

Student Name: _____

MATHEMATICAL METHODS (CAS)

Unit 3

Targeted Evaluation Task for School-assessed Coursework 2



2012 Test (multiple choice, short answer, extended response) on Differentiation for Outcomes 1 & 3

Recommended writing time*: 60 minutes

Total number of marks available: 40 marks

TASK BOOK

* The recommended writing time is a guide to the time students should take to complete this task. Teachers may wish to alter this time and can do so at their own discretion.

Conditions and restrictions

- Students are permitted to bring into the room for this task: pens, pencils, highlighters, erasers, sharpeners and rulers, an approved CAS calculator.
- Students are NOT permitted to bring into the room for this task: blank sheets of paper and/or white out liquid/tape.
- A Calculator may be used for Sections 2 and 3 – Multiple Choice and Analysis Task Questions
- No calculator is permitted to be used in Section 1 –Short Answer Questions

Materials supplied

- Question and answer book of 10 pages.

Instructions

- Print your name in the space provided on the top of the front page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the room for this task.

For any question worth more than 1 mark, relevant working must be shown.

SECTION 1- Short-answer Questions

Instructions for Section 1
For any question worth more than 1 mark, relevant working must be shown.

Question 1

Consider the function $f(x) = 2x^3 + x^2 - 4x$.

- a. Find the derivative function, $f'(x)$.

_____ 1 mark

- b. Find the x -coordinates of the maximum and minimum turning points of $y = f(x)$.

_____ 2 marks

- c. Describe, in terms of x , the region or regions where $f(x)$ is increasing.

_____ 1 mark

Question 2

For the graph of $y = xe^{-2x}$ find:

- a. The gradient at $x = 1$.

_____ 2 marks

- b. The gradient of the normal to the graph at $x = 1$.

_____ 1 mark

c. The equation of the normal to the graph at $x = 1$.

2 marks

Question 3

The displacement of a car, in metres, is given by the function $f(t) = \frac{t^2}{3} + \sin\left(\frac{t}{8}\right)$, where t is in seconds. Give your answers in this question correct to 3 decimal places.

a. Use calculus to find the instantaneous velocity of the car at $t = 5$ s

2 marks

b. Find the average velocity of the car between $t = 5$ s and $t = 10$ s.

2 marks

Question 4

Find, in terms of a , the minimum value of $y = ax^2 - 4ax + 20$.

2 marks

Section 2: Multiple Choice Questions**Question 1**

The gradient of the graph of $y = f(x)$ at $x = a$ is given by:

- A. $\frac{f'(a)}{f(a)}$
- B. $f(a) - f'(a)$
- C. The gradient of the tangent to $y = f(x)$ at $x = a$
- D. $\lim_{h \rightarrow 0} f(a+h) - f(a)$
- E. $\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{f(h)}$

Question 2

$\lim_{x \rightarrow 1} \frac{(3x+1)(x-1)}{x^2 e^x - x e^x}$ is

- A. $4e^{-1}$
- B. 4
- C. $4e$
- D. 0
- E. Undefined

Question 3

$\frac{3}{x}$ could **not** be the derivative of which one of the following expressions?

- A. $3\log_e(2x) + 2$
- B. $\log_e(3x)$
- C. $\frac{\log_e(x^9)}{3}$
- D. $\log_e(4x^3) - 1$
- E. $-\log_e\left(\frac{1}{x^3}\right)$

Question 4

If $y = \frac{e^{3x}}{x^2}$ then $\frac{dy}{dx} =$

- A. $3x^2e^{3x} + 2xe^{3x}$
- B. $\frac{3e^{3x}}{2x}$
- C. $\frac{3e^{3x} - 2x}{x^4}$
- D. $\frac{3x^2e^{3x} + 2xe^{3x}}{x^4}$
- E. $\frac{e^{3x}(3x - 2)}{x^3}$

Question 5

If $f(x) = 6\sqrt{3-x^2}$ then $f'(x) =$

- A. $6\sqrt{3-2x}$
- B. $-12x\sqrt{3-x^2}$
- C. $\frac{-12x}{\sqrt{3-x^2}}$
- D. $\frac{-6x}{\sqrt{3-x^2}}$
- E. $\frac{6}{\sqrt{3-x^2}}$

Question 6

The equation of the tangent of the graph of $y = 5x^2 - 8x + 2$ at $x = 1$ is:

- A. $y = -3x + 2$
- B. $y = 2x + 2$
- C. $y = 3x + 2$
- D. $y = 2x - 3$
- E. $y = 2x - 1$

Question 7

If a quartic function has a maximum turning point at $x = -2$ and a stationary point of inflection at $x = 3$ then:

- A. $f(x)$ is increasing for $\{x : x < -2\}$ and decreasing for $\{x : x > -2\}$
- B. The only region where $f(x)$ is decreasing is $\{x : -2 < x < 3\}$.
- C. $f(x)$ is increasing for $\{x : x < -2\}$ and decreasing for $\{x : -2 < x < 3\} \cup \{x : x > 3\}$.
- D. $f(x)$ is decreasing for $\{x : x < -2\}$ and increasing for $\{x : -2 < x < 3\} \cup \{x : x > 3\}$.
- E. $f'(x) = 0$ at $x = -2$ only.

Question 8

The derivative of $x^2 \cos 4x$ is:

- A. $2x \cos 4x + 4x^2 \sin 4x$
- B. $2x(\cos 4x - 2x \sin 4x)$
- C. $2x(\cos 4x - x \cos 4)$
- D. $2x \cos 4x$
- E. $-8x \sin 4x$

Questions 9 and 10 refer to the following information

Consider the graph of $f(x) = 2xe^{-0.5x} \sin 2x$.

Question 9

The gradient of the graph when $x = 0.4$ is closest to:

- A. 0.009
- B. 0.041
- C. 0.470
- D. 1.852
- E. 3.464

Question 10

The coordinates of the second local minimum to the right of the y-axis are:

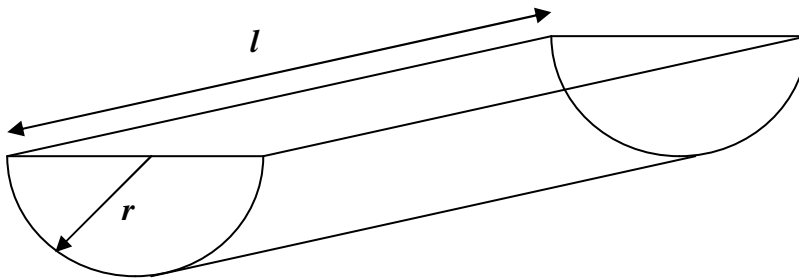
- A. (0.926, 1.120)
- B. (2.338, -1.452)
- C. (3.142, 0)
- D. (3.867, 1.111)
- E. (5.420, -0.713)

SECTION 3 - Analysis Questions

Instructions for Section 3
For any question worth more than 1 mark, relevant working must be shown.

Question 1

A water trough is in the shape of a semi-cylinder with length, l , and radius, r , as shown below.



- a.** If the volume of the trough is $62500\pi \text{ cm}^3$,
- i.** Find the length of the trough, l , in terms of r .

1 mark

- ii.** Find the external surface of the trough in terms of r .

2 marks

- iii.** Find the exact value of r for which the surface area is a minimum.

2 marks

iv. Find the minimum external surface area correct to the nearest cm^2 .

1 mark

v. Find the magnitude of the rate of change of the external area with respect to the radius when $l = 75$ cm, correct to 2 decimal places

2 marks

b. If the external surface area is 20000 cm^2 ,

i. Find the length of the trough, l , in terms of r .

1 mark

ii. Find the volume of the trough in terms of r .

2 marks

iii. Find the maximum value that r can take, correct to 2 decimal places.

1 mark

iv. Find the exact value of r for which the volume is a maximum.

2 marks

v. Find the maximum volume correct to the nearest cm^3 .

1 mark

END OF TASK BOOK