Neap:

Trial Examination 2012

VCE Mathematical Methods (CAS) Units 3 & 4

Written Examination 2

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 2 hours

Student's Name: ______________________________

Teacher's Name: ______________________________

Structure of Booklet

Section	Number of questions	Number of questions to be answered	Number of marks
1	22	22	22
2	4	4	58
			Total 80

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator (memory DOES NOT need to be cleared) and, if desired, one scientific calculator. For approved computer-based CAS, their full functionality may be used.

Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

Question and answer booklet of 20 pages, with detachable sheet of miscellaneous formulas in the centrefold.
Answer sheet for multiple-choice questions.

Instructions

Detach the formula sheet from the centre of this book during reading time.

Write **your name** and **teacher's name** in the space provided above on this page.

All written responses must be in English.

At the end of the examination

Place the answer sheet for multiple-choice questions inside the front cover of this booklet.

Students are NOT permitted to bring mobile phones and/or any other electronic communication devices into the examination room.

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2012 VCE Mathematical Methods (CAS) Units 3 & 4 Written Examination 2.

SECTION 1

Instructions for Section 1

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.

Question 1

The curve with equation $y = 3 + 2\cos\left(\frac{x}{4}\right)$ has

A. an amplitude of 5 and a period of $\frac{\pi}{2}$

B. an amplitude of 2 and a period of $\frac{\pi}{2}$

C. an amplitude of 2 and a period of 8π

D. an amplitude of 3 and a period of 8π

E. an amplitude of 2 and a period of 8

Question 2

The function g satisfies the functional equation $g(x + y) = g(x)g(y)$.

The rule for the function is

A. $g(x) = x^2$

B. $g(x) = x$

C. $g(x) = \cos(x)$

D. $g(x) = e^x$

E. $g(x) = \sin(x)$

Question 3

The maximum value of the function $g(x) = \left|\sin(x) - \frac{1}{2}\right|$ is

A. $\frac{3\pi}{2}$

B. $\frac{\pi}{2}$

C. $\frac{3}{2}$

D. 1

E. $\frac{1}{2}$

Question 4

When the area, A m^2, of a contracting circle is decreasing twice as fast as its radius, r m, the radius is

A. $\frac{1}{\pi}$

B. $\frac{1}{4\pi}$

C. $\frac{1}{4}$

D. 1

E. π

Question 5

The maximal domain D of the function $f: D \to R, f(x) = \log_e(x^2 - 8)$ is

A. $x \in R$

B. $|x| \le 2\sqrt{2}$

C. $|x| < 2\sqrt{2}$

D. $|x| \ge 2\sqrt{2}$

E. $|x| > 2\sqrt{2}$

Question 6

The simultaneous linear equations

$$px - 2y = 0$$
$$3x - (p + 1)y = 0$$

where p is a real constant, have a unique solution provided that

A. $p \in \{-3, 2\}$

B. $p \in R\backslash\{-3, 2\}$

C. $p \in \{-2, 3\}$

D. $p \in R\backslash\{-2, 3\}$

E. $p \in R\backslash\{0\}$

Question 7

If $\int_1^9 g(x)dx = 4$ and $\int_9^3 g(x)dx = 6$, then $\int_1^3 g(x)dx$ is equal to

A. -1

B. 0

C. 1

D. 9

E. 10

Question 8

The continuous random variable X has a normal distribution with mean μ and standard deviation 2.

If the random variable Z has the standard normal distribution, then $\Pr(X > \mu + 3)$ is equal to

