2010 Assessment Report

VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

2010 Mathematical Methods (CAS) GA 3: Examination 2

GENERAL COMMENTS

There were 15 610 students who sat the Mathematical Methods (CAS) examination in 2010 (8517 students sat the Mathematical Methods examination in 2009). Marks ranged from 0 to 80 out of a possible score of 80. Student responses showed that the paper was accessible and that it provided an opportunity for students to demonstrate what they knew and could do.

The mean score for 2010 was 41. The median score for the paper was 41 marks. Of the whole cohort, 10% of students scored 83% or more of the available marks, and 23% scored 71% or more of the available marks.

The mean score for the multiple-choice section was 13.8 (out of 22). Question 3 was answered correctly by 87% of students. Less than 50% of students obtained the correct answers for Questions 7, 12, 18, 20, 21 and 22. Students had difficulty in identifying whether events are mutually exclusive or independent in a given context, and identifying the conditions under which simple systems of simultaneous linear equations including a parameter have or do not have a unique solution.

Students must ensure that they:

- read questions carefully
- give answers to the required accuracy
- transcribe the correct equation
- provide answers to all parts of the question
- answer the question that is being asked.

After completing a question students should reread the question. Exact answers were required for the intercepts in Question 1aii., while Question 1aiii. required the answer correct to three decimal places. Many students omitted the '+1' constant from the rule of the function *g* in Question 1ai., while others did not state the domain. In Question 2a., some students found the probability that the third statue was 'Superior', not 'Regular', and others did not state the final answer in Question 2c.

As stated in the instructions, students **must** show appropriate working for questions worth more than one mark. There was a noticeable increase in the number of students showing working, but a number of students gave answers only or insufficient working for Questions 1aiv. and 2e. Writing out the mathematical expression to be evaluated or equation to be solved is considered sufficient working. Technology syntax should not be used for this purpose. Students should be encouraged to attempt to write out an expression or equation, even if they think their answer to a previous question is incorrect, as marks can be awarded; for example, in Questions 1aiv., 1biii., 3b., 3c., 3f. and 4d. There was a number of 'show that' questions on this year's examination – Questions 1biii., 1biv., 2di., 3b., and 3d. – and students must make sure that they show sufficient working for these questions.

Most students used correct mathematical notation this year; in particular, dx was used properly in Question 1aiv. Technology syntax should not be used, although it was seen occasionally in Question 1ai. Incorrect units were used in Question 3 and a surprising number of algebraic errors was made in Questions 1aiv., 2di. and 3c. Brackets were not used well in Question 1aiv. and most of Question 3. Students need to ensure their writing and graphs are legible. For questions worth one mark, the final answer needs to be clear, especially where there is related working on the page. This can be done by highlighting, underlining, annotating or using a box.

It was pleasing to see a variety of approaches involving various combinations of by-hand and technology-assisted methods being used to solve questions like Questions 1aiii., 1aiv., all parts of Question 2, 3a., 3e., 4d. and 4f. In particular, a matrix method was used by some students to solve Question 2dii. to find the expected value. However, some students did not use the appropriate syntax to separate the coefficient a from the variable x in Question 4b. when using technology. Students should be familiar with the functionality of their technology and be able to use a variety of approaches to formulating and solving problems.

SPECIFIC INFORMATION

Section 1

The table below indicates the percentage of students who chose each option. The correct answer is indicated by shading.

