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Unit 3 and 4 Mathematical Methods (CAS): Exam 1

Practice Exam Question and Answer Booklet

Duration: 15 minutes reading time, 1 hour writing time

Structure of book:

Number of questions	Number of questions to be answered	Number of marks
10	10	45

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers and rulers.
- Students are not permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.
- No calculators or notes are allowed in this examination.

Materials supplied:
- This question and answer booklet of 7 pages, including a detachable formula sheet.

Instructions:
- You must complete all questions of the examination.
- Write all your answers in the spaces provided in this booklet.

Instructions

Answer all questions in the spaces provided.
In all questions where a numerical answer is required an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working must be shown.
Unless otherwise indicated, the diagrams in this book are not drawn to scale.

Questions

Question 1

a. Differentiate $e^{\sin(3x)}$ with respect to x.

2 marks

b. For $f(x) = \sin(2x)\, e^{3x}$, find $f'\left(\frac{\pi}{2}\right)$.

2 marks

Question 2

a. Give the antiderivative of $\frac{1}{x}$ that has value 10 at $x = 3$.

2 marks

b. For what values of c does the inequality $\int_1^c 3x^2 dx > 2$ hold?

2 marks

Question 3

Let $f: (3, \infty) \to R$ where $f(x) = \log_e\left(\frac{x-3}{2}\right) + 5$.

a. Find $f^{-1}(x)$.

3 marks

b. Find $f\big(g(x)\big)$, where $g(x) = f^{-1}(x)$.

2 marks

Question 4

The transformation $T: R^2 \rightarrow R^2$ is defined by:

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \boldsymbol{A}\begin{bmatrix} x \\ y \end{bmatrix} + \boldsymbol{b}$$

The image of the curve $y = \cos(x)$ under the transformation T has equation $y = \pi\cos\left(2x + \frac{\pi}{2}\right)$. Find the matrices $\boldsymbol{A}$ and $\boldsymbol{b}$.

4 marks

Question 5

Solve $\tan(3x + 3\pi) = \frac{1}{\sqrt{3}}$ where $x \in [-\pi, \pi]$.

3 marks

Question 6

Let X be normally distributed with $\mu = \sigma = 27.2$.

a. Now let aX be a transformed version of X. If $\Pr(aX \geq 108.8) = 0.025$, what is the value of a?

3 marks

b. If $\Pr(X \leq 0) = \Pr(aX \leq b)$, what is the value of b?

2 marks

Question 7

Let $f(x) = \frac{1}{\theta} e^{\frac{-x}{\theta}}$ where $x \in [0, \infty)$ and $\theta > 0$.

a. Show that $f(x)$ is a probability density function.

2 marks

b. Find $\Pr(X \leq \theta | X \leq 3\theta)$.

2 marks

Question 8

Let X be a discrete random variable with probability:

x	-1	0	1	2
$\Pr(x = x)$	0.5	0.1	0.3	0.1

a. Find $E(X)$.

1 mark

b. Find $E(X^2)$.

1 mark

c. Find $Var(2X)$.

2 marks

Question 9

a. Find the tangent to the curve $y = f(x) = x^2 - 4$ at $x = 3$.

2 marks

b. Use the relationship $f(x + h) \approx f(x) + hf'(x)$, for a small positive value of h,to find an approximate value for $f(3.02)$.

2 marks

Question 10

For the following questions, assume that light from the Sun's core travels at a rate of 3×10^8 m/s in all directions, and that that the Sun is spherical with surface area $S = 4\pi r^2$, where r represents the distance of the light from the sun's core.

a. Find $\frac{dS}{dt}$ in terms of r, where S now represents the surface area of the sphere formed by the light escaping from the sun's core, and t is the time since it left the sun's core.

3 marks

b. Hence, or otherwise, find the radius r at which S will be increasing at a rate of $10^{19}\ \mathrm{m^2/s}$.

3 marks

c. Find the ratio of the amount of light in $1\ \mathrm{m^2}$ at $r = 10^{10}$ m to the amount of light in $1\ \mathrm{m^2}$ at $r = 10^{12}$ m, assuming that the amount of light is proportional to the surface area of the sphere formed by the light escaping from the sun's core.

2 marks

Formula sheet

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin A$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}(x^n) = nx^{n-1}$

$\int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$

$\frac{d}{dx}(e^{ax}) = ae^{ax}$

$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$

$\frac{d}{dx}(\log_e x) = \frac{1}{x}$

$\int \frac{1}{x} dx = \log_e|x| + c$

$\frac{d}{dx}(\sin(ax)) = a\cos(ax)$

$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + c$

$\frac{d}{dx}(\cos(ax)) = -a\sin(ax)$

$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + c$

$\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = a\sec^2(ax)$

product rule $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{\left(v\frac{du}{dx} - u\frac{dv}{dx}\right)}{v^2}$

chain rule $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

approximation $f(x+h) = f(x) + hf'(x)$

Probability

$\Pr(A) = 1 - \Pr(A')$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

transition matrices $S_n = T^n \times S_0$

mean $\mu = E(X)$

variance $var(X) = \sigma^2 = E((X-\mu)^2) = E(X^2) - \mu^2$

probability distribution		mean	variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \Sigma xp(x)$	$\sigma^2 = \Sigma(x-\mu)^2 p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)dx$	$\mu = \int_{-\infty}^{\infty} xf(x)dx$	$\sigma^2 = \int_{-\infty}^{\infty}(x-\mu)^2 f(x)dx$