A. $\Pr(Z < 2)$

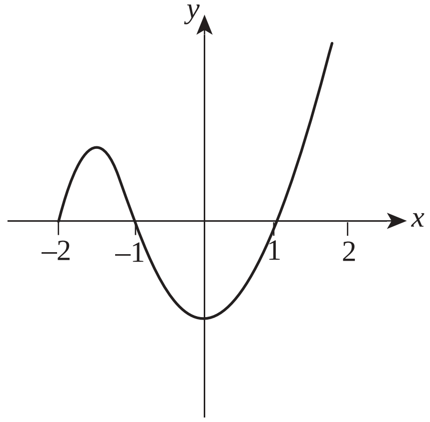
B. $\Pr(Z < -3)$

C. $\Pr\left(Z < -\frac{3}{2}\right)$

D. $\Pr\left(Z < \frac{3}{2}\right)$

E. $\Pr(Z > 2)$

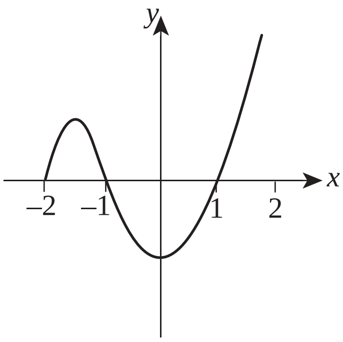
Question 9



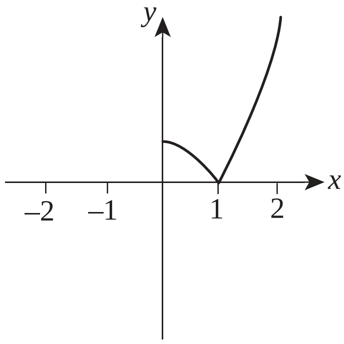
The graph of $y = h(x)$ is shown above.

The graph of $y = h(|x|)$ is

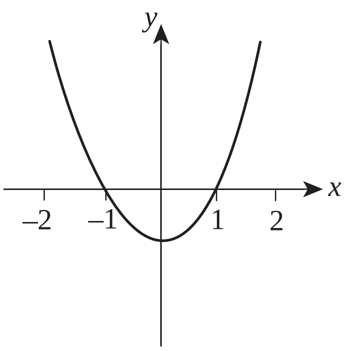
A.



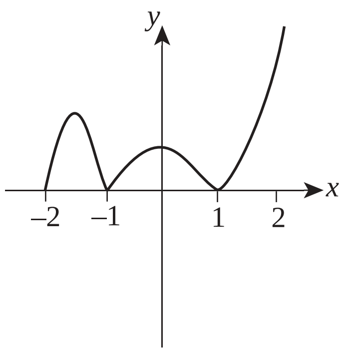
B.



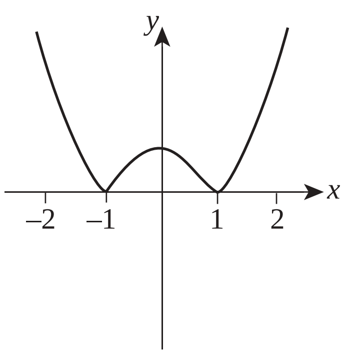
C.



D.



E.



Question 10

Consider the following random variables:

i The number of goals scored in a netball game

ii The time, in seconds, it takes for a computer to shut down

iii The amount of water passing through a pipe

iv The amount of money donated to a charity

v The weight of chicken eggs

Which of the above are continuous random variables?

A. ii, iii and v
B. ii and v
C. ii, iii, iv and v
D. i and iv
E. i, ii, iii, iv and v

Question 11

If $p(x) = (x + 2)^2(x - k)$ and the remainder is 36 when $p(x)$ is divided by $x - 1$, then k is equal to

A. 6
B. 4
C. 3
D. –3
E. –4

Question 12

If the equation $e^{2x} = b$ has solution $x = \log_e(3)$ for $b > 0$, then the value of b is

A. $\log_e(3)$
B. $\log_e(9)$
C. 3
D. 9
E. e^3

Question 13

A transformation $T: R^2 \to R^2$ that maps the curve $y = g(x)$ onto the curve with equation $y = 1 - 4g(2x + 3)$ is given by

A. $T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

B. $T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} & 0 \\ 0 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} \frac{3}{2} \\ 1 \end{bmatrix}$

C. $T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 & -4 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

D. $T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -\frac{3}{2} \\ -1 \end{bmatrix}$

E. $T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -\frac{3}{2} \\ 1 \end{bmatrix}$

Question 14

The graph of $y = kx - 1$ intersects the graph of $y = x^2 + 5x$ at two distinct points for

A. $3 \le k \le 7$

B. $k < 3$ or $k > 7$

C. $k = 3$

D. $k = 5$

E. $4 \le k \le 6$

Question 15

f and g are differentiable functions, such that

$f(1) = 4$, $f'(1) = 6$, $f'(4) = -8$,

$g(1) = 4$, $g'(1) = -6$, $g'(4) = 10$.

If $h(x) = f(g(x))$, then $h'(1)$ is equal to

A. 60

B. 48

C. 24

D. –36

E. –80

Question 16

The set of values of d for which the graph of $y = x^3 - 6x^2 + d$ has three distinct x-intercepts is

A. $d > 0$

B. $d < 0$

C. $d = 0, 32$

D. $0 < d < 32$

E. $d \in R$

Question 17

A discrete random variable X has a mean of 8 and standard deviation 8.

The expected value of X^2, $E(X^2)$ is equal to

A. 128

B. 72

C. 64

D. 56

E. 16

Question 18

A secondary school canteen serves either an apple or an orange each morning of a school week (Monday–Friday). A survey found that 20% of students who eat an orange on a particular day eat an apple the next day, while 25% of students who eat an apple on a particular day eat an orange the next day.

On a particular Monday morning, 800 students eat an apple and 800 eat an orange.

The number of students predicted to eat an apple on Wednesday of that week is

A. 738

B. 760

C. 800

D. 840

E. 862

Question 19

X is a binomial random variable with parameters n and p.

If $\Pr(X = 0) = k$, then $\Pr(X = 1)$ is equal to

A. $\dfrac{n(1-p)k}{p}$

B. $\dfrac{1-p}{npk}$

C. $\dfrac{npk}{1-p}$

D. $\dfrac{n(n-1)p^2k}{2(1-p)^2}$

E. $\dfrac{pk}{1-p}$

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Question 20

If the graph of $y = \dfrac{mx + n}{x + p}$ has a vertical asymptote $x = -4$ and a horizontal asymptote $y = 3$, then $m + p$ is equal to

A. -7
B. -1
C. 0
D. 1
E. 7

Question 21

w is a differentiable function, such that $w(x) < 0$ for $x \in R$.

If $v'(x) = (x^2 - 9)w(x)$, which one of the following is true?

A. v has a local maximum at $x = -3$ and a local minimum at $x = 3$
B. v has a local minimum at $x = -3$ and a local maximum at $x = 3$
C. v has local minima at $x = -3$ and $x = 3$
D. v has local maxima at $x = -3$ and $x = 3$
E. v does not have a local maximum or a local minimum.

Question 22

If $g'(x)$ and $h'(x)$ exist, and $g'(x) > h'(x)$ for $x \in R$, then the graph of $y = g(x)$ and the graph of $y = h(x)$

A. do not intersect.
B. intersect exactly once.
C. could intersect more than once.
D. intersect no more than once.
E. have a common tangent at each point of intersection.

END OF SECTION 1

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Question 1

On a particular day, the temperature, T °C, at time t hours after midnight can be modelled by

$$T(t) = 21 - 4\cos\left(\frac{\pi t}{12}\right),\ 0 \le t \le 24$$

a. Find the maximum temperature and the time of day that it occurs.

2 marks

b. Find the average rate of decrease in temperature from 2 pm to 10 pm.

2 marks

c. State the average temperature over the 24-hour period.

1 mark

d. Find the average temperature between 8 am and 4 pm, correct to one decimal place.

3 marks

e. Find the value of t for which the temperature is increasing most rapidly.

Hence find this maximum rate of increase in temperature, correct to two decimal places.

3 marks

The following day, the temperature, W°C, at time t hours after midnight can be modelled by

$$W(t) = -\frac{1}{18}(t-12)^2 + 25, \quad 0 \le t \le 24.$$

The difference in temperature between the two models, D°C, is given by

$$D(t) = W(t) - T(t), \quad 0 \le t \le 24.$$

f. Find the values of t for which the maximum difference in temperature occurs, correct to two decimal places.

Hence find the maximum difference in temperature, correct to one decimal place.

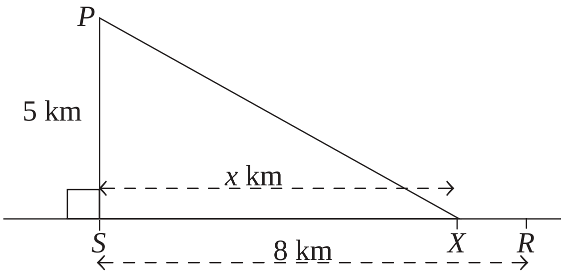
4 marks
Total 15 marks

Question 2

An offshore oil platform is located in the ocean at a point *P*, which is 5 km from the nearest point *S* on a stretch of straight coastline.

An oil refinery is located at a point *R*, 8 km along the coast from point *S*.

An oil company needs to lay straight-line sections of pipeline from the platform to the refinery. The cost of laying pipeline is $1 000 000 per km under water, and $750 000 per km over land.



The oil company decides to investigate three possible routes for the pipeline.

1. Pipe the oil directly from *P* to *R*.
2. Pipe the oil from *P* to *S* to *R*.
3. Pipe the oil from *P* to a point *X* on the coast, *x* km from *S* in the direction of *R*, and then from *X* to *R*.

a. Show that the cost, *C* million dollars, of laying the pipeline in terms of *x* is given by $C(x) = \sqrt{x^2 + 25} + \dfrac{3(8 - x)}{4}$.

1 mark

b. Find, correct to the nearest ten thousand dollars, the cost of laying pipeline directly from *P* to *R*.

1 mark

c. Find the cost of laying pipeline from P to S to R.

1 mark

The oil company decides to lay pipeline from P to X to R.

d. Find $C'(x)$.

1 mark

e. Find the exact distance from S to X for which the least expensive pipeline occurs.

2 marks

f. Hence find the least expensive pipeline. Give your answer to the nearest ten thousand dollars.

1 mark

The relative cost, k, where $k > 1$, is defined as the cost of laying pipeline under water, compared to the cost of laying pipeline over land.

The cost of laying pipeline over land is T million dollars per km.

g. Find the exact set of values of k for which the cost of laying pipeline directly from P to R is the least expensive.

4 marks

Total 11 marks

Question 3

Petra has been selected to compete in a javelin throwing competition.

In training for the competition, the distance X metres thrown by Petra was found to be normally distributed with mean 59.5 and standard deviation 3.

a. Find, correct to four decimal places,

i. $\Pr(59.5 \le X \le 63)$

ii. $\Pr(X \ge 63 | X \ge 59.5)$

1 + 3 = 4 marks

In training, 75% of Petra's throws were longer than x metres.

b. Find the value of x, giving your answer correct to the nearest tenth of a metre.

2 marks

In her final training session prior to competition, Petra completed five throws. The length of any throw was independent of that of any other throw.

c. Find the probability that Petra's five completed throws were between 56.5 metres and 62.5 metres, correct to four decimal places.

2 marks

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Petra's main rival, Louise, has also been selected to compete in the javelin competition.

In training, the distance Y metres thrown by Louise was found to be normally distributed with mean 60.5 and standard deviation 1.9.

In the first round of competition, each athlete must attempt five throws. The length of any throw is independent of that of any other throw. To qualify for the next round of competition, an athlete must record at least one throw of 63 metres.

d. **i.** Show that Petra is more likely to qualify with her first throw than Louise is with her first throw.

ii. Find the probability that **only one** of Petra or Louise qualify for the next round, correct to four decimal places.

3 + 5 = 8 marks
Total 16 marks

Question 4

The function f is defined by $f: (-\infty, -2) \rightarrow R, f(x) = \log_e((x-1)^2 - k)$, where k is a real constant.

a. Find the maximum value of k.

3 marks

Let $k = 8$.

b. The tangent to the graph of f at point P passes through the points $(3\log_e(2), -3)$ and $(5\log_e(2), -2\log_e(2) - 3)$.

Find the exact coordinates of P.

4 marks

c. Show that there are no solutions to the equation $f(x) = 0$.

2 marks

The inverse function of f is f^{-1}.

d. Find the rule for f^{-1}.

2 marks

e. State both the domain and range of f^{-1}.

2 marks

The function g is defined by $g\colon [2, \infty) \to R, g(x) = \sqrt{2x-4}$.

f. Find $g^{-1}(f^{-1}(x))$.

Express your answer in the form $\dfrac{e^x - p\sqrt{e^x + 8} + q}{p}$ where p and q are integers.

3 marks

Total 16 marks

END OF QUESTION AND ANSWER BOOKLET