QATs

Quality Assessment Tasks

UNIT 4 ■ OUTCOMES 1, 2 & 3

VCE Mathematical Methods (CAS)

SCHOOL-ASSESSED COURSEWORK

Introduction

Outcome 1
Define and explain key concepts as specified in the content from the areas of study, and apply a range of related mathematical routines and procedures.

Outcome 2
Apply mathematical processes in non-routine contexts, and analyse and discuss these applications of mathematics.

Outcome 3
Select and appropriately use technology to develop mathematical ideas, produce results and carry out analysis in situations requiring problem-solving, modeling or investigative techniques or approaches.

Task

Analysis Task

The task has been designed to allow achievement up to and including the highest level in the performance Descriptors. It covers a broad range of **key knowledge** and **key skills** over the three outcomes for Unit 4.

It will contribute 20 out of the total (40) marks allocated for SAC in Unit 4.

This task will be marked out of 80 and then will be converted to a proportion of the contribution of this task to SAC in this unit.

The marks for each part are indicated in brackets. Answer in space provided or as directed.

You have 120 minutes over no more than two days.

You can access your logbook and an approved CAS calculator.

CAS TECHNOLOGY Indicates where use of the technology is specifically required in order to answer the question.
Your teacher will advise you of any variation to these conditions.

NAME:

Task

PART 1

You have one hour for this section. Circle the letter of the correct response.

Question 1

Two events A and B can be represented on a Karnaugh Map as follows:

	B	**B'**
A	7	5
A'	3	5

The probability of A is:

A $\frac{7}{20}$

B $\frac{7}{12}$

C $\frac{3}{5}$

D $\frac{1}{2}$

E $\frac{3}{8}$

Question 2

CAS TECHNOLOGY

A binomial distribution has a probability of success p = 0.4. If the experiment is trialled 6 times, then the median and mode of the distribution are:

A 2.4 and 2

B 2 and 2

C 2 and 2.4

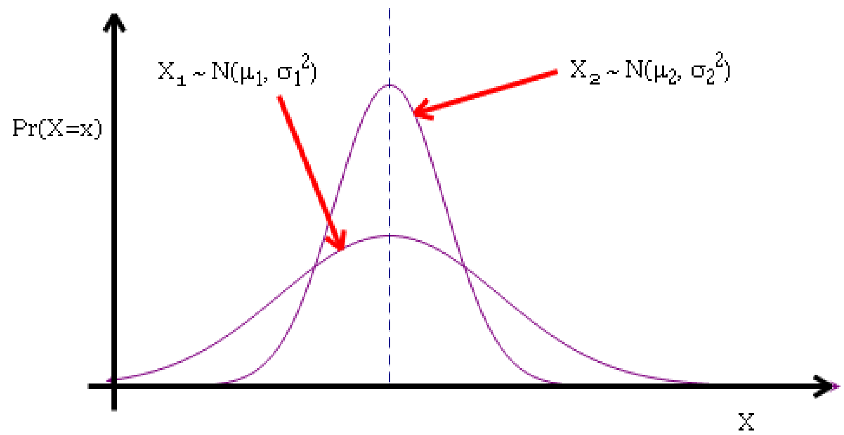
D 2.4 and 3.6

E 2.4 and 1.44

Task

Question 3

The diagram below shows the graphs of two normal distribution curves with means of μ_1 and μ_2 and standard deviations of σ_1 and σ_2 respectively.



Which one of the following is true?

A $\mu_1 < \mu_2$ and $\sigma_1 < \sigma_2$

B $\mu_1 < \mu_2$ and $\sigma_1 = \sigma_2$

C $\mu_1 > \mu_2$ and $\sigma_1 < \sigma_2$

D $\mu_1 > \mu_2$ and $\sigma_1 > \sigma_2$

E $\mu_1 = \mu_2$ and $\sigma_1 > \sigma_2$

Task

Question 4

The function f is a probability density function with

$$f(x) = \begin{cases} k\,x^3 & 0 \le x \le k \\ 0 & elsewhere \end{cases}$$

Hence the value of k is equal to:

A $\frac{1}{216}$

B $\sqrt[5]{4}$

C 216

D 64

E $\frac{1}{\sqrt[5]{4}}$

Question 5

If E(X) = 3 and Var(X) = 25 then E(-2X+1) and Var(-2X + 1) are:

A 3 and 25

B -5 and -50

C 7 and 51

D 5 and 100

E -5 and 100

Task

Question 6
A random variable X has a normal distribution with mean 6 and standard deviation 0.35. If the random variable Z has the standard normal distribution, then the probability that X is less than 4.95 is

A Pr(Z > 1.05)

B Pr(Z < 1.05)

C Pr(Z < 1)

D Pr(Z > 3)

E Pr(Z > -2)

Question 7
A binomial random variable of n trials has a probability of success p which is less than its probability of failure q. Which of the following statements is true?

A $\sigma = npq$

B $\mu = nq$

C The distribution is positively skewed

D $p + q < 1$

E $p > 0.5$

CAS TECHNOLOGY

Question 8
A random variable *f* has a probability density function

$$f(x) = \begin{cases} e^{-x} & 0 \le x \le \infty \\ 0 & \textit{elsewhere} \end{cases}$$

Then E(X) is equal to:

A 1

B 0

C $ln(2)$

D $\frac{1}{2}$

E $1 - \ln(2)$

Task

Question 9
A random variable X has a probability distribution as given in the table:

X	0	1	3	5	7
Pr(X = x)	0.25	0.1	a	0.2	b

If the mean of X is 2.65 then the values of a and b respectively are:

A 0.04 and 0.5

B 0.4 and 0.05

C 0.2 and 0.25

D 0.05 and 0.4

E 0.5 and 0.4

Question 10
The sample space of an unbiased eight sided die is {1, 2, 3, 4, 5, 6, 7, 8}. All outcomes are equally likely. For which pair of events are the events independent?

A {2, 4, 6, 8} and {1, 3, 5, 7}

B {1, 3, 5, 7} and {1, 6}

C {2, 4, 6} and {1, 2, 3, 4}

D {3, 4, 5} and {3, 4, 6, 7}

E {1, 2, 4, 6, 7} and (3, 5, 7, 8}

Task

Question 11

A box contains 8 hard centred chocolates and 12 soft centred chocolates. Four chocolates are removed from the box without replacement. The probability that they are all soft centres is:

A $\frac{33}{343}$

B $\frac{11}{141}$

C $\frac{188}{1628}$

D $\frac{81}{625}$

E $\frac{297}{4000}$

CAS TECHNOLOGY

Question 12

A fair coin is tossed twelve times. The probability , correct to four decimal places, of getting no more than 3 heads is

A 0.0730

B 0.0193

C 0.0537

D 0.9807

E 0.0161

Task

CAS TECHNOLOGY

Question 13

A coin is biased in such a way that the probability of success, p, is three times the probability of failure, q, for any given toss of the coin. If the coin is tossed 8 times then the mean and variance of the distribution respectively are:

A 6 and 1.5

B 2 and 1.5

C 1.5 and 2

D 1.5 and 6

E 0.25 and 0.75

Question 14

A second coin has a probability $p = 0.3$ of landing heads up on any given throw. If the coin is tossed 4 times then the probability of obtaining either two or three heads is:

A 0.2646

B 0.0756

C 0.3402

D 0.6598

E 0.3483

Question 15

The events **A** and **B** are mutually exclusive events of a sample space with

$$\Pr(\mathbf{A}) = m \text{ and } \Pr(\mathbf{B}) = n \qquad \text{where } 0 < m < 1 \text{ and } 0 < n < 1$$

$\Pr(\boldsymbol{A}' \cap \boldsymbol{B}) =$

A $(n - 1)m$

B $n(m - 1)$

C $n\,m$

D m

E n

Task

Question 16

The times in minutes for students to complete a Mathematical Methods trial exam are normally distributed with a mean of 102 minutes and a standard deviation of 8 minutes. If the duration of the examination 120 minutes, the proportion of students who do not complete the examination is closest to:

A 0.0122

B 0.1499

C 0.9878

D 0.2023

E 0.7977

CAS TECHNOLOGY

Question 17

A continuous random variable X has a probability density function given by

$$f(x)=\begin{cases} \frac{1}{3}\cos\left(\frac{x}{3}\right) & \text{if } 0<x<\frac{3\pi}{2} \\ 0 & \text{elsewhere} \end{cases}$$

The variance of X is:

A $\frac{3(\pi-2)}{2}$

B $\frac{1}{3}$

C $9(\pi-3)$

D $\frac{3\pi}{2}$

E $3\sqrt{\pi-3}$

Task

Question 18

A ride at an amusement park has limits on the size of passengers. The minimum height is 120cm and the maximum height is 210 cm. These extremes are equidistant from the mean value. It has been determined that this will allow 99.73% of visitors to the park to ride the attraction. The mean and standard deviation of heights in the population respectively are closest to:

A 99.73 cm and 20 cm

B 165cm and 30 cm

C 162.5 cm and 20 cm

D 165 cm and 15 cm

E Not enough information

Question 19

Footy cards are sold in packs of 6. The probability of any one card in a pack being an Essendon card is $\frac{1}{18}$. The team on each card is independent of the teams on any other card. When a card is removed from the pack it is not replaced. The probability that the second card removed is an Essendon card is:

A $\frac{1}{3}$

B $\frac{5}{18}$

C $\frac{1}{324}$

D $\frac{1}{306}$

E $\frac{1}{18}$

Task

Question 20

For a given Binomial Distribution, the probability of success, p, is $\frac{1}{3}$. The probability of obtaining a run length of 6 successes is:

A $\frac{2}{2187}$

B $\frac{64}{2187}$

C $\frac{64}{729}$

D $\frac{665}{729}$

E $\frac{2185}{2187}$

Question 21

A variable X has a normal distribution and with N(15.2, 1.96). Given that the Pr(X < a) = 0.4 the magnitude of a is:

