The Mathematical Association of Victoria

Trial Exam 2012

MATHEMATICAL METHODS (CAS)

WRITTEN EXAMINATION 1

STUDENT NAME ____

Reading time: 15 minutes
Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
11	11	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

Materials supplied
- Question and answer book of 9 pages, with a detachable sheet of miscellaneous formulas at the back
- Working space is provided throughout the book.

Instructions
- Detach the formula sheet from the back of this book during reading time.
- Write your name in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

Let $g(x)=\cos(x)$ and $f(x)=\log_e(2x-2)$.

a. State the maximal domain of *f*.

1 mark

b. Find the rule for $g\left(f(x)\right)$.

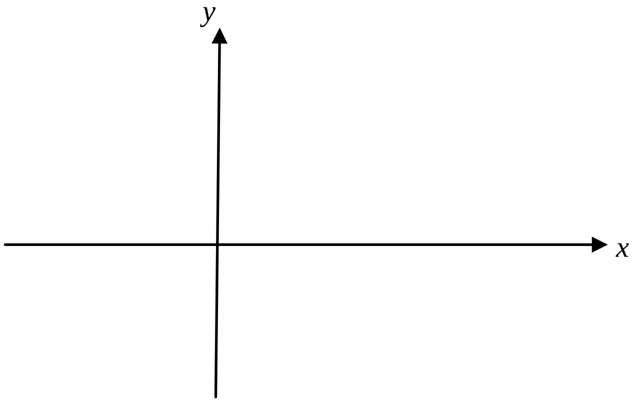
1 mark

c. Find $g'\left(f(x)\right)f'(x)$.

2 marks

Question 2

Sketch the graph of $y=\left(\frac{1}{4}\right)^{x-1}-1$ on the set of axes below. Clearly label any asymptotes with their equations and axes-intercepts with their coordinates.



3 marks

Question 3

Solve the equation $2\log_e(x-2)-\log_e(x+1)=\log_e(2)$ for x.

3 marks

Question 4

a. Differentiate $(x+2)\sqrt{(x-1)}$, giving your answer as a single fraction.

3 marks

b. Hence, find an antiderivative of $\dfrac{x}{\sqrt{(x-1)}}$.

2 marks

Question 5

Find the area bounded by the curve of f with equation $f(x)=\frac{1}{2-4x}$, the y-axis and the line $y = 2$. Write your answer in the form $\frac{a-\log_e(b)}{b}$, where a and b are positive integers.

5 marks

Working Space

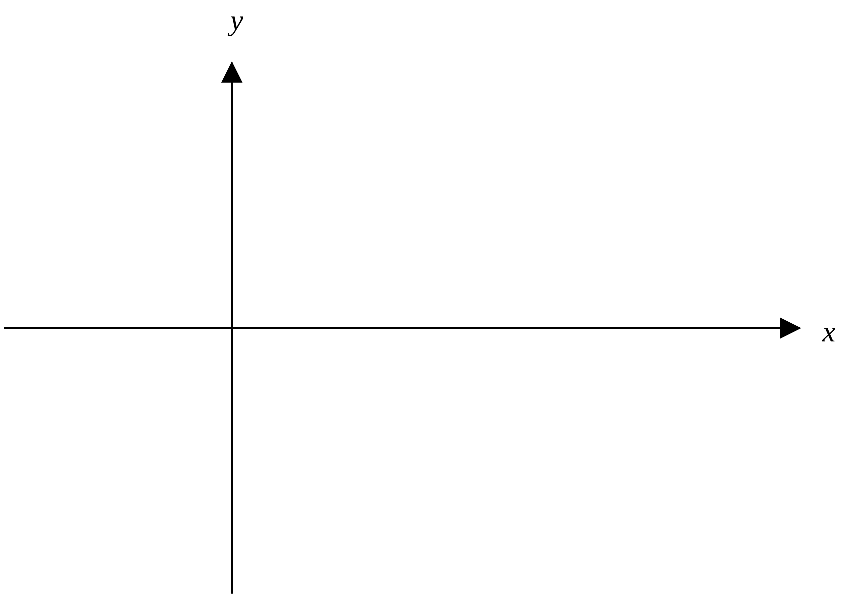
Question 6

If $h(x)=e^{(x-1)}$, show that $h(x+y)\times h(x-y)=\left(h(x)\right)^2$.

2 marks

Question 7

On the axes provided, sketch the graph of the function $f:[-\pi,3\pi]\rightarrow R$ where $f(x)=\frac{1}{2}\sin\left(\frac{x}{2}\right)$. Label the endpoints with their coordinates.



2 marks

Question 8

Find the values of $a \in \left[0, \frac{2}{3}\right]$ for which $x = \frac{1}{6}$ is the solution to the equation $2\sin(3\pi(x-a)) = \sqrt{3}$.

3 marks

Question 9

If $X \sim \text{Bi}(5, 0.1)$, find Pr(X = 3).

2 marks

Question 10

The probability density function of a random variable X has a density function given by

$$f(x) = \begin{cases} |x-1| & 0 \le x \le 1 \\ 0.1 & 1 < x \le a \\ 0 & \text{elsewhere} \end{cases}$$

a. Find the value of a.

1 mark

b. Find $\Pr(X > 0.5)$.

1 mark

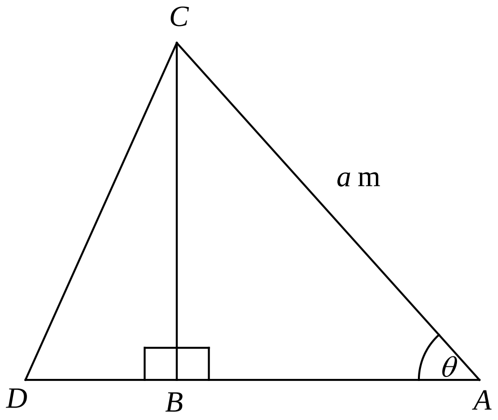
c. Find $\Pr(X > 2 \mid X > 0.5)$.

2 marks

Question 11

The diagram represents two sails of a toy yacht, where triangle *ABC* and triangle *BCD* are the two sails. The length of *AC* is *a* metres and $\angle BAC = \theta^c$.

Also $AB = 2BD$ and $\angle CBD = \angle CBA = \dfrac{\pi}{2}$.



a. Write *BC* and *AD* in terms of *a* and θ.

2 marks

b. Write the total sail area, $T\text{ m}^2$, in terms of a and θ.

1 mark

c. **i.** Find $\dfrac{dT}{d\theta}$.

ii. **Hence**, find the maximum total sail area, when $a = 4\text{ m}$.

1+ 3 = 4 marks

END OF QUESTION AND ANSWER BOOK

MATHEMATICAL METHODS (CAS)

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

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Mathematical Methods (CAS)
Formulas

Mensuration

area of a trapezium: $\frac{1}{2}(a+b)h$

curved surface area of a cylinder: $2\pi rh$

volume of a cylinder: $\pi r^2 h$

volume of a cone: $\frac{1}{3}\pi r^2 h$

volume of a pyramid: $\frac{1}{3}Ah$

volume of a sphere: $\frac{4}{3}\pi r^3$

area of a triangle: $\frac{1}{2}bc\sin A$

Calculus

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

$$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$$

$$\frac{d}{dx}\left(\sin(ax)\right) = a\ \cos(ax)$$

$$\frac{d}{dx}\left(\cos(ax)\right) = -a\ \sin(ax)$$

$$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\ \sec^2(ax)$$

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax} + c$$

$$\int \frac{1}{x}\,dx = \log_e|x| + c$$

$$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + c$$

$$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + c$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

approximation: $f(x+h) \approx f(x) + hf'(x)$

Probability

$\Pr(A) = 1 - \Pr(A')$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

transition matrices: $S_n = T^n \times S_0$

mean: $\mu = \mathrm{E}(X)$

variance: $\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X-\mu)^2) = \mathrm{E}(X^2) - \mu^2$

probability distribution		mean	variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x\,p(x)$	$\sigma^2 = \sum (x-\mu)^2\,p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)\,dx$	$\mu = \int_{-\infty}^{\infty} x\,f(x)\,dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx$

END OF FORMULA SHEET