**Student Name:** _____

# MATHEMATICAL METHODS (CAS)

## Unit 3
## Targeted Evaluation Task for School-assessed Coursework 1

## 2012 Test (multiple choice, short answer, extended response) on Functions for Outcomes 1 & 3

Recommended writing time*: 60 minutes

Total number of marks available: 40 marks

**TASK BOOK**

* The recommended writing time is a guide to the time students should take to complete this task. Teachers may wish to alter this time and can do so at their own discretion.

**Conditions and restrictions**

- Students are permitted to bring into the room for this task: pens, pencils, highlighters, erasers, sharpeners and rulers, an approved CAS calculator.
- Students are NOT permitted to bring into the room for this task: blank sheets of paper and/or white out liquid/tape.
- An approved CAS calculator may be used for sections 2 and 3 – Multiple Choice and Analysis Task Questions
- No calculator is permitted for use in section 1 –Short Answer Questions

**Materials supplied**

- Question and answer book of 11 pages.

**Instructions**

- Print your name in the space provided on the top of the front page.
- All written responses must be in English.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the room for this task.**

**SECTION 1- Short-answer Questions**

**Instructions for Section 1**

**For any question worth more than 1 mark, relevant working must be shown.**

**Question 1**

**a.** Describe the transformations required to transform the graph of $y = \sin\left(x - \frac{\pi}{6}\right)$ to the graph of $y = 4\sin 2\left(x - \frac{2\pi}{3}\right) + 5$.

2 marks

**b.** Find the exact solutions of $4\sin 2\left(x - \frac{2\pi}{3}\right) + 5 = 7$ over the domain $[0, \pi]$

4 marks

**Question 2**

Show that $2\log_a 2x + \log_a 4x - 5\log_a x = -2\log_a\left(\frac{x}{4}\right)$

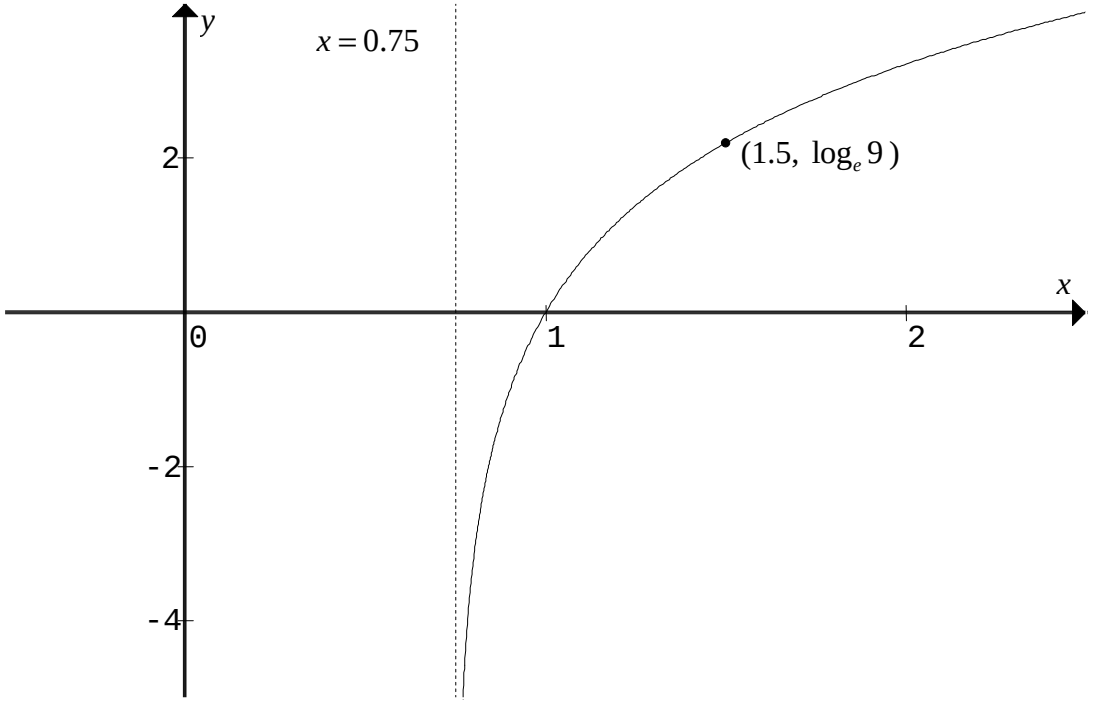
2 marks

**Question 3**

Find the exact *x*-intercepts of the graph of $y = 2x^4 + 5x^3 + x^2$.

3 marks

**Question 4**




A graph of a function of the form $f(x) = a\log_e(bx + c)$ is shown above. The graph has a vertical asymptote at $x = 0.75$, an $x$-intercept of 1 and passes through the point $(1.5, \log_e 9)$.

**a.** Find the values of $b$ and $c$.

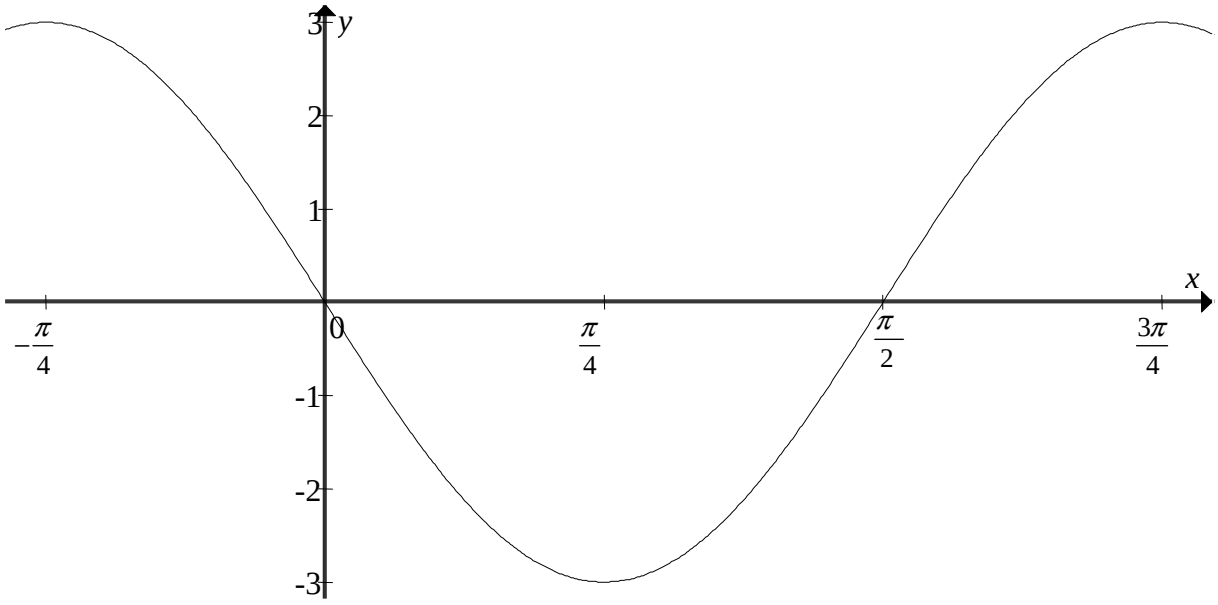
3 marks

**b.** Find the value of $a$.

1 mark

**Section 2: Multiple Choice Questions**

**Question 1**
The equation of the following graph could be:




**A.** $y = -3\sin x$

**B.** $y = -\frac{\pi}{2}\sin 3x$

**C.** $y = 3\sin\left(x - \frac{\pi}{2}\right)$

**D.** $y = 3\cos 2\left(x + \frac{\pi}{4}\right)$

**E.** $y = 3\cos 2\left(x - \frac{\pi}{4}\right)$

**Question 2**
Which of the following functions **does not** have an inverse function?

**A.** $f(x) = 3x - 4,\ x \in R$

**B.** $f(x) = (x+1)^2 - 2,\ x \le 0$

**C.** $f(x) = \cos\left(\frac{x}{2}\right),\ 0 \le x \le 2\pi$

**D.** $f(x) = 2e^{-3x},\ x \ge 0$

**E.** $f(x) = x^3,\ x \in R$

*The following information relates to Questions 3 and 4.*

Given $f(x) = \sqrt{x-4}$ and $g(x) = x^2$.

**Question 3**

$g[f(7)] =$

**A.** $\sqrt{3}$
**B.** $\sqrt{45}$
**C.** 49
**D.** 9
**E.** 3

**Question 4**

The maximal implied domain of $f[g(x)]$ is:

**A.** $\{x : x \leq -4\} \cup \{x : x \geq 4\}$
**B.** $\{x : x \geq 2\}$
**C.** $\{x : x \leq -2\} \cup \{x : x \geq 2\}$
**D.** $\{x : x \geq 4\}$
**E.** $\{x : x \geq 0\}$

**Question 5**

The rule for the inverse function of $f(x) = \sqrt{x+5} - 2$ would be:

**A.** $f^{-1}(x) = (x-2)^2 - 5$
**B.** $f^{-1}(x) = (x+5)^2 - 2$
**C.** $f^{-1}(x) = (x+2)^2 - 5$
**D.** $f^{-1}(x) = (x-5)^2 + 2$
**E.** $f^{-1}(x) = -2(x+5)^2$

**Question 6**

If $f(x) = (x-2)^2 + 3,\ 0 \leq x \leq 5$, the domain of $f^{-1}(x)$ would be:

**A.** [0, 5]
**B.** [2, 3]
**C.** [3, 5]
**D.** [3, 12]
**E.** [7, 12]

**Question 7**

The $x$-intercept of $y = \frac{1}{2}\log_e(x-1)+3$ is:

**A.** $e^{-6}+1$
**B.** $e^{-\frac{3}{2}}+1$
**C.** $1$
**D.** $2e^{-\frac{1}{3}}$
**E.** $2e^{-3}+1$

**Question 8**

If the function $f(x) = \log_e(x+3)$ is dilated away from the $x$-axis, which of the following would remain unchanged?

**A.** The $x$-intercept only
**B.** The $y$-intercept only
**C.** The asymptote only
**D.** The $x$-intercept and the $y$-intercept
**E.** The $x$-intercept and the asymptote

**Question 9**

If the graph of $y = e^x + 3$ is

- Reflected about the $x$-axis
- Translated $-2$ units parallel to the $x$-axis
- Translated 1 unit parallel to the $y$-axis

in that order, the equation of the resulting graph would be:

**A.** $y = e^{-(x-2)}+4$
**B.** $y = -e^{(x+2)}-2$
**C.** $y = -e^{(x+2)}+4$
**D.** $y = -e^{(x-1)}-5$
**E.** $y = e^{-(x-1)}+1$

**Question 10**

The graph of $y = \left|(x-2)(x-4)\right|+3,\ 0 \le x \le 5$ will have:

**A.** A minimum value at (3, -1)
**B.** A minimum value at (3, 2)
**C.** A range of [0, 1]
**D.** Minimum values at (2, 3) and (4, 3) and a range of [3, 4]
**E.** Minimum values at (2, 3) and (4, 3) and a range of [3, 11]

**SECTION 3- Analysis Questions**

**Instructions for Section 3**

**For any question worth more than 1 mark, relevant working must be shown.**

**Question 1**

**a.** A population of bacteria is increasing according to the function $f_1(t) = Ae^{kt}$, $t \geq 0$, where $t$ is the time in hours after 9 am.

At 11 am there are 5000 bacteria and at 2 pm there are 12500 bacteria.

**i.** Find the value of $k$ correct to 4 decimal places.

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2 marks

**ii.** Use your value of $k$ above to find the value of $A$ correct to the nearest whole number.

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1 mark

**iii.** Using your values for $A$ and $k$, find the time, to the nearest minute, when the bacteria population will be 30000.

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2 marks

**b.** A second bacteria population grows according to the function
$f_2(t) = Be^{nt}$, $t \geq 0$, where $t$ is the time in hours after 9 am.

**i.** Write down the value of $B$ if, at 9 am, the number of bacteria in this population is one third of number of bacteria in the population in part **a.**

1 mark

**ii.** Write down the value of $n$ if this population has the same initial population as the population in part **a.** but grows at twice the rate of the population in part **a.**

1 mark

**c.** Two other populations of bacteria are growing according to the functions
$g(x) = 2e^{2t} + 5,\ t \geq 0$ and
$h(x) = 11e^{t},\ t \geq 0$
Find the exact time (or times), $t$, when the two population are equal.

5 marks

**d.** An antibiotic solution is introduced into one of the bacteria populations. The bacteria start decreasing according to the function, $P = 16000e^{-0.2t} + 500,\ t \geq 0$

**i.** Find the rule of the function that gives $t$ in terms of $P$.

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1 mark

**ii.** Find the time, $t$, when the population of the bacteria is one tenth of its initial value. Give your answer in hours correct to 2 decimal places.

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2 marks

**END OF TASK BOOK**