Neap:

Trial Examination 2011

VCE Mathematical Methods (CAS) Units 1 & 2

Written Examination 1

Suggested Solutions

Question 1

a. $13m^2 = 5m$

$13m^2 - 5m = 0$ M1

$m(13m - 5) = 0$

$m = 0$ or $m = \frac{5}{13}$ A1

b. $\frac{4^{2m+1} - 4^{2m}}{4^{2m} + 4^{2m-1}} \times m = \frac{12}{5}$

$\frac{4^{2m}(4^1 - 1)}{4^{2m}(1 + 4^{-1})} \times m = \frac{12}{5}$ M1

$\frac{3}{1 + 4^{-1}} \times m = \frac{12}{5}$

$\frac{3}{1 + \frac{1}{4}} \times m = \frac{12}{5}$

$\frac{3}{\frac{5}{4}} \times m = \frac{12}{5}$

$\frac{12}{5} \times m = \frac{12}{5}$

$m = 1$ A1

c. $\log_5(2m - 1) < 1$

$\log_5(2m - 1) < \log_5(5)$

$2m - 1 < 5$

$2m < 6$

$m < 3$ A1

On the other hand, domain of $\log_5(2m - 1)$ is $2m - 1 > 0$ then $2m > 1$ thus $m > 0.5$

Therefore $0.5 < m < 3$. A1

Question 2

$\left(4x - \frac{1}{2x}\right)^2 = (4x)^2 + 2(4x)\left(\frac{1}{2x}\right) + \left(\frac{1}{2x}\right)^2$

$= 16x^2 - 4 + \frac{1}{4x^2}$ A1

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Question 3

The vertical asymptote is: $10 - x = 0$

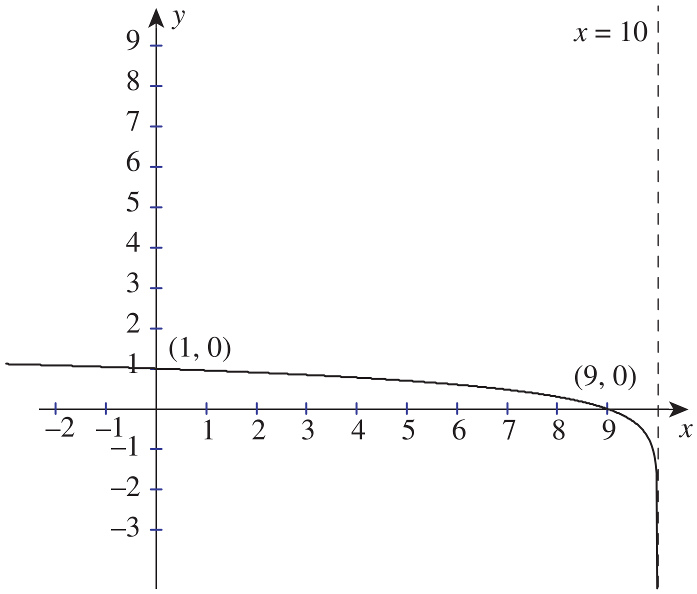
$x = 10$

The y-intercept is: $y = \log_{10}(10 - 0) = \log_{10}(10) = 1$ A1

The x-intercept is: $0 = \log_{10}(10 - x)$

$1 = 10 - x$

$x = 9$ A1



A1

Question 4

a. $f(x) = 2x^3 + x^2$ M1

$f'(x) = 2(3)x^{3-1} + 2x^{2-1}$

$f'(x) = 6x^2 + 2x$ A1

b. $f'(x) = 6x^2 + 2x$

$f'(4) = 6(4)^2 + 2(4) = 96 + 8 = 104$ A1

Question 5

The graph of $y = ax^2 + bx + c$ has turning point with coordinates $\left(-\frac{b}{2a}, c-\frac{b^2}{4a}\right)$.

In this case, $a = -1$, $b = 8$, and $c = -15$, thus the turning point has coordinates

$\left(-\frac{8}{2(-1)}, -15-\frac{8^2}{4(-1)}\right)$ M1

$= (4,1)$ A1

The distance, *d*, between (4,1) (0,0) is

$d = \sqrt{(4-0)^2 + (1-0)^2}$

$= \sqrt{17}$ A1

Alternatively, solve by completing the square.

$y = -x^2 + 8x - 15$

$= -(x^2 - 8x + 15)$

$= -(x^2 - 8x + 16 - 16 + 15)$ M1

$= -((x-4)^2 - 1)$

$= -(x-4)^2 + 1$

The turning point has coordinates (4,1) A1

The distance, *d*, between (4,1) and (0,0) is

$d = \sqrt{(4-0)^2 + (1-0)^2}$

$= \sqrt{17}$ A1

Other methods of finding the turning point included calculus and using symmetry property that the x*-coordinate of the turning point is half way between the* x*-intercepts.*

Question 6

$$F \times \begin{bmatrix} 4 & 5 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 6 & 7 \\ 24 & 33 \end{bmatrix}$$

$$F \times \begin{bmatrix} 4 & 5 \\ 2 & 3 \end{bmatrix} \times \begin{bmatrix} 4 & 5 \\ 2 & 3 \end{bmatrix}^{-1} = \begin{bmatrix} 6 & 7 \\ 24 & 33 \end{bmatrix} \times \begin{bmatrix} 4 & 5 \\ 2 & 3 \end{bmatrix}^{-1}$$ M1

$$F = \begin{bmatrix} 6 & 7 \\ 24 & 33 \end{bmatrix} \times \begin{bmatrix} 4 & 5 \\ 2 & 3 \end{bmatrix}^{-1}$$

$$F = \begin{bmatrix} 6 & 7 \\ 24 & 33 \end{bmatrix} \times \frac{1}{4 \times 3 - 5 \times 2} \times \begin{bmatrix} 3 & -5 \\ -2 & 4 \end{bmatrix}, \; det(M) = 2$$

$$F = \frac{1}{2} \times \begin{bmatrix} 6 & 7 \\ 24 & 33 \end{bmatrix} \times \begin{bmatrix} 3 & -5 \\ -2 & 4 \end{bmatrix}$$

$$F = \frac{1}{2} \times \begin{bmatrix} 6 \times 3 + 7 \times (-2) & 6 \times (-5) + 7 \times 4 \\ 24 \times 3 + 33 \times (-2) & 24 \times (-5) + 33 \times 4 \end{bmatrix}$$ M1

$$F = \frac{1}{2} \times \begin{bmatrix} 4 & -2 \\ 6 & 12 \end{bmatrix}$$

$$F = \begin{bmatrix} 2 & -1 \\ 3 & 6 \end{bmatrix}$$ A1

Alternatively, the solution can be found by solving the simultaneous equations.

Question 7

$$f(x) = \int\left(-\frac{x^{\frac{1}{2}}}{2} + 1\right)dx$$

$$= \int\left(-\frac{x^{\frac{1}{2}}}{2}\right)dx + \int 1\,dx$$

$$= -\frac{1}{2}\int x^{\frac{1}{2}}dx + x + c$$ M1

$$-\frac{1}{2}\int x^{\frac{1}{2}}dx + x + c = -\frac{1}{2}\frac{x^{\frac{1}{2}+1}}{\left(\frac{1}{2}+1\right)} + x + c$$

$$= -\frac{1}{2}\frac{x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} + x + c$$

$$= -\frac{1}{2} \times \frac{2}{3} \times x^{\frac{3}{2}} + x + c$$

$$= -\frac{1}{3}x^{\frac{3}{2}} + x + c$$ A1

Hence $f(x) = -\frac{1}{3}x^{\frac{3}{2}} + x + C$ and if $f(x)$ passes through (4, 0) then $f(4) = 0$

So $-\frac{1}{3}4^{\frac{3}{2}} + 4 + c = 0$

$$-\frac{1}{3}2^3 + 4 + c = 0$$

$$-\frac{8}{3} + 4 + c = 0$$

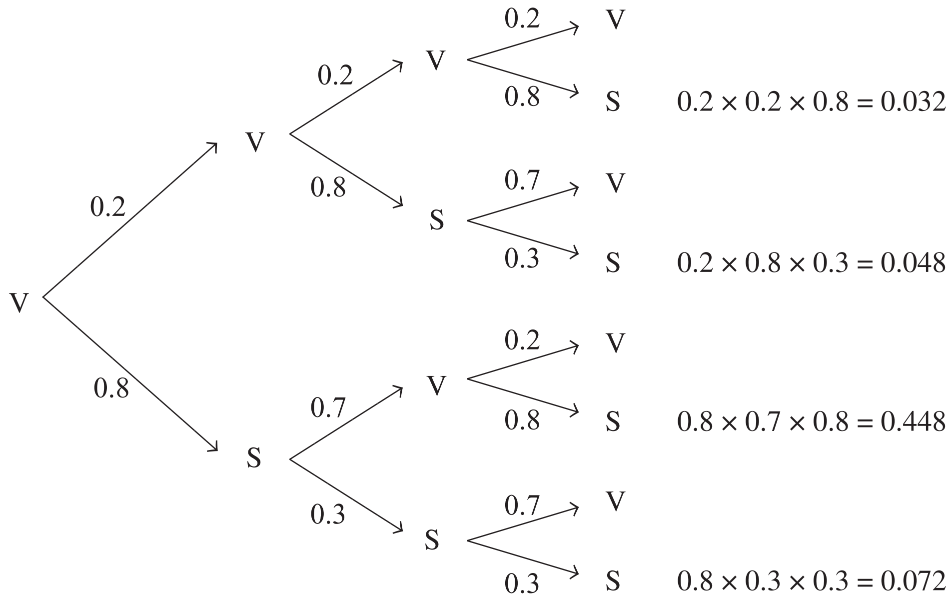
$$\frac{4}{3} + c = 0$$

$$c = -\frac{4}{3}$$

Therefore $f(x) = -\frac{1}{3}x^{\frac{3}{2}} + x - \frac{4}{3}$ A1

Question 8

21 days is 3 weeks so:



$\Pr(\text{S in 21 days}) = 0.032 + 0.048 + 0.448 + 0.072 = 0.6$

tree diagram M1
correct results A1
correct final result A1

Alternatively:

$\Pr(VVS) + \Pr(VSS) + \Pr(SVS) + \Pr(SSS)$ M1

$= 0.2 \times 0.2 \times 0.8 + 0.2 \times 0.8 \times 0.3 + 0.8 \times 0.7 \times 0.8 + 0.8 \times 0.3 \times 0.3$ A1

$= 0.6$ A1

Question 9

a. $3\sin\left(\frac{x}{2}\right)-\frac{3}{2}=0$

$$3\sin\left(\frac{x}{2}\right)=\frac{3}{2}$$

$$\sin\left(\frac{x}{2}\right)=\frac{1}{2}$$

$$\frac{x}{2}=\frac{\pi}{6} \text{ or } \frac{x}{2}=\pi-\frac{\pi}{6}$$

$$x=\frac{2\pi}{6} \text{ or } \frac{x}{2}=\frac{5\pi}{6}$$

$$x=\frac{\pi}{3}$$ M1

$$\text{or } x=\frac{5\pi}{3}$$ M1

b. Average rate of change is $\frac{f(\pi)-f(-\pi)}{\pi-(-\pi)}$ M1

$$=\frac{-3-3}{\pi+\pi}=\frac{-6}{2\pi}=-\frac{3}{\pi}$$ A1

c.

$$\cos(2\pi-x)-\cos(\pi+x)$$

$$=\cos(x)-(-\cos(x))$$

$$=\cos(x)+\cos(x)$$ M1

$$=0.7+0.7$$

$$=1.4$$ A1

Question 10

a. According to factor theorem if $x+2$ is a factor of $p(x)$ then we can say $p(-2)=0$

$p(-2)=(-2)^3-6(-2)^2-9(-2)+m=-8-24+18+m=-14+m$

So $-14+m=0$

$m=14$ A1

b. If $x+2$ is a factor of $p(x)=x^3-6x^2-9x+14$, we may find the other factor by long division:

$$\begin{array}{r} x^2-8x+7 \\ x+2\,\overline{)\,x^3-6x^2-9x+14} \\ -(x^3+2x^2) \\ \hline -8x^2-9x+14 \\ -(-8x^2+16x) \\ \hline 7x+14 \\ -(7x+14) \\ \hline 0 \end{array}$$ M1

Thus $x^3-6x^2-9x+14=(x+2)(x^2-8x+7)=(x+2)(x-1)(x-7)$

Therefore linear factors are $(x+2)$, $(x-1)$, and $(x-7)$. A1

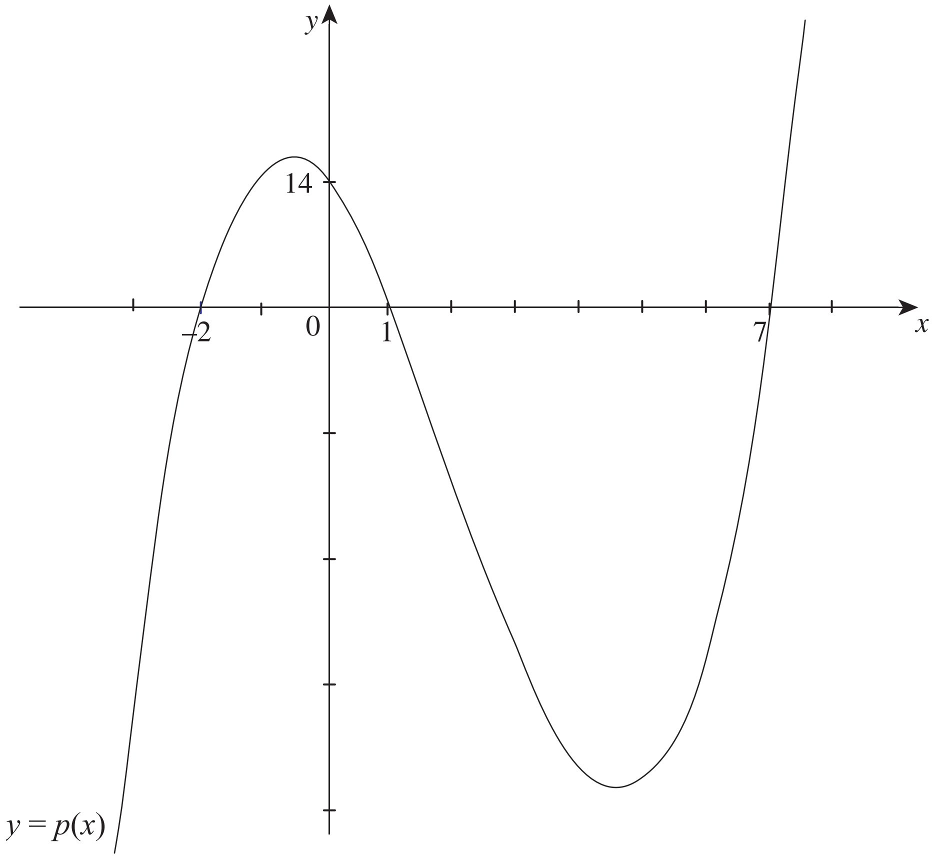
c. $y = p(x)$ where $p(x) = x^3 - 6x^2 - 9x + 14 = (x+2)(x-1)(x-7)$

x-intercepts, $p(x) = 0$: $(x+2)(x-1)(x-7) = 0$

so $x = -2$, $x = 1$, and $x = 7$

y-intercept, $y = p(0)$: $y = 0^3 - 6(0^2) - 9(0) + 14$

so the y-intercept is at (0,14) or $y = 14$ x- *and* y-*intercepts* A1



correct graph shape A1

Question 11

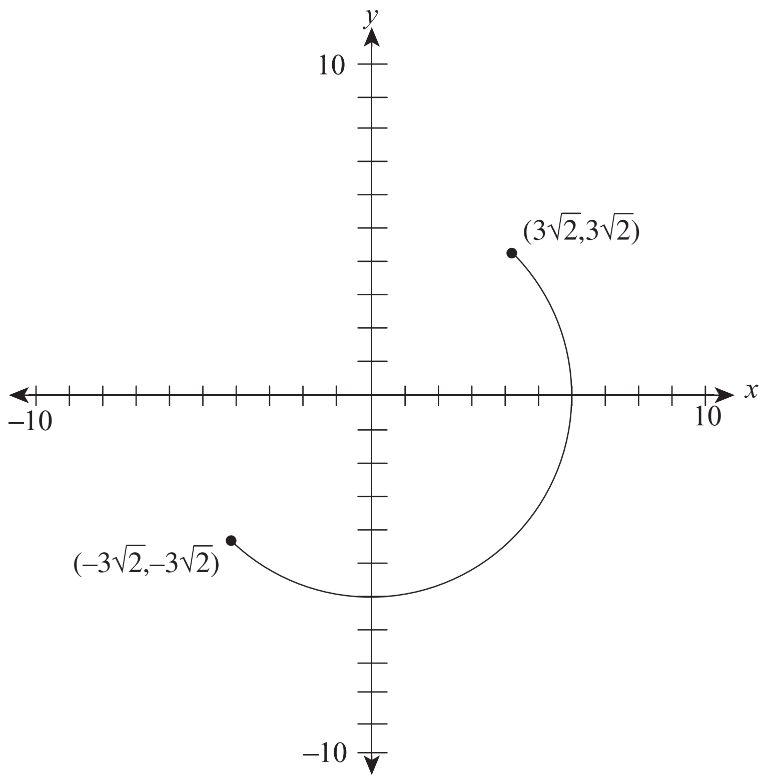
$x^2 + y^2 = 36$ is a circle where the centre is located at (0,0) and the radius is 6.

The points on the circle with the x-coordinate $-3\sqrt{2}$ are:

$$\begin{aligned} x^2 + y^2 &= 36 \\ (-3\sqrt{2})^2 + y^2 &= 36 \\ 18 + y^2 &= 36 \\ y^2 &= 18 \\ y &= \pm\sqrt{18} \\ y &= \pm 3\sqrt{2} \end{aligned}$$ A1

Similarly, the points with y-coordinates $3\sqrt{2}$ will have x-coordinate as $\pm 3\sqrt{2}$.

Since $x \geq -3\sqrt{2}$ and $y \leq 3\sqrt{2}$, the section of the circle that is required starts at $(-3\sqrt{2}, -3\sqrt{2})$ and, moving in an anti-clockwise direction, ends at $(3\sqrt{2}, 3\sqrt{2})$.



correct shape A1
correct section of the graph with endpoints labelled A1

Range of the relation is $[-6, 3\sqrt{2}]$. A1