The Mathematical Association of Victoria
Trial Exam 2011

MATHEMATICAL METHODS (CAS)

STUDENT NAME ____

Written Examination 1

Reading time: 15 minutes
Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
10	**10**	**40**

Note
- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

Materials supplied
- Question and answer book of 8 pages, with a detachable sheet of miscellaneous formulas at the back.
- Working space is provided throughout the book.

Instructions
- Detach the formula sheet from the back of this book during reading time.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown. Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

a. $\frac{d}{dx}(x\tan(x))$.

2 marks

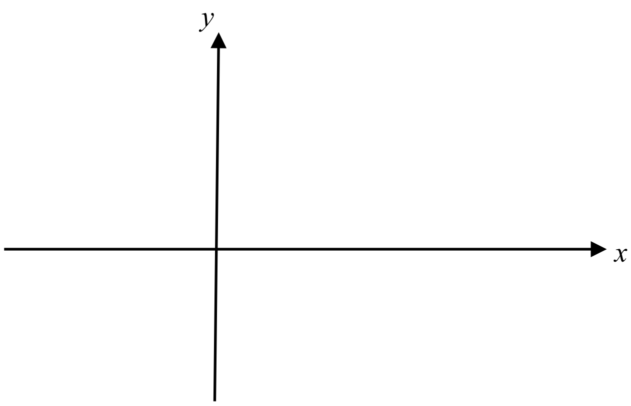
b. **i.** $\frac{d}{dx}\left(e^{2x}+2x\right)$.

ii. Hence, find an antiderivative of $\frac{4(e^{2x}+1)}{e^{2x}+2x}$.

1 + 1 = 2 marks

Question 2

a. Sketch the graph of $f:[-1,3]$, where $f(x)=(x-1)^{\frac{2}{3}}+2$ on the set of axes below. Clearly label the endpoints, intercept and sharp point with their coordinates.



2 marks

TURN OVER

b. Find the average value of *f*, over the interval $[0, 2]$.

3 marks

Question 3

For what value(s) of *k*, where *k* is a real constant, do the simultaneous equations

$$kx + 2y = 6 \text{ and}$$

$$3x + (k-1)y = 6$$

have no solution?

3 marks

Question 4

Solve $2\log_2(x-1) + \log_2(x+1) = 0$ for *x*.

4 marks

Question 5

Let $f : R \rightarrow R$, where $f(x) = 1 - e^{-x}$.

a. Find f^{-1}.

3 marks

b. State the coordinates of the point where $f = f^{-1}$.

1 mark

Question 6

If $f(x) = x^3 + 3$, $g(x) = |x - 1|$ and $h(x) = g(f(x))$, define $h'(x)$ as a hybrid function.

3 marks

TURN OVER

Question 7

A transformation is described by the equation $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & 3 \end{bmatrix}\left(\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \end{bmatrix}\right)$.

a. Find the image of the curve with equation $y = \dfrac{2}{x+1} - 1$ under this transformation. Give your answer in the form $y = \dfrac{a}{x} + b$ where a and b are real constants.

3 marks

b. Hence, describe how the graph of $y = \dfrac{2}{x+1} - 1$ can be transformed to the graph of the image.

3 marks

Question 8

The table below represents a probability distribution of a random variable X.

x	0	1	2	3	4
$\Pr(X = x)$	p	$3p$	q	0.03	0.01

a. If $2p - q = 0$, show that the value of p is 0.16.

2 marks

b. Find $\Pr(X < 2 \mid X < 3)$.

1 mark

Question 9

The length of time, t (hours) that certain sea anemones survive is a random variable whose probability density function can be modelled by $f(t) = \begin{cases} \dfrac{k}{2}\left(\cos\left(\dfrac{\pi}{3}t\right)+1\right) & 0 \le t \le 3 \\ 0 & \text{elsewhere} \end{cases}$

and k is a real constant.

a. Show that $k = \dfrac{2}{3}$.

2 marks

b. Evaluate the probability that a particular sea anemone will survive for more than two years.

2 marks

TURN OVER

Question 10

Solve $\frac{d}{dx}\sqrt{\left(1+\cos^3(2x)\right)}=0$ for $0 \le x \le \pi$.

4 marks

END OF QUESTION AND ANSWER BOOK

Mathematical Methods (CAS)
Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$	volume of a pyramid:	$\frac{1}{3}Ah$
curved surface area of a cylinder:	$2\pi rh$	volume of a sphere:	$\frac{4}{3}\pi r^3$
volume of a cylinder:	$\pi r^2 h$	area of a triangle:	$\frac{1}{2}bc\sin A$
volume of a cone:	$\frac{1}{3}\pi r^2 h$		

Calculus

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

$$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$$

$$\frac{d}{dx}\left(\sin(ax)\right) = a\ \cos(ax)$$

$$\frac{d}{dx}\left(\cos(ax)\right) = -a\ \sin(ax)$$

$$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\ \sec^2(ax)$$

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$$

$$\int \frac{1}{x} dx = \log_e|x| + c$$

$$\int \sin(ax)dx = -\frac{1}{a}\cos(ax) + c$$

$$\int \cos(ax)dx = \frac{1}{a}\sin(ax) + c$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$ quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$ approximation: $f(x+h) \approx f(x) + hf'(x)$

Probability

$\Pr(A) = 1 - \Pr(A')$ $\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$ transition matrices: $S_n = T^n \times S_0$

mean: $\mu = \mathrm{E}(X)$ variance: $\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X-\mu)^2) = \mathrm{E}(X^2) - \mu^2$

probability distribution		mean	variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x\,p(x)$	$\sigma^2 = \sum (x-\mu)^2\,p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)dx$	$\mu = \int_{-\infty}^{\infty} x\,f(x)dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx$

END OF FORMULA SHEET