A 14.7034

B 14.7718

C 14.8706

D 14.8453

E -2.4509

Task

Question 22

For a particular game of chance, a player has shown that his chance of winning a round is related to whether he won or lost the last round. The transformation matrix, T, for this player is:

$$T = \begin{bmatrix} 0.3 & 0.75 \\ 0.7 & 0.25 \end{bmatrix}$$

The steady state probability that this player wins the nth game is:

A $\frac{15}{29}$

B $\frac{14}{29}$

C $\frac{3}{4}$

D $\frac{1}{4}$

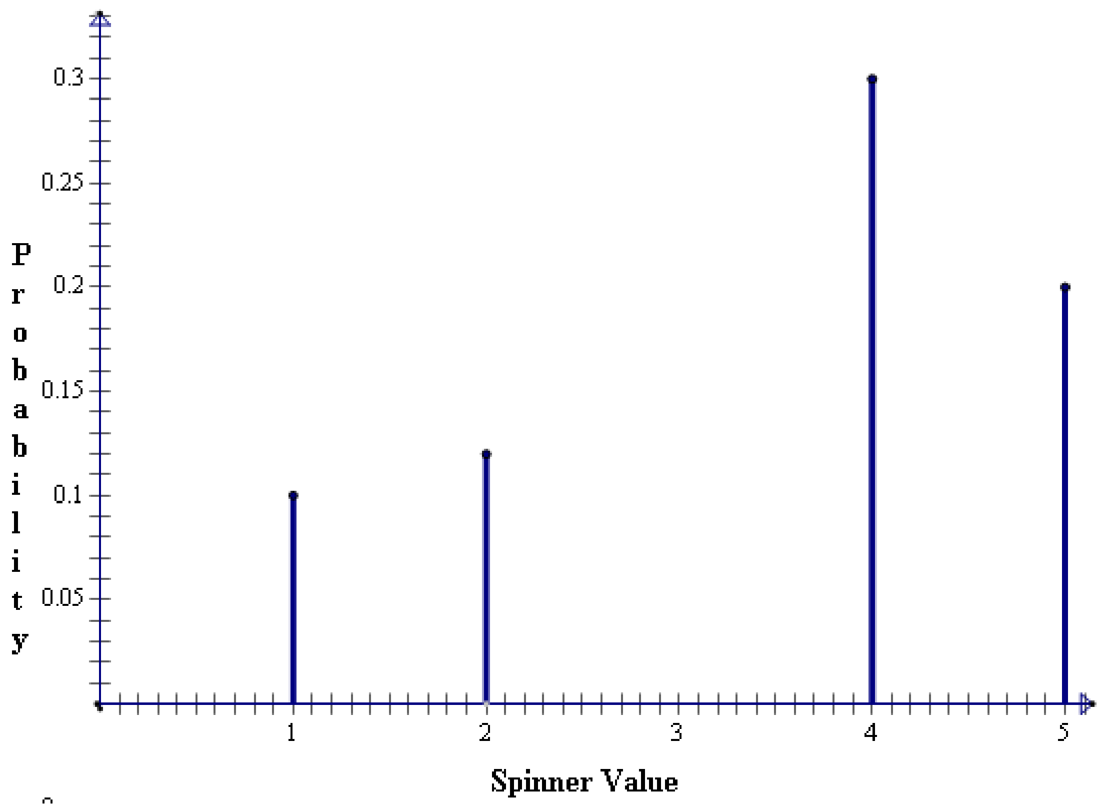
E $\frac{3}{10}$

(22 x 1= 22 marks)

Task

Question 23

A game of chance relies on a spinner that has five outcomes, numbered 1 to 5. A graph of the probabilities of each outcome is shown but is missing *one* of the points.



a) Fill in the missing entry, clearly showing the probability.

(1 mark)

b) Hence show that the Expected value of the game is 3.38.

(2 marks)

Task

c) Write down the value of the median score.

(1 mark)

d) Write down the value of the modal score.

(1 mark)

e) Paddy has tried to work out the variance of results for the game. Unfortunately there is an error in his calculations. His working is given below. Identify the error, *circle it* and *write a note* describing the nature of the error and find the correct variance.

E(X) = 3.38

x^2	1	4	9	16	25
X^2.Pr(X)	0.1	0.48	2.52	4.8	5

Hence Var(X) = 11.9 – 11.4244 = 0.4756

(3 marks)

f) The game costs \$2 per round to play and pays \$4 for a win. What is the long term expected profit for owner of this game?

(2 marks)

Task

Evonne decides to play the spinner game three times. If she wins more than once she will then play a fourth time, otherwise she will walk away.

g) Evonne decides to bet on the number 4 turning up in each round. What is the probability that she will win all three rounds?

(1 mark)

h) What is the probability that she will win AT MOST only one round?

(3 marks)

i) What is the probability that Evonne will play a fourth round of the game?

(1 mark)

j) If Evonne were to bet on the number 3, would this improve or worsen her chance of winning a round? Show calculations to support your answer.

(3 marks)

End of Part 1

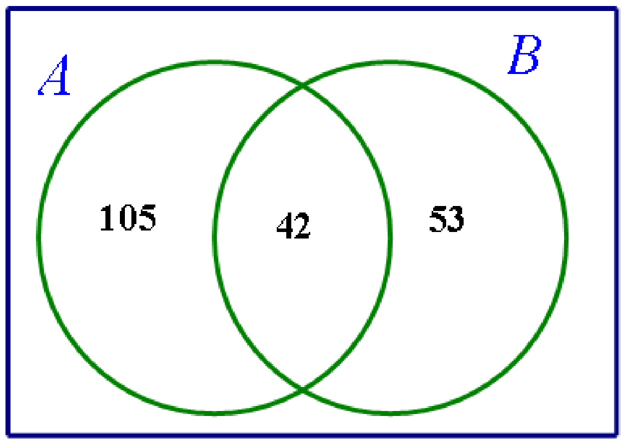
Task

PART 2

You have one hour for this section.

Question 24: **Binomial Distribution (10 marks)**

Jordan is basketball player for her school. A record of her performance over the season has been kept and part of it is represented in the Venn Diagram shown.



In 200 attempts at shooting a goal, her results have been divided into two sets. Set A is she scored a goal and set B is she attempted a 3-point shot.

a) What percentage of her shots for the season scored a goal? Quote your answer to two decimal places.

(1 mark)

b) Given that she scored a goal, what is the probability that it was a 3-pointer?

(2 marks)

Task

Her season results are being used to find the probability that she takes 8 shots at goal in the grand final, she will score a goal in at least two of her attempts.

c) Write down an expression for finding this probability.

(2 marks)

d) Now calculate the probability.

(1 mark)

In another analysis of Jordan's results, it has been found that if she scored a goal on her last attempt then the probability of scoring a goal on her next attempt is 0.65. However, if she missed the goal on her last attempt the probability of scoring on her next attempt goes down to 0.4.

Her first attempt at scoring a goal in the grand final was successful.

e) What is the probability, correct to four decimal places, that her next 7 attempts at scoring a goal in the match will be successful?

(1 mark)

Task

f) What is the probability that exactly one of her next three attempts at goal is successful?

(2 marks)

g) What percentage of Jordan's attempts at scoring, in the long term, are successful?

(1 mark)

Task

Question 25 (13 marks)
Exposure to solar radiation when outside over a 24 hour period can be modelled by the equation

$$R(x) = -A\left[\cos(nx) - 1\right]$$

It is known that the period of this function is 24 hours starting at midnight.

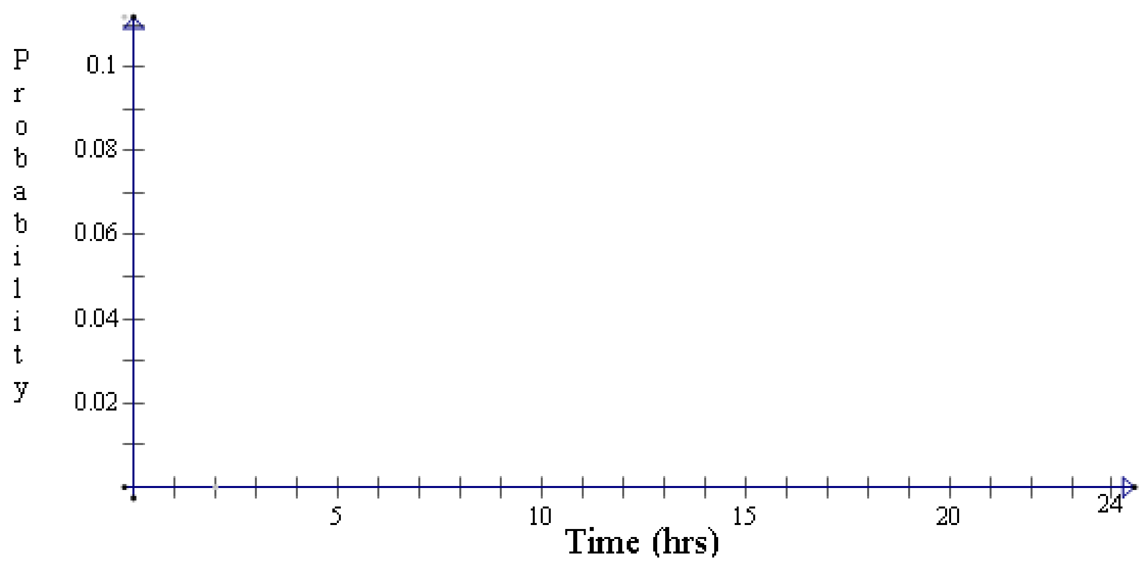
a) Write down the value of n for this function.

(1 mark)

b) In order for this function to act as a probability density function the area underneath the graph must equal 1. Use calculus to show that the value of A is $\frac{1}{24}$.

(2 marks)

c) Sketch a graph of this function for the domain [0, 24].



(3 marks)

d) What is the maximum value of the function?

(1 mark)

Task

Average healthy exposure to solar radiation is in a band between 40% and 70% of total daily radiation from the Sun.

e) A road worker on night shift is outside from 6pm until 6am. What percentage of daily solar radiation are they exposed to? (Quote your answer to 2 decimal places.)

(2 marks)

f) How much below the minimum healthy radiation dose is the worker exposed to?

(1 mark)

CAS TECHNOLOGY

g) To obtain a healthy average dose of radiation the worker should spend more time outdoors during daylight hours. Starting at 6am, how much longer must the worker stay outdoors continually, in order to reach a minimum of 40% radiation exposure? Quote your time in hours to 3 decimal places.

Note, this question requires the use of a calculator. Explain briefly your method for determining your result.

(3 marks)

Task

Question 26: Normal Distribution (10 marks)

Reams of photocopying paper have 500 sheets and the weight of the paper (including its packaging) is normally distributed with a mean of 2.5 kg and a standard deviation of 0.01 kg. The paper is sold in boxes containing four reams.

a) Reams that are under 2.47 kg or over 2.53 kg are rejected and not allowed to go to stores. What is the probability that a randomly selected ream of paper will be rejected? Quote your answer to 4 decimal places.

(2 marks)

The manufacturer would like each ream to weigh between 2.485 kg and 2.515 kg.

b) What is the probability, again correct to 4 decimal places, that a randomly selected ream will be in this weight range?

(2 marks)

Task

c) What is the probability that a ream of paper that has been accepted is NOT within the manufacturer's desired range of 2.485 kg and 2.515 kg?

(1 mark)

d) A box of 4 reams is selected at random. What is the probability, correct to four decimal places, that at least one of the reams has a weight outside the manufacturer's desired range of 2.485 kg and 2.515 kg?

(2 marks)

e) What are the mean and standard deviation of four reams of paper in a box?

(3 marks)

Task

Question 27 (7 marks)

A continuous random variable is to be made up of three parts:

$$f(x)=\begin{cases} -0.1(x-2)^2+0.4 & 0\le x\le 2 \\ mx+\dfrac{26}{35} & 2\le x\le A \\ 0 & \text{elsewhere} \end{cases}$$

a) Find the area bounded by the curve for $0\le x\le 2$.

(1 mark)

b) Hence determine the area under the second section of the curve $mx+\dfrac{26}{35}$.

(1 mark)

c) Given that the straight line section of the function passes through the point $\left(2,\dfrac{2}{5}\right)$, find the value of the x-intercept, A, that makes this function a probability density function.

(2 marks)

Task

d) Hence show that the gradient of the linear section is $\frac{-6}{35}$

(1 mark)

CAS TECHNOLOGY

e) Finally, find the median value of the distribution.

(2 marks)

End Part 2

Teacher Advice

This is the Analysis Task, as suggested to be undertaken in weeks 12 and 13 in the sample teaching sequence on page 193 of the VCAA Study Design.

This task contributes 20 of the 40 SAC marks in Unit 4.
The coursework scores for this task are:

Outcome 1	8 marks	40%
Outcome 2	7 marks	35%
Outcome 3	5 marks	25%
TOTAL	20 marks	

This weighting can be used in the conversion of their mark out of 50.
For example, a score of 40 results in:

OUTCOME 1	OUTCOME 2	OUTCOME 3
40/80*20*0.4	40/80*20*0.35	40/80*20*0.25
= 4	= 3.5	= 2.5
	= 4 (rounded up)	= 4 (rounded up)

The above can be established in an Excel file.
This QAT has been designed to meet the highest level in the performance descriptors provided by VCAA for each outcome in unit 3 in the Assessment Handbook 2006-14.

Notes on the two parts of the task

PART 1 is based on the traditional Probability sections of the course: Discrete, Binomial and Normal Distributions. Each of the questions is worth either 13 or 14 marks, the same as extended response questions in Examination 2.

PART 2 is on continuous probability distributions. Each of the four questions is based on a different type of function: Linear, Quadratic, Exponential and Cosine. This allows for graphing, algebra and integration to be incorporated into the task.

Solution Pathway

PART 1

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
C	B	E	B	E	D	C	A	B	B	A	A	B	C	E	A	C	D	E	A	D	B

Question 1

The total number of events is 7+5+3+5 = 20.

The number of favourable events is 7+5 = 12.

$\Pr(A) = \frac{12}{20} = \frac{3}{5}$ Hence C.

Question 2

Calculate the values of the probabilities for x = 0 to 6.

X	0	1	2	3	4	5	6
Pr(X=x)	0.04666	0.1866	0.3110	0.2765	0.1382	0.0369	0.0041

The mode is 2 since it is the x value with the highest probability. The median is also 2 since the sum of probabilities for x = 0 and 1 is 0.2333 and for x = 0 to 2 is 0.54426. Hence B

Question 3

Normal distribution curves are all symmetrical with their mean at the highest point of the curve. Both of these curves have their maximums at the same place. Therefore . $\mu_1 = \mu_2$ The first curve is wider

$\sigma_1 > \sigma_2$

than the second. Hence E.

Solution Pathway

Question 4

The area under the curve must equal 1. Integrate the equation and solve for k.

$$\int_0^k k\,x^3\,dx = \left[\frac{k\,x^4}{4}\right]_0^k = 1$$

$$\frac{k^5}{4} - 0 = 1$$

$$k^5 = 4$$

$$k = \sqrt[5]{4}$$

Hence B.

Question 5

$$\begin{aligned} E(-2X+1) &= 2E(X) + 1 \\ &= -2\times 3+1 \\ &= -5 \end{aligned}$$

$$\begin{aligned} VAR(-2X+1) &= 4\times Var(X) \\ &= 4\times 25 \\ &= 100 \end{aligned}$$

Hence answer is E.

Question 6

Use $z = \dfrac{x-\mu}{\sigma}$

$$\begin{aligned} z &= \frac{4.95-6}{0.35} \\ &= \frac{-1.05}{0.35} \\ &= -3 \end{aligned}$$

$$\textit{Hence}, \Pr(Z < -3)$$

This option does not appear but is equivalent to Pr(Z>3). Answer D.

Solution Pathway

Question 7

If p < q then there is a long tail to the right of the graph. Hence positive skew. Answer C.

Question 8

$$\boldsymbol{E}(\boldsymbol{X}) = \int_0^{\infty} \boldsymbol{x}\,\boldsymbol{e}^{-x}\,\boldsymbol{dx}$$
$$= 1$$

Hence A.

Question 9

The sum of the probabilities is 1

Therefore a + b + 0.55 = 1

So a + b = 0.45

a = 0.45 - b

The expected value is 2.65 = 0.1 + 3a + 5 + 7b

So 2.65 = 1.1 + 3a + 7b

1.55 = 3a + 7b (2)

Substitute (1) into (2)

1.55 = 3(0.45 – b) + 7b

1.55 = 1.35 + 4b

0.2 = 4b

b = 0.05 and a = 0.4. Hence B.

Question 10

For answer B $\Pr(\boldsymbol{A} \cap \boldsymbol{B}) = \Pr(1) = \frac{1}{8}$, and $\Pr(\boldsymbol{A}) \times \Pr(\boldsymbol{B}) = \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$ so B is correct.

Solution Pathway

Question 11

$\frac{12}{20} \times \frac{11}{19} \times \frac{10}{18} \times \frac{9}{17} = \frac{33}{343}$ Hence A

Question 12

$$\begin{aligned} \Pr(0) + \Pr(1) + \Pr(2) + \Pr(3) &= \boldsymbol{binomCdf}(12, 0.5, 0, 3) \\ &= 0.0730 \end{aligned}$$ Hence A.

Question 13

p + q = 1

But p = 3q

Therefore p = 0.75

Mean = n p = $8 \times 0.75 = 6$

Variance = n p q = $8 \times 0.75 \times 0.25 = 1.5$

Hence D.

Question 14

$\boldsymbol{binomCdf}(4, 0.3, 2, 3) = 0.3402$ Hence C.

Question 15

A and B are mutually exclusive events, hence there is not intersection. $\Pr(\boldsymbol{A}' \cap \boldsymbol{B}) = \boldsymbol{Pr}(\boldsymbol{B}) = \boldsymbol{n}$ Hence E

Question 16

Binomcdf(120, ∞ , 102, 8) = 0.0122. Hence A.

Solution Pathway

Question 17

$$\mu = \int_0^{\frac{3\pi}{2}} x\,\frac{1}{3}\cos\left(\frac{x}{3}\right)dx = \frac{3(\pi-2)}{2}$$

$$Var(X) = \int_0^{\frac{3\pi}{2}} \left(x - \frac{3(\pi-2)}{2}\right)^2 \frac{1}{3}\cos\left(\frac{x}{3}\right)dx = 9(\pi-3)$$

Hence C.

Question 18

Mean is midway between the extremes. $\mu = \dfrac{210+120}{2} = 165\,\text{cm}$

99.73% is 3 standard deviations either side of the mean, hence 6 standard deviations in all.

$\dfrac{210-120}{6} = \dfrac{90}{6} = 15$ Hence D.

Question 19

The key to this question is that probabilities are independent. Even though there are 6 cards per pack the packs have been filled randomly from a much larger collection. Hence the probability of a second card being an Essendon card is still $\frac{1}{18}$. Hence E.

Question 20

Run length = $p^n q$ where p is the probability of success, q the probability of failure and n the length of the run required.

$$p^n q = \left(\frac{1}{3}\right)^6 \left(\frac{2}{3}\right)$$
$$= \frac{2}{2187}$$

Answer A.

Question 21

15.2 is the mean and 1.96 is the variance so standard deviation is 1.4. Therefore, *invNorm*(0.4,15.2,1.4) = 14.8453. Answer D.

Solution Pathway

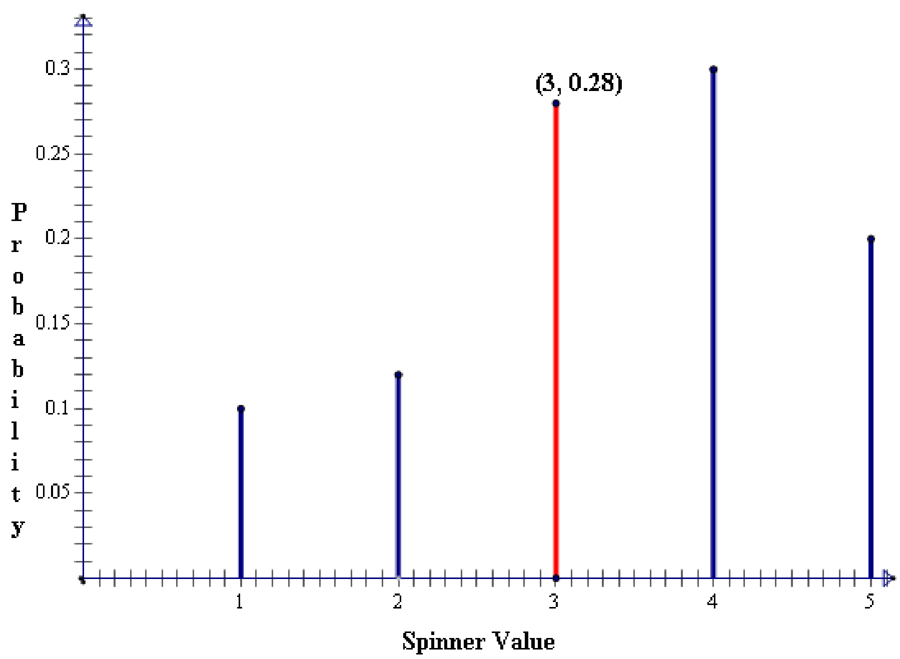
Question 22

For steady state Pr(W)= $\frac{a}{a+b}$. From the transformation matrix, a = 0.7, b = 0.75

Hence Pr(W) = $\frac{14}{29}$. Answer B

Question 23

a) Subtracting the known probabilities from 1 provides Pr(3) = 0.28



b) $$E(X) = 1\times 0.1 + 2\times 0.12 + 3\times 0.28 + 4\times 0.3 + 5\times 0.2$$
$$= 3.38$$

The working line must be shown to award marks.

c) Add the probabilities until the total is more than 0.5. In this case Pr(1, 2, 3) = Pr(4, 5) = 0.5

Hence median is $\frac{3+4}{2} = 3.5$

d) The modal value is 4

Solution Pathway

e) The sum of the second row in the table is 12.9 not 11.9. (1 mark)

Circling or highlighting the 11.9 (1 mark)

Var(X) = 1.4756 (1 mark)

f) E(2X-4) = 2E(X) – 4

=2 x 3.38-4

= 6.76 – 4

= $2.76

g) $\Pr(3\,\boldsymbol{wins}) = 0.3 \times 0.3 \times 0.3$

$= 0.027$

h) $\Pr(\boldsymbol{at\ most}\,1\,\boldsymbol{win}) = \Pr(3\,\boldsymbol{losses}) + \Pr(2\,\boldsymbol{losses})$

$= (0.7)^3 + 3 \times 0.3 \times (0.7)^2$

$= 0.784$

i) Pr(4th round) = 1 – 0.784 = 0.216

j) Worse since the probability for a 3 being spun is less than the probability of a 4 being spun.

PART 2

Question 24

a) $\dfrac{105+42}{200} \times 100 = 73.5\%$

b) $\dfrac{42}{147} = 0.2857$ (1 mark for identifying the correct calculation. Q mark for the calculation itself.

c) $1-\left(\dfrac{53}{200}\right)^8 - 28\left(\dfrac{147}{200}\right)\left(\dfrac{53}{200}\right)^7$ 1 mark for partially correct.

d) 0.9981

e) $(0.65)^7 = 0.0280$

f) $0.65 \times 0.35 \times 0.6 + 0.6 \times 0.4 \times 0.65 + 0.6 \times 0.6 \times 0.4 = 0.4365$

Solution Pathway

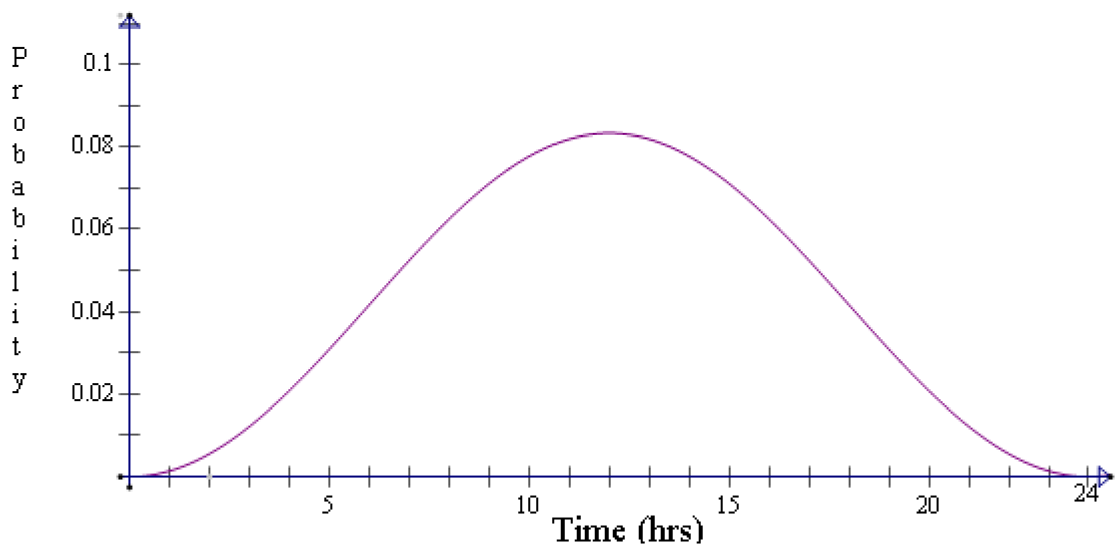
g) $\frac{0.65}{0.65+0.4} = \frac{0.65}{1.05} = 61.9\%$

Question 25

a)
$$\boldsymbol{n} = \frac{2\pi}{\boldsymbol{period}} = \frac{2\pi}{24} = \frac{\pi}{12}$$

b)
$$1 = \int_0^{24} -\boldsymbol{A}\left[\cos\left(\frac{\pi \boldsymbol{x}}{12}\right) - 1\right]\boldsymbol{dx}$$
$$1 = -\boldsymbol{A}\left[\frac{12}{\pi}\sin\left(\frac{\pi \boldsymbol{x}}{12}\right) - \boldsymbol{x}\right]_0^{24}$$
$$1 = -\boldsymbol{A}[-24 - 0]$$
$$\boldsymbol{A} = \frac{1}{24}$$

c)



k, period – 1 mark, maximum – 1 mark

Solution Pathway

d) $\frac{1}{12}$

e)

$$2\int_0^6 -\frac{1}{24}\left[\cos\left(\frac{\pi x}{12}\right)-1\right]dx = 18.17\%$$

f) 40-18.17 = 21.83%.

g) Use the calculator to find t in the following expression by trial and error.

$$\int_6^t -\frac{1}{24}\left[\cos\left(\frac{\pi x}{12}\right)-1\right]dx$$

$t = 9.635$

Hence the extra time needed outdoors is 9.635-6 = 3.635 hrs

Expression – 1 mark. t- 1 mark. Final answer 1 mark.

Question 26

a) *normCdf*(2.47, 2.53,2.5, 0.01)=0.9973.

b) *normCdf*(2.485, 2.515,2.5, 0.01)=0.8864.

c) 1-0.8864 = 0.1136.

d) $1-(0.8864)^4 = 0.3827$.

e) $\boldsymbol{mean} = 4\times 2.5 = 10\,\text{kg}$ (2 marks)

Standard deviation = 0.04 kg (1 mark)

Solution Pathway

Question 27

a)

$$\int_0^2 -0.1(\boldsymbol{x}-2)^2+0.4\,\boldsymbol{dx}=\left[\frac{\boldsymbol{x}^2}{5}-\frac{\boldsymbol{x}^3}{30}\right]_0^2$$
$$=\frac{8}{15}$$

b) $1-\frac{8}{15}=\frac{7}{15}$

c) Use area of a triangle formula:

$$\frac{7}{15}=\frac{1}{2}\times\frac{}{5}\times \boldsymbol{base}$$
$$\boldsymbol{base}=\frac{70}{30}$$
$$=\frac{14}{6}$$

Now the x-intercept is $2+\frac{14}{6}=\frac{26}{6}=4\frac{1}{3}$

d) The linear section passes through the points $\left(2,\frac{2}{5}\right)$ and $\left(\frac{26}{6},0\right)$. The gradient of the line is then

$$\boldsymbol{m}=\frac{0-\frac{2}{5}}{\frac{26}{6}-2}$$
$$=\frac{\frac{-2}{5}}{\frac{14}{6}}$$
$$=\frac{-2}{5}\times\frac{6}{14}$$
$$=\frac{-6}{35}$$

Solution Pathway

e) Median occurs between 0 and 2 since $\frac{8}{15} > \frac{1}{2}$

Hence use $\int_0^m -0.1(x-2)^2 + 0.4\,dx = \frac{1}{2}$

$$\left[\frac{x^2}{5} - \frac{x^3}{30}\right]_0^m = \frac{1}{2}$$

$$\frac{m^2}{5} - \frac{m^3}{30} = \frac{1}{2}$$

$$6m^2 - m^3 = 15$$

$$m = -1.422 \text{ or } 1.92 \text{ or } 5.51$$

Only one of these results in within the domain therefore median = 1.92.