Question	% A	% B	% C	% D	% E	% No Answer	Comments
1	7	26	2	60	5	0	$f(x) = 4\tan\left(\frac{x}{3}\right)$, Period = $\frac{\pi}{\frac{1}{3}} = 3\pi$ Option B was obtained if 2π was used instead of π giving $\frac{2\pi}{\frac{1}{3}} = 6\pi$.
2	9	6	9	6	69	1	
3	87	5	2	4	2	0	
4	1	7	2	83	7	0	
5	8	80	3	6	3	0	
6	4	14	79	2	1	0	$g(x) = \int (x^2 - 2x)dx = \frac{x^3}{3} - x^2 + c$, $g(1) = 0$ $g(x) = \frac{x^3}{3} - x^2 + \frac{2}{3}$ Option B was obtained if the constant was not considered.
7	13	9	17	47	13	1	$(m-1)x + 5y = 7$ and $3x + (m-3)y = 0.7m$ have to represent the same answer line for infinitely many solutions. For this to be the case $\frac{m-1}{3} = \frac{5}{m-3} = \frac{7}{0.7m}$. Hence m = 6.
8	8	2	7	4	79	0	
9	60	13	13	10	4	0	$f(x) = x^3 - 3x^2 + 3$, $f'(x) = 0$, x = 0 (local maximum) or x = 2 (local minimum) The domain of f is $(-\infty, 0]$ for the inverse function to exist. f must have 1:1 correspondence. Hence a = 0. The answer could also be obtained graphically as the x values of the turning points are integers.
10	10	12	66	8	3	0	
11	7	14	54	11	12	1	Solve $\int_{\frac{3\pi}{4}}^{a} (\cos(2x))dx = \frac{1}{4}$ for a. $a \approx 2.88$
12	5	44	10	38	3	0	$X \sim \text{Bi}(15, 0.6), \Pr(X < 7) = \Pr(X \le 6) \approx 0.0950$ Option D is obtained if 7 is used as the upper bound.
13	6	51	17	15	10	1	$\Pr(-2 < Z < 1) = \Pr(-1 < Z < 2)$ $= \Pr(20 - 1\times 6 < X < 20 + 2\times 6)$ $= \Pr(14 < X < 32)$
14	81	10	3	2	3	0	
15	1	4	83	10	2	0	
16	9	3	3	78	6	0	
17	12	60	8	9	11	1	The graph has a stationary point of inflection at x = 4 because $f'(4) = 0$ and $f'(x) > 0$ for $2 < x < 4$ and x > 4.

Question	% A	% B	% C	% D	% E	% No Answer	Comments
18	7	10	10	45	28	0	By inspection of the graph near zero, the function is not differentiable.
19	5	5	2	77	11	0	
20	19	28	20	25	7	1	$2\int_0^{5a}\left(f\left(\frac{x}{5}\right)+3\right)dx = 2\times 5a + 2\int_0^{5a}(3)dx = 40a$ Option B was obtained if the antiderivative of 3 was not found. $2\left(\int_0^{5a}\left(f\left(\frac{x}{5}\right)\right)dx+3\right) = 2\times 5a + 2\times 3 = 10a+6$
21	29	10	43	8	10	0	If events A and B are mutually exclusive then $\Pr(A\cap B)=0$. Hence $\Pr(A'\cap B') = 1-(p+q)$. Option A was obtained if A and B are independent events. $\Pr(A'\cap B') = \Pr(A')\times\Pr(B') = (1-p)(1-q)$
22	10	20	19	22	29	1	$\int_3^{ab+2}\left(\frac{1}{x-2}\right)dx = \int_3^{a+2}\left(\frac{1}{x-2}\right)dx + \int_3^{b+2}\left(\frac{1}{x-2}\right)dx$ as $\log_e(ab) = \log_e(a)+\log_e(b)$. Alternatively, a suitable choice of values could be used to test the relationship.

Section 2

Question 1

Questions 1bi. and 1bii. were answered extremely well.

1ai.

Marks	0	1	2	3	Average
%	8	8	27	57	2.3

$g(x) = 2\log_e(x+4)+1$, let $y = 2\log_e(x+4)+1$, inverse swap x and y

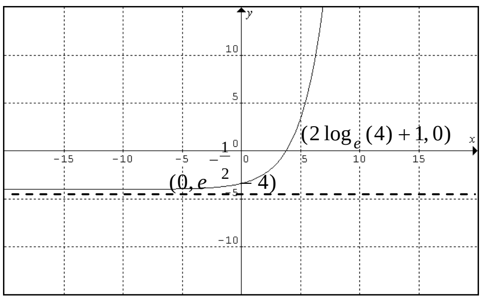
$x = 2\log_e(y+4)+1$, $g^{-1}(x) = e^{\left(\frac{x-1}{2}\right)} - 4$. The domain is R.

