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Student Name……

MATHEMATICAL METHODS (CAS) UNITS 3 & 4

TRIAL EXAMINATION 1

2011

Reading Time: 15 minutes
Writing time: 1 hour

Instructions to students

This exam consists of 11 questions.
All questions should be answered in the spaces provided.
There is a total of 40 marks available.
The marks allocated to each of the questions are indicated throughout.
Students may **not** bring any calculators or notes into the exam.
Where an exact answer is required a decimal approximation will not be accepted.
Where more than one mark is allocated to a question, appropriate working must be shown.
Diagrams in this trial exam are not drawn to scale.
A formula sheet can be found on page 11 of this exam.

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Question 1

a. Let $y = \sqrt{2x^2 - 1}$. Find $\frac{dy}{dx}$.

2 marks

b. Let $f(x) = \frac{x}{e^{3x}}$. Find $f'(1)$.

2 marks

Question 2

Solve $\log_e(3) + 2\log_e(x) = \log_e(4x)$ for x.

3 marks

Question 3

Let $g:(2,\infty)\to R,\ g(x)=3\log_e(x-2)$.

a. Find g^{-1}, the inverse function of g.

3 marks

b. If $h(x)=g^{-1}(g(x))$, find

i. the rule for h

ii. the domain of h.

$1+1=2$ marks

Question 4

Let X be a random variable with a normal distribution with mean 10 and variance 4 and let Z be a random variable with the standard normal distribution.

a. Find $\Pr(X > 12)$.

1 mark

b. Find $\Pr(X > 12 | X > 10)$.

2 marks

c. Find a such that $\Pr(Z > a) = \Pr(X < 5)$.

2 marks

Question 5

Solve the equation $\sin\left(\frac{x}{2}\right)+\frac{1}{\sqrt{3}}\cos\left(\frac{x}{2}\right)=0$ for $x \in R$.

3 marks

Question 6

At a fast food outlet, 60% of orders placed at the drive-through counter include chips. It can be assumed that an order placed by one drive-through customer is independent of an order placed by another drive-through customer.

a. What is the probability that out of the next three orders placed at the drive-through counter at least two will include chips?

2 marks

b. How many orders would need to have been placed at the drive-through counter if the probability that at least one order included chips was equal to 0.84,

3 marks

Question 7

A spherical balloon is being inflated. Its volume is increasing at the rate of $2cm^3$ per second. Find the rate in cm/sec, at which the radius of the balloon is increasing when the radius is 4cm.

3 marks

Question 8

Let $g: R \setminus \{0\} \to R,\ g(x) = 1 + \frac{1}{x}$.

Show that $4g(2u) - g(-u) = 3g(u)$.

2 marks

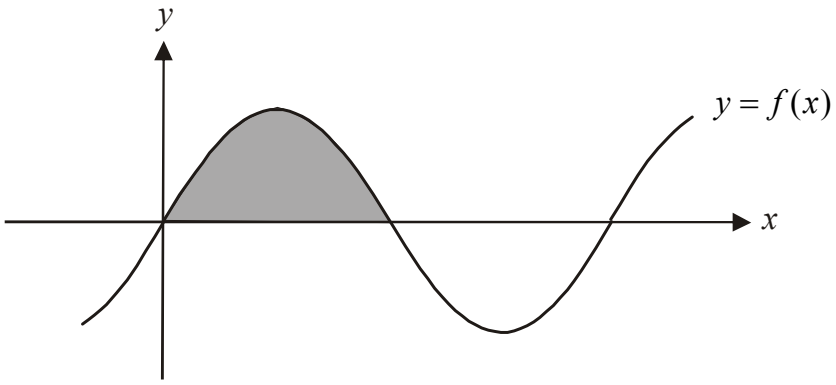
Question 9

Given that $f(x+h) \approx f(x) + hf'(x)$, where h is small, find an approximate value of $\sqrt{9.03}$.

3 marks

Question 10

Part of the graph of the function $f : R \to R,\ f(x) = a\sin(2x)$ where a is a positive constant is shown below.



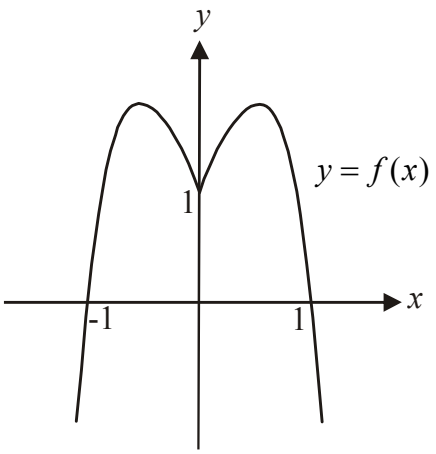
The shaded region represents an area of 4 square units.
Find the value of a.

4 marks

Question 11

Let $f : R \to R, f(x) = 2|x| - 3x^4 + 1$.

The graph of $y = f(x)$ is shown below.



a. Write down the domain of the derivative function $f'(x)$.

_____

1 mark

b. Find the rule for $f'(x)$.

_____

_____

_____

_____

_____

2 marks

Mathematical Methods (CAS) Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$	volume of a pyramid:	$\frac{1}{3}Ah$
curved surface area of a cylinder:	$2\pi rh$	volume of a sphere:	$\frac{4}{3}\pi r^3$
volume of a cylinder:	$\pi r^2 h$	area of a triangle:	$\frac{1}{2}bc\sin A$
volume of a cone:	$\frac{1}{3}\pi r^2 h$		

Calculus

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1} \qquad \int x^n dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax} \qquad \int e^{ax} dx = \frac{1}{a}e^{ax} + c$$

$$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x} \qquad \int \frac{1}{x} dx = \log_e|x| + c$$

$$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax) \qquad \int \sin(ax)dx = -\frac{1}{a}\cos(ax) + c$$

$$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax) \qquad \int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + c$$

$$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\sec^2(ax)$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

approximation: $f(x+h) \approx f(x) + hf'(x)$

Probability

$\Pr(A) = 1 - \Pr(A')$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

transition matrices: $S_n = T^n \times S_0$

mean: $\mu = \mathrm{E}(X)$

variance: $\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X-\mu)^2) = \mathrm{E}(X^2) - \mu^2$

probability distribution		mean	variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \Sigma x\, p(x)$	$\sigma^2 = \Sigma (x-\mu)^2 p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)dx$	$\mu = \int_{-\infty}^{\infty} x f(x)dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx$