model.

At a port the depth of water, *d*, in the shallowest section of the channel leading into the port can be modelled using the function

$$d_1(t) = a\sin bt + c$$

where *t* is the time in hours after 9.00 am on Sept 2 2012 and *a*, *b* and *c* are real constants. The minimum depth of water is 0.5 m, the maximum depth of water is 9.5 m and the high tide occurs every 12 hours and 30 minutes.

Question 1

a. Using the information given above find the exact values of *a*, *b* and *c*.

3 marks

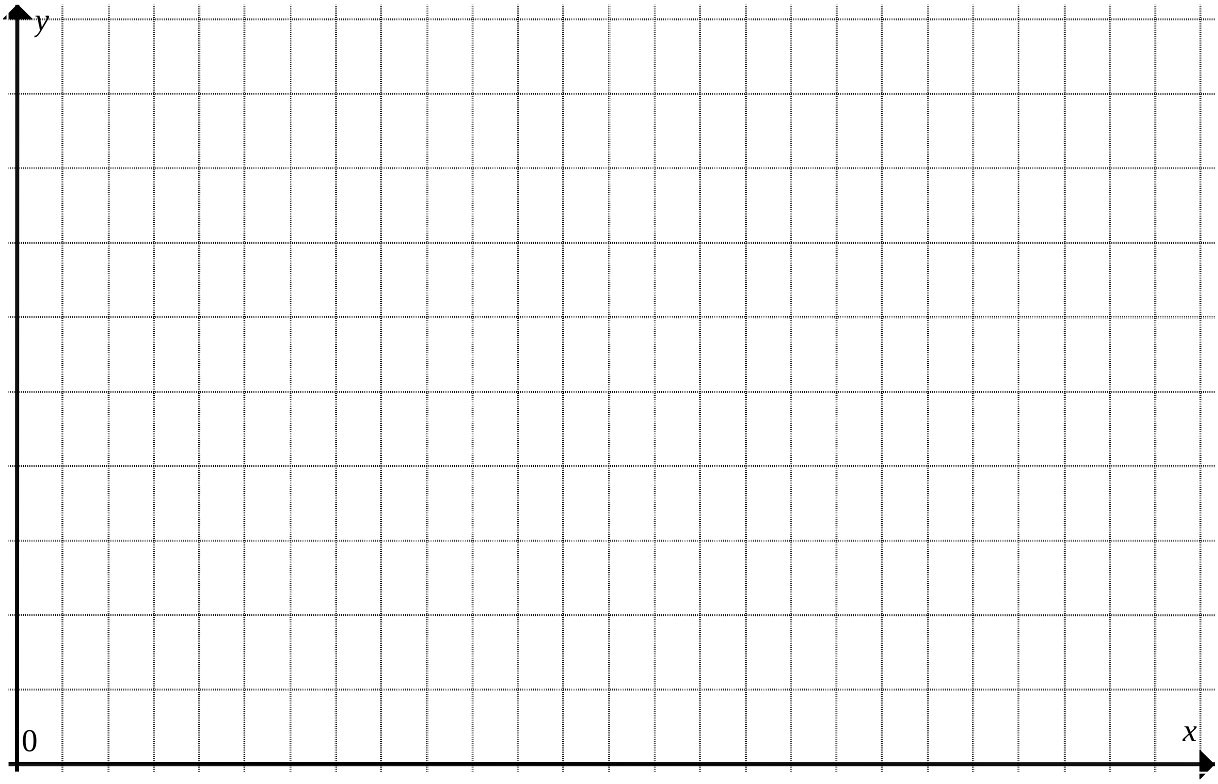
b. At what times (to the nearest minute) will the first maximum depth and the first minimum depth occur?

2 marks

c. What will be the depth of water (to the nearest cm) at 6.15 am on Sept 4?

2 marks

d. Sketch a graph of $d_1(t)$ for $0 \le t \le 25$ on the set of axes below. Mark in an appropriate scale and clearly label all turning points.



4 marks

Question 2

a. Find the first 4 times (to the nearest minute) when the depth of the water is 8.2 m. Mark these co-ordinates on the graph above.

5 marks

b. A particular ship requires a minimum water depth of 3.5 m to enter the port. Use your graphics calculator to find the percentage of time (correct to 1 decimal place) that this ship can enter the port within the first 48 hours after 9.00 am on Sept 2. Describe how you calculated this value.

4 marks

Question 3

a. Find the rule for the rate of change of $d_1(t)$.

1 mark

b. What is the maximum rate of change of $d_1(t)$ correct to 2 decimal places?

1 mark

c. If the rate of change of the water depth is large then the speed of the water flowing in and out of the port through the channel can make it difficult to manoeuvre some ships and hence make it unsafe to enter the port. The ship described in **Question 2 b** can only enter when the magnitude of the rate of change of the depth of water is less than 2 metres per hour. During the first 10 hours after 9.00 am Sept 2, what is the total time that this ship can safely enter the port (to the nearest minute)? Describe how you obtained your answer.

4 marks

The heights of the low and high tides do not actually stay constant but vary from day to day due to the changing relative positions of the sun and moon. A more refined model for the depth of water in the port that takes account of this is:

$$d_2(t) = 4.5\cos\left(\frac{\pi t}{168}\right)\sin\left(\frac{4\pi t}{25}\right) + 5$$

where, once again, t is the time in hours after 9.00 am Sept 2.

Question 4

a. Graph $y = d_2(t)$ on your graphics calculator and describe the way the difference between the heights of successive high and low tides varies with time.

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2 marks

b. Find the exact period of the variation described in part **a.** Show your working.

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2 marks

c. Find the heights (in metres) of the lowest high tide and the highest low tide predicted by $d_2(t)$ and the values of t (in hours) at which they occur. Give your answers correct to 2 decimal places.

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2 marks

Question 5

a. Find $d_2'(t)$

2 marks

b. Graph $y = d_2'(t)$ on a graphics calculator and find, during the first 4 days after 9.00 am Sept 2:

i. The coordinates of the first minimum point of $d_2'(t)$ correct to 3 decimal places.

ii. The latest time, to the nearest minute, when the magnitude of the rate of change of the depth of water is at least 2 metres per hour.

1 + 2 = 3 marks

c. Show that $d_2(t)$ will have turning points when $\tan\left(\frac{\pi t}{168}\right)\tan\left(\frac{4\pi t}{25}\right) = \frac{672}{25}$.

3 marks

END OF TASK BOOK