This question was well done. Some students did not include '+ 1' in the equation and others did not include the domain.

Technology syntax such as $y = e^{\wedge}(\frac{x-1}{2}) - 4$ should not have been used.

1aii.

Marks	0	1	2	3	Average
%	13	14	29	44	2.1



$y = -4$

Some students drew the asymptote at $y = -5$. The asymptotic behaviour of the graph had to be shown. Exact answers were required for the intercepts and if they were given in coordinate form the zeroes had to be in the correct positions. A clear curve with no breaks was required.

1aiii.

Marks	0	1	2	Average
%	16	18	66	**1.5**

Solve $g^{-1}(x) = g(x)$, $g^{-1}(x) = x$ or $g(x) = x$; $x = -3.914$ or $x = 5.503$

This question was done quite well. Most students solved $g^{-1}(x) = x$ or $g(x) = x$. A small number of students solved $g'(x) = g(x)$, while others did not give their answers correct to three decimal places.

1aiv.

Marks	0	1	2	Average
%	38	21	41	**1**

$$\int_{-3.91432...}^{5.50327...} \left(g(x) - g^{-1}(x)\right)dx \text{ or } 2\int_{-3.91432...}^{5.50327...} \left(2\log_e(x+4) + 1 - x\right)dx \text{ or}$$

$$\int_{-3.91432...}^{5.50327...} \left(2\log_e(x+4) + 1 - (e^{\frac{x-1}{2}} - 4)\right)dx \text{, Area } \approx 52.63$$

The correct answer could be obtained if three decimal places were used. A common error was

$$\int_{-3.91432...}^{5.50327...} \left(2\log_e(x+4) + 1 - e^{\frac{x-1}{2}} - 4)\right)dx$$

. Some students partitioned the area into different sections, rather than just using a single area based on the difference between the two functions. This would have been very time-consuming. Most students had dx at the end of the expression.

1bi.

Marks	0	1	Average
%	11	89	**0.9**

Vertical asymptote, $a = 1$

1bii.

Marks	0	1	Average
%	15	85	**0.9**

y-intercept, $c = 1$

1biii.

Marks	0	1	2	Average
%	22	2	75	**1.5**

$$10 = k\log_e(p+1) + 1, \quad \frac{10-1}{k} = \log_e(p+1), \quad k = \frac{9}{\log_e(p+1)}$$

This was a 'show that' question. Students were required to substitute their answers from Questions 1bi. and 1bii. and the point $(p, 10)$ into f.

1biv.

Marks	0	1	Average
%	48	52	**0.5**

$$f(x) = \frac{9}{\log_e(p+1)}\log_e(x+1)+1,\ f'(x) = \frac{9}{(x+1)\log_e(p+1)},\ f'(p) = \frac{9}{(p+1)\log_e(p+1)}$$

This was a 'show that' question. Students were required to show the derivative and then substitute $x = p$.

1bv.

Marks	0	1	2	Average
%	64	6	30	**0.7**

$$f'(p) = \frac{10-0}{p+1},\ p = e^{\frac{9}{10}} - 1$$

Many students worked out the equation of the tangent, which was unnecessary and very time-consuming, instead of equating the gradient of the segment with the derivative at $x = p$.

Question 2

2a.

Marks	0	1	2	Average
%	34	17	49	**1.2**

$$\begin{array}{c} \\ S_{i+1} \\ R_{i+1} \end{array}\begin{array}{c} \begin{array}{cc} S_i & R_i \end{array} \\ \begin{bmatrix} 0.9 & 0.7 \\ 0.1 & 0.3 \end{bmatrix}^2 \end{array}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.88 \\ 0.12 \end{bmatrix} \text{ or } 1\times 0.9\times 0.1 + 1\times 0.1\times 0.3,\ 0.12 \text{ or } \frac{3}{25}$$

Students should clearly indicate the final answer. Many students found the probability that the fourth statue, not the third, was 'Regular', while others gave 0.88 as the answer. Some students left their answer in terms of p.

2b.

Marks	0	1	Average
%	26	74	**0.8**

$(0.9)^3 = 0.729$

This question was answered well.

2c.

