MATHEMATICAL METHODS (CAS)

Unit 3

Targeted Evaluation Task for School-assessed Coursework 1

TSSM™
Creating VCE Success

2012 Test (multiple choice, short answer, extended response) on Functions for Outcomes 1 & 3

SOLUTIONS & RESPONSE GUIDE

SECTION 1- Short-answer Questions

Question 1

a.

- Translation of $\frac{\pi}{2}$ units parallel to the *x*-axis.
- Dilation by a factor of $\frac{1}{2}$ away from the *y*-axis.
- Dilation by a factor of 4 away from the *x*-axis.
- Translation of 5 units parallel to the *y*-axis.

($\frac{1}{2}$ mark for each correct transformation)

b. $4\sin 2\left(x-\frac{2\pi}{3}\right)=2$

$\sin 2\left(x-\frac{2\pi}{3}\right)=\frac{1}{2},\ 0\le x\le \pi$

(1 mark)

$\sin 2\left(x-\frac{2\pi}{3}\right)=\frac{1}{2},\ -\frac{2\pi}{3}\le x-\frac{2\pi}{3}\le \frac{\pi}{3}$

$\sin 2\left(x-\frac{2\pi}{3}\right)=\frac{1}{2},\ -\frac{4\pi}{3}\le 2\left(x-\frac{2\pi}{3}\right)\le \frac{2\pi}{3}$

(1 mark)

$2\left(x-\frac{2\pi}{3}\right)=-\frac{7\pi}{6},\ \frac{\pi}{6}$

(1 mark)

$x=\frac{\pi}{12},\ \frac{3\pi}{4}$

(1 mark)

Question 2

$\text{LHS}=\log_a 4x^2+\log_a 4x-\log_a x^5$

$=\log_a\left(\frac{16x^3}{x^5}\right)=\log_a\left(\frac{16}{x^2}\right)$

(1 mark)

$=\log_a\left(\frac{4}{x}\right)^2$

$=\log_a\left(\frac{x}{4}\right)^{-2}$

$=-2\log_a\left(\frac{x}{4}\right)=\text{RHS}$

(1 mark)

Question 3

$2x^4 + 5x^3 + x^2 = 0$

$x^2(2x^2 + 5x + 1) = 0$

$x = 0$ is one solution and $2x^2 + 5x + 1 = 0$ will give the other solutions

(1 mark)

Using the quadratic rule on $2x^2 + 5x + 1 = 0$

$$x = \frac{-5 \pm \sqrt{25-8}}{2\times 2} = \frac{-5 \pm \sqrt{17}}{4}$$

$\therefore$ the other two solutions are

$$x = \frac{-5-\sqrt{17}}{4} \text{ and } \frac{-5+\sqrt{17}}{4}$$

(2 mark)

Question 4

a. The asymptote at $x = 0.75$ means that

$0.75b + c = 0$

$c = -0.75b$ (1)

(1 mark)

The x-intercept at 1 means that

$b + c = 1$ (2)

(1 mark)

And combining equations (1) and (2)

$b - 0.75b = 1$

$b = 4$

$c = -3$

(1 mark)

b. $y = a\log_e(4x-3)$ using values from part **a.**

Substituting in the given point values

$a\log_e 3 = \log_e 9 = \log_e 3^2 = 2\log_e 3$

$a = 2$

(1 mark)

SECTION 2 Multiple-choice questions (1 mark each)

Question 1

Answer: D

Explanation

Graph can be considered a cosine graph of the form $y = a\cos n(x+b)$. a = amplitude = 3.
Period, T = π and $n = \frac{2\pi}{T} = \frac{2\pi}{\pi} = 2$. There is a horizontal shift of $-\frac{\pi}{4}$ units, so b = $\frac{\pi}{4}$

Question 2

Answer: B

Explanation

A function will have an inverse if and only if it is a one-to-one function. All the functions are one-to-one over their given domains except for B.

Question 3

Answer: E

Explanation

$g[f(x)] = (\sqrt{x-4})^2 = x-4$

$\therefore g[f(7)] = 7-4 = 3$

Question 4

Answer: C

Explanation

$f[g(x)] = \sqrt{x^2-4}$ which is defined for $x^2 - 4 \geq 0$

$\therefore x^2 \geq 4$

$x \geq |2|$

$x \leq -2$ or $x \geq 2$

Question 5

Answer: C

Explanation

Let $y = \sqrt{x+5} - 2$ and interchange *x* and *y*.

$x = \sqrt{y+5} - 2$

$(x+2)^2 = y+5$

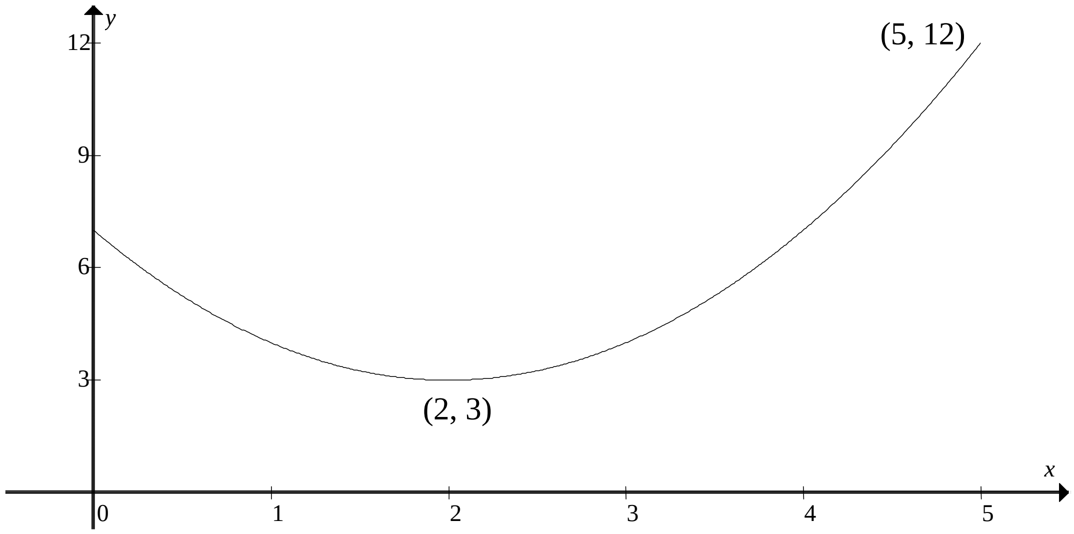
$y = (x+2)^2 - 5$

Question 6

Answer: D

Explanation

$dom\ f^{-1}(x) = ran\ f(x) = [3,12]$ from the graph of $y = f(x)$ below.



Question 7

Answer: A

Explanation

Let $0 = \frac{1}{2}\log_e(x-1)+3$

$-6 = \log_e(x-1)$

$e^{-6} = x-1$

$x = e^{-6}+1$

Question 8

Answer: E

Explanation

The *y*-intercept will change but the other two will remain the same.

Question 9

Answer: B

Explanation

Reflection gives $y = -(e^x+3) = -e^x - 3$

First translation gives $y = -e^{(x+2)} - 3$

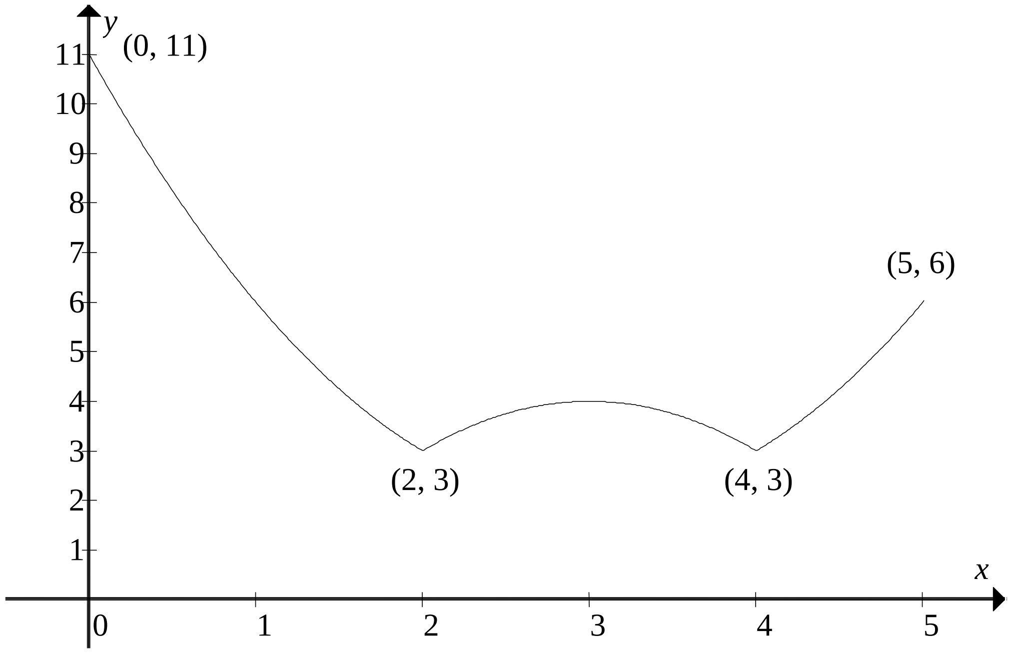
Second translation gives $y = -e^{(x+2)} - 2$

Question 10

Answer: E

Explanation

The graph of $y = |(x-2)(x-4)| + 3,\ 0 \le x \le 5$ is shown below.



The *y*-values range from 3 to 11.

SECTION 3- Analysis Questions

a.

i. Substituting the given values into the function

$Ae^{2k} = 5000 \quad (1)$

$Ae^{5k} = 12500 \quad (2)$

(1 mark)

Dividing (2) by (1)

$e^{3k} = 2.5$

$k = \frac{\log_e 2.5}{3} = 0.3054$

(1 mark)

ii. Substituting for k in equation (1) and rearranging

$A = \frac{5000}{e^{0.6108}} = 2715$

(1 mark)

iii. $2715e^{0.3054t} = 30000$

$0.3054t = \log_e\left(\frac{30000}{2715}\right)$

$t = \frac{\log_e\left(\frac{30000}{2715}\right)}{0.3054} = 7.866 \text{ hrs}$

(1 mark)

$t = 7$ hrs 52 min

Time will be 4.52 pm

(1 mark)

b.

i. $B = \frac{A}{3} = 905$

(1 mark)

ii. $n = 2k = 0.6108$

(1 mark)

c. Let $g(x) = h(x)$

$2e^{2t} + 5 = 11e^{t}$

$2e^{2t} - 11e^{t} + 5 = 0$

(1 mark)

Let $e^{t} = x$

$2x^2 - 11x + 5 = 0$

(1 mark)

$(2x-1)(x-5) = 0$

$x = \frac{1}{2}$ or 5

(1 mark)

$e^t = \frac{1}{2}$ or 5

$t = \log_e \frac{1}{2}$ or $\log_e 5$

(1 mark)

But $\log_e \frac{1}{2} < 0$ and $t \geq 0$

$\therefore t = \log_e 5$ is the only solution

(1 mark)

d.

i. $\frac{P-500}{16000} = e^{-0.2t}$

$t = -5\log_e\left(\frac{P-500}{16000}\right)$

(1 mark)

ii. Initial population $= 16000 + 500 = 16500$ and $\frac{16500}{10} = 1650$

(1 mark)

$t = -5\log_e\left(\frac{1650-500}{16000}\right) = 13.16$ hrs

(1 mark)