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**‘2016 Examination Package’ - Trial Examination 2 of 5**

**STUDENT NUMBER**

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Figures | | | | | | | | | | Letter |
| Words | | | | | | | | | | |

# MATHEMATICAL METHODS

## Units 3 & 4 – Written examination 2

## (TSSM’s *2012 trial exam updated for the current study design)*

Reading time: 15 minutes
Writing time: 2 hours

## QUESTION & ANSWER BOOK

**Structure of book**

| *Section* | *Number of questions* | *Number of questions to be answered* | *Number of marks* |
|---|---|---|---|
| 1 | 22 | 22 | 22 |
| 2 | 5 | 5 | 58 |
| | | | Total 80 |

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator (memory DOES NOT need to be cleared) and, if desired, one scientific calculator
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

**Materials supplied**

- Question and answer book of 22 pages including answer sheet for multiple-choice questions.

**Instructions**

- Print your name in the space provided on the top of this page and the multiple-choice answer sheet.
- All written responses must be in English.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the examination room.**

## SECTION 1 –Multiple-choice questions

| Instructions for Section 1 |
|---|
| Answer all questions on the answer sheet provided for multiple choice questions. |
| Choose the response that is **correct** for the question. |
| A correct answer scores 1, an incorrect answer scores 0. |
| Marks will **not** be deducted for incorrect answers. |
| No marks will be given if more than one answer is completed for any question. |

### Question 1

The amplitude and period of the graph $y = -2cos3\left(x - \frac{\pi}{3}\right), 0 \le x \le 2\pi$ is:

**A.** $3\ and\ \pi$

**B.** $2\ and\ \frac{2\pi}{3}$

**C.** $3\ and\ \frac{2\pi}{3}$

**D.** $-2\ and\ \frac{2\pi}{3}$

**E.** $\frac{1}{2}\ and\ \frac{\pi}{3}$

### Question 2

Let $f: R\setminus\{-1\} \to R,\ f(x) = \frac{kx-4}{x+1}$. The positive value of $k$ for which there is only one value of $x$ satisfying $f(x) = x$ is:

**A.** 3

**B.** 4

**C.** 5

**D.** 6

**E.** 7

### Question 3

The implied domain of the inverse of the function $f: (3, \infty) \to R,\ f(x) = log_e(x-3)$ is:

**A.** $[3, \infty)$

**B.** $(3, \infty)$

**C.** $(-3, \infty)$

**D.** $(0, \infty)$

**E.** $R$

**SECTION 1** - continued

**Question 4**

If $f(x) = e^{x} - e^{-x}, x \in R$, the function $\left(f(x)\right)^3$ is the same as:

**A.** $f(x^3)$

**B.** $f(3x) - 3f(x)$

**C.** $3f(x)$

**D.** $f(3x) + f(x)$

**E.** $f(x) - 3f(x)$

**Question 5**

If $\frac{f(1+h)-f(1)}{h} = 4h + 8$, then $f'(1)$ equals:

**A.** 6

**B.** 10

**C.** 12

**D.** 0

**E.** 8

**Question 6**

If $y = ax^2 + bx$ has a maximum at (2 , 3), the values of *a* and *b* are:

**A.** $\frac{3}{4}\ and -3$

**B.** $-\frac{3}{4}\ and -3$

**C.** $\frac{3}{4}\ and\ 3$

**D.** $-\frac{3}{4}\ and\ 3$

**E.** $3\ and\ -\frac{3}{4}$

**Question 7**

If $y = F(x)$and $\frac{dy}{dx} = f(x)$, then $\int_2^3 f(x)dx$ equals:

**A.** $F(3) - F(2)$

**B.** $f(3) - f(2)$

**C.** $f(3) - F(2)$

**D.** $F(3) - f(2)$

**E.** $f(x) + c, where\ c\ is\ a\ constant$

**SECTION 1** - continued

**TURN OVER**

**Question 8**

A line with gradient $m$, $m < 0$, passes through the point (1 , 2). The value of $m$ for which the area enclosed by the line and the two axes is a minimum is:

**A.** $-6$

**B.** $-5$

**C.** $-4$

**D.** $-3$

**E.** $-2$

**Question 9**

If $\int_0^a (3x - 6)dx = 0$, then the value(s) of $a$ is (are):

**A.** $0\ and\ 4$

**B.** $0\ only$

**C.** $4\ only$

**D.** $-3\ and\ 4$

**E.** $-3\ only$

**Question 10**

The scores on a test are normally distributed with mean $\mu$ and standard deviation $\sigma$. About 95% of the scores on the test are between 42 and 58. Which one of the following is true?

**A.** $\mu = 46\ and\ \sigma = 4$

**B.** $\mu = 55\ and\ \sigma = 5$

**C.** $\mu = 50\ and\ \sigma = 4$

**D.** $\mu = 50\ and\ \sigma = 5$

**E.** $\mu = 52\ and\ \sigma = 10$

**SECTION 1** - continued

**Question 11**

If $x = 5$ is a solution of the equation $log_e(ax + 3) = 4$, then the exact value of $a$ is:

**A.** $\frac{e^4}{5} + 3$

**B.** $\frac{e^4-3}{5}$

**C.** $\frac{e^4}{5} - 3$

**D.** $\frac{log_a(4)-3}{5}$

**E.** $\frac{e}{3}$

**Question 12**

The continuous random variable $X$ has a normal distribution with mean 5.8 and variance 1.69. The continuous random variable $Z$ has the standard normal distribution. The probability that $X$ is less than 4.5 is equal to

**A.** $\Pr(Z > -1)$

**B.** $\Pr(Z < 1)$

**C.** $\Pr(Z > 1)$

**D.** $1 - \Pr(Z < -1)$

**E.** $\Pr(-1 < Z < 1)$

**SECTION 1** – continued
**TURN OVER**

**Question 13**

The following transformations of the plane as given by
$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 3 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 2 \\ -4 \end{bmatrix}$ are:

**A.** dilation by factor 3 from *x*-axis, reflection in *y*-axis, translation 2 units to the right and 4 units down

**B.** dilation by factor 3 from *y*-axis, reflection in *x*-axis, translation 2 units to the right and 4 units down

**C.** dilation by factor $\frac{1}{3}$ from *y*-axis, reflection in *x*-axis, translation 2 units to the left and 4 units up

**D.** dilation by factor $\frac{1}{3}$ from *y*-axis, reflection in *x*-axis, translation 2 units to the right and 4 units down

**E.** dilation by factor 3 from *x*-axis, reflection in *x*-axis, translation 2 units to the left and 4 units down

**Question 14**

If $f(x) = e^x$ *and* $g(x) = \sin(x)$ *and* $h(x) = f(g(x))$, which of the following statements about $h(x)$ is correct?

**A.** The maximal domain of $h$ is $R^+$

**B.** Local minimum turning points occur at $x = (2n-1)\pi + \frac{\pi}{2},\ n \in Z$

**C.** $h(x) = \sin(e^x)$

**D.** Range of $h$ is $[0, e]$

**E.** The minimum turning points are on the *x* axis.

**SECTION 1** – continued

**Question 15**

The average value of the function $y = \tan(x)$ for the interval $[\frac{\pi}{8}, \frac{\pi}{4}]$ is:

**A.** $\frac{8}{\pi} log_e(\frac{\sqrt{2}+2}{2})$

**B.** $\frac{1}{2} log_e\left(\frac{\sqrt{2}+2}{2}\right)$

**C.** $\frac{8}{\pi}$

**D.** $\frac{4}{\pi} log_e(\frac{\sqrt{2}+2}{2})$

**E.** $\infty$

**Question 16**

A graph has a rule $f(x) = -(x+2)(x-2)^3 + 2$. At $x = 2$, the graph has a:

**A.** *x*-axis intercept

**B.** stationary point of inflection

**C.** *y*-axis intercept

**D.** local maximum

**E.** local minimum

**Question 17**

The sum of the solutions of $\tan(2x) = \sqrt{3}\ for\ 0 \leq x \leq \pi$ is:

**A.** $\frac{5\pi}{3}$

**B.** $\frac{11\pi}{3}$

**C.** $\frac{\pi}{6}$

**D.** $\frac{5\pi}{6}$

**E.** $\frac{22\pi}{3}$

**SECTION 1** – continued
**TURN OVER**

**Question 18**

The length of the wires sold at a store is a random variable *X* metres with a probability density function $f(x) = \begin{cases} 4e^{-4x}, & for\ x > 0 \\ 0, & elsewhere \end{cases}$. The value of $k$ such that 80% of wires are less than *k* metres is closest to:

**A.** 0.056
**B.** 0.402
**C.** 0.438
**D.** 0.893
**E.** 1.609

**Question 19**

A binomial random variable has a mean of 80 and standard deviation of 4. The values of *n* and *p* respectively are:

**A.** $100\ and\ \frac{1}{5}$
**B.** $100\ and\ \frac{4}{5}$
**C.** $200\ and\ \frac{2}{5}$
**D.** $400\ and\ \frac{4}{5}$
**E.** $1600\ and\ \frac{1}{20}$

**Question 20**

The coordinates of the turning point of $y = log_e(2x) - 2x$ are:

**A.** $\left(\frac{1}{4}, log_e\frac{1}{2} - \frac{1}{2}\right)$
**B.** $\left(\frac{1}{4}, 0\right)$
**C.** $\left(\frac{1}{2}, -1\right)$
**D.** $\left(\frac{1}{2}, log_e 2\right)$
**E.** $\left(-\frac{1}{2}, -1\right)$

**SECTION 1** – continued

**Question 21**

Given that $\log_a 5 + 2\log_a(2x+1) = \log_a 45$, then **all** the values of $x$ which satisfy the equation are:

**A.** $2$

**B.** $1, 2$

**C.** $-2,1$

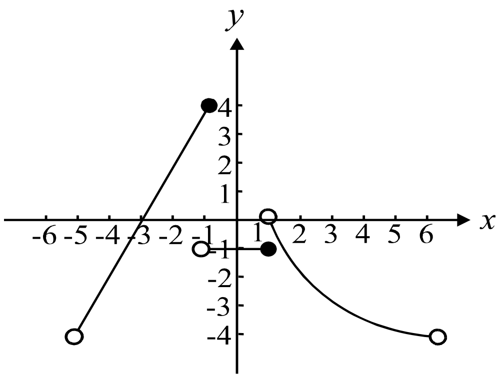
**D.** $0,1$

**E.** $1$

**SECTION 1** – continued
**TURN OVER**

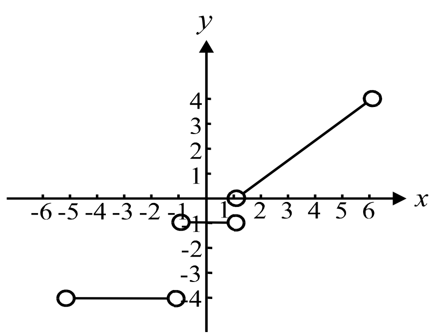
**Question 22**

The graph of the function $g$ is shown below:

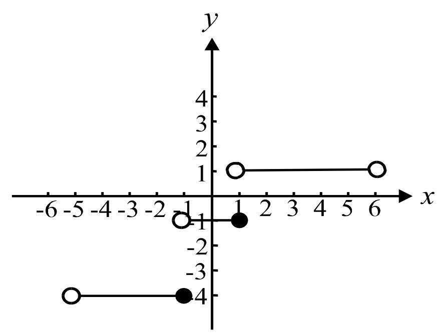


The gradient function of $g$ is closest to:

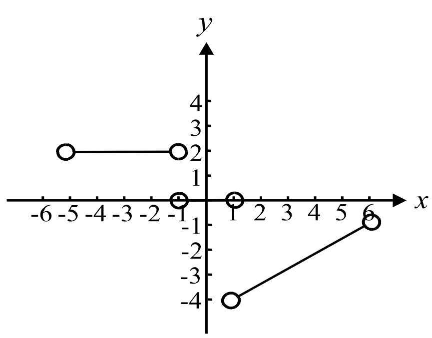
**A.**



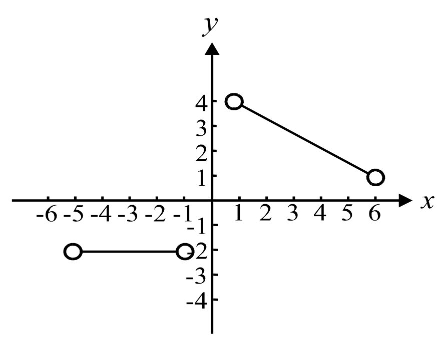
**B.**



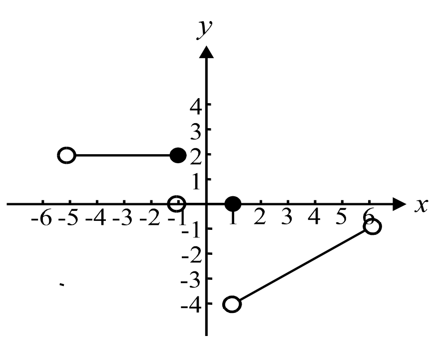
**C.**



**D.**



**E.**



**END OF SECTION 1**

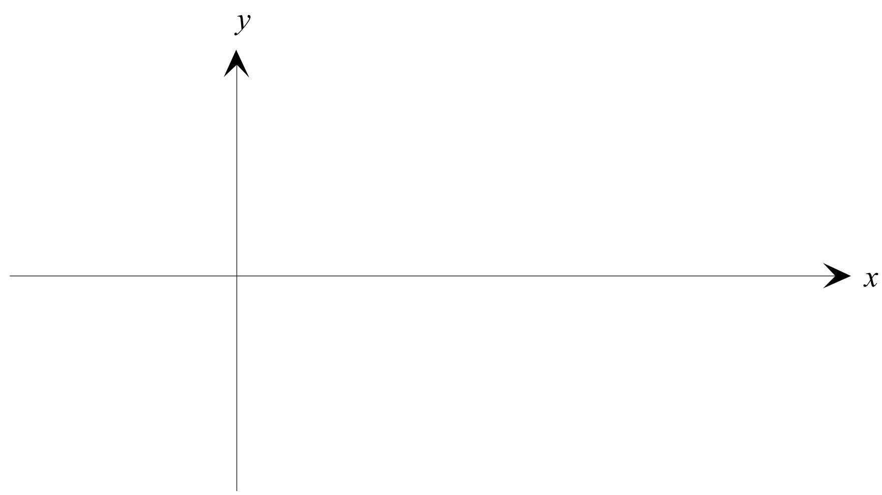
## SECTION 2

| **Instructions for Section 2** |
|---|
| Answer **all** questions in the spaces provided. |
| A decimal approximation will not be accepted if an **exact** answer is required to a question. |
| In questions where more than one mark is available, appropriate working **must** be shown. |
| Where an instruction to **use calculus** is stated for a question, you must show an appropriate derivative or anti-derivative. |
| Unless otherwise indicated, the diagrams in this book are **not** drawn to scale. |

### Question 1

Consider the function $f: R \rightarrow R$, $f(x) = \frac{1}{2}x^2(x-8) + 12$

**a.** Sketch the graph of $y = f(x)$ on the axes below.
Label intersection points with the axes and stationary points with their exact coordinates.




3 marks

**SECTION 2 – Question 1** - continued
**TURN OVER**

**b.** Find the exact area of the region enclosed by the graph of $y = f(x)$ and the $x$ axis.

2 marks

Given $g$: $R \rightarrow R$, $g(x) = ax^2(x - 8)$ where $a > 0$

**c.** Show that the equation of the tangent to the curve $y = g(x)$, in terms of $a$, at the point where $x = 1$ is $y = 6a - 13ax$

3 marks

**SECTION 2** – **Question 1** - continued

**d.** Hence, find, in terms of $a$, the area of the region bounded by the curve $y = g(x)$ and the tangent to the curve, $y = 6a - 13ax$.

3 marks

**e.** Find the value of $a$ so that the tangent to the curve $y = g(x)$ at $(1, g(1))$ is also the normal to the curve at the second point of intersection of the tangent with the curve.

3 marks

Total 14 marks

**SECTION 2** – continued

**TURN OVER**

**Question 2**

John keeps hens in his backyard. He regularly records the weights of the eggs that they lay and finds that the weights are normally distributed with a mean of 61 grams and a standard deviation of 8 grams.

One afternoon John checks to find a fresh laid egg in the hen coop.

**a.** Calculate the probability, correct to four decimal places, that the egg weighs more than 67 grams.

1 mark

**b.** Calculate the probability, correct to four decimal places, that the egg weighs more than 67 grams, given that he knows it weighs more than 61 grams.

2 marks

**c.** Find the standardised value for the weight of an egg that is 59 grams.

1 mark

**d.** Hence, calculate the probability, correct to four decimal places, that the egg weighs less than 59 grams.

1 mark

**SECTION 2 – Question 2** - continued

The next morning, John finds 6 freshly laid eggs in the coop.

**e.** Find the probability that at least two of the eggs weigh more than 67 grams.

1 mark

John's neighbors, Kath and Kim, also keep hens, and they lay eggs whose weights are normally distributed with a standard deviation of only 2 grams.
Kath and Kim brag that 98% of their eggs weigh more than 67 grams.

**f.** Find the mean weight of Kath and Kim's eggs be? Give your answer correct to four decimal places.

2 marks

Total 8 marks

**Question 3**

Cooper is learning to swim. In his first 10 attempts at swimming 50m unassisted he is successful 7 times.

**a.** Find the sample proportion of successfully completing the 50m swim.

1 mark

**b.** In his next 4 attempts at swimming, Cooper improves his sample proportion of successfully completing the 50m to $\hat{p} = a$. If the sample proportion has an approximately normal distribution and the standard deviation is equal to 0.2, find the value of $a$.

2 marks

**c.** Use your answer from part **b.** to state the 95% confidence interval for *p*.

1 mark

After more training the length of time Cooper can now swim follows a normal distribution, with a mean time of 55 minutes. The probability that he stays in the water for more than 80 minutes is 0.04.

**d.** Find the standard deviation of the time he spends in water, correct to 2 decimal places.

2 marks

**SECTION 2** – **Question 3** - continued

**e.** Hence find the probability that he is in the water for less than one hour, given that he stays in at least 30 minutes, correct to 2 decimal places.

2 marks

Total 8 marks

**Question 4**

The number of mice, *M*, during a 12 week period in a farm can be modeled by the following rule

$$M(t) = 120 + 40sin\left(\frac{\pi}{3}\left(t - \frac{3}{2}\right)\right)$$

Where *t* is the number of weeks from when the mice were first counted.

**a.** State the minimum and maximum number of mice in the 12-week period.

2 marks

**b.** How many mice are there after 5 weeks?

1 mark

**SECTION 2 – Question 4** - continued
**TURN OVER**

**c.** At what time(s) are there 140 mice? Show all working out.

3 marks

The farmer decides to travel during the weeks when the mice numbers are 150 or more.

**d.** For how long does he stay away in total during the 12-week period, correct to two decimal places?

2 marks

**e.** Consider the two-week period when the mice numbers are at their lowest level. What is the maximum number of mice on the farm during this period?

2 marks

**SECTION 2 – Question 4** - continued

**f.** **i.** Show that the derivative can be expressed as $M'(t) = \frac{40\pi}{3}\sin\left(\frac{\pi t}{3}\right)$ .

1 mark

**ii.** Hence find the times when the mice are increasing at a rate of $\frac{20\pi}{3}$ mice per week.

2 marks

Total 13 marks

**Question 5**

A cuboid has dimensions $x$ metres, $h$ metres and $4x$ metres. The cuboid is made of 240 metres of wire.

**a.** Find $h$ in terms of $x$.

2 marks

**b.** Find the volume, $V\text{m}^3$ , of the cuboid in terms of $x$.

1 mark

**SECTION 2 – Question 5** - continued
**TURN OVER**

**c.** Find the volume when $x = 11$.

2 marks

**d.** Find the possible values of $x$ for which the cuboid will exist.

2 marks

**e.** Find the possible values of $x$ when $V = 1620$, correct to two decimal places.

1 mark

**f.** Find the maximum volume of the cuboid and the value of $x$ when this exists.

2 marks

Total 10 marks

**SECTION 2** – continued

**Question 6**

**a.** Find the points of intersection of $y = 2\cos(3x)$ and $y = 1$ for $x \in [0, \pi]$.

2 marks

**b.** Determine the value(s) of k for which the straight line with equation $y = x - 2k$ ($k \in R$) doesn't intersect the parabola with equation $y = x^2 - 2x + 1$.

3 marks

Total 5 marks

**END OF QUESTION AND ANSWER BOOK**

# MULTIPLE CHOICE ANSWER SHEET

**Student Name**:___________________________________

Circle the letter that corresponds to each correct answer.

| Question | | | | | |
|---|---|---|---|---|---|
| 1 | A | B | C | D | E |
| 2 | A | B | C | D | E |
| 3 | A | B | C | D | E |
| 4 | A | B | C | D | E |
| 5 | A | B | C | D | E |
| 6 | A | B | C | D | E |
| 7 | A | B | C | D | E |
| 8 | A | B | C | D | E |
| 9 | A | B | C | D | E |
| 10 | A | B | C | D | E |
| 11 | A | B | C | D | E |
| 12 | A | B | C | D | E |
| 13 | A | B | C | D | E |
| 14 | A | B | C | D | E |
| 15 | A | B | C | D | E |
| 16 | A | B | C | D | E |
| 17 | A | B | C | D | E |
| 18 | A | B | C | D | E |
| 19 | A | B | C | D | E |
| 20 | A | B | C | D | E |
| 21 | A | B | C | D | E |
| 22 | A | B | C | D | E |