Marks	0	1	Average
%	44	56	**0.6**

$$\begin{bmatrix} 0.9 & 0.7 \\ 0.1 & 0.3 \end{bmatrix}\begin{bmatrix} x \\ 1-x \end{bmatrix} = \begin{bmatrix} x \\ 1-x \end{bmatrix} \text{ or } \begin{bmatrix} 0.9 & 0.7 \\ 0.1 & 0.3 \end{bmatrix}^{50}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.875 \\ 0.125 \end{bmatrix}$$

or $\frac{0.7}{0.1+0.7} = \frac{7}{8} = 0.875$.

A number of different methods could be used to solve this question. The final answer needed to be clearly indicated.

2di.

Marks	0	1	2	3	Average
%	44	12	8	36	**1.4**

$$\begin{bmatrix} p & p-0.2 \\ 1-p & -p+1.2 \end{bmatrix}^2\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.7 \\ 0.3 \end{bmatrix},\ \text{or } 1\times p\times p + 1\times(1-p)(p-0.2) = 0.7$$

$$p = \frac{3}{4} = 0.75$$

A number of different approaches were used to solve this question but it was not done well by most students. For 'show that' questions, appropriate working must be shown. Some students made an algebraic error, having $-p + 0.8$, instead of $-p + 1.2$.

2dii.

Marks	0	1	2	3	4	Average
%	63	14	12	2	10	**0.9**

x	0	1	2
$\Pr(X = x)$	0.2025	0.3850	0.4125

$E(X) = 0\times 0.2025 + 1\times 0.385 + 2\times 0.4125 = 1.21$

This question was not well answered. Some students calculated the expected value using matrices

$\begin{bmatrix}0\\1\end{bmatrix}+\begin{bmatrix}0.75 & 0.55\\0.25 & 0.45\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}+\begin{bmatrix}0.75 & 0.55\\0.25 & 0.45\end{bmatrix}^2\begin{bmatrix}0\\1\end{bmatrix}=\begin{bmatrix}1.21\\1.79\end{bmatrix}$. The resultant matrix provided both the expected value of 'Superior' statues, 1.21 and the expected value of 'Regular' statues, 1.79. The matrix method is effective for large groups of statues, such as five consecutive statues, as it is less time-consuming.

2e.

Marks	0	1	2	3	Average
%	58	18	9	15	**0.8**

$\Pr(X \geq 2) \geq 0.9$, $\Pr(X = 0) + \Pr(X = 1) \leq 0.1$, $0.8^n + \binom{n}{1}(0.2)(0.8)^{n-1} \leq 0.1$, $n = 18$

or $\Pr(X \geq 2) \geq 0.9$, $X \sim \text{Bi}(n, 0.2)$, $n = 18$

Several different approaches were used to solve this question. Some students used trial and error, while others used a table of values. Some students used $p = 0.8$.

Question 3

3ai.

Marks	0	1	Average
%	48	52	**0.5**

$\cos(x) = \dfrac{AZ}{10}$, $AZ = 10\cos(x)$, $AB = 20\cos(x)$

Equivalent forms of the answer were acceptable. Some students used the sine rule and others used $180 - 2x$ when x was in radians. $AB = \dfrac{\cos(x)}{20}$ was a common incorrect answer.

3aii.

Marks	0	1	Average
%	46	54	**0.6**

$\sin(x) = \dfrac{WZ}{10}$, $WZ = 10\sin(x)$

Equivalent forms of the answer were acceptable.

3b.

Marks	0	1	2	Average
%	47	10	42	1

$A_{base} = 400\cos^2(x)$, $A_{triangle} = 10\cos(x)10\sin(x)$,

$A_{triangles} = 400\cos(x)\sin(x)$, $S = 400\cos(x)\sin(x) + 400\cos^2(x)$,

$S = 400(\cos(x)\sin(x) + \cos^2(x))$

This was a 'show that' question and adequate working was required. There was poor use of brackets and $\cos x^2$ was often seen.

3c.

Marks	0	1	2	Average
%	68	6	26	0.6

$h = \sqrt{(WZ)^2 - \left(\frac{AB}{2}\right)^2}$, $h = 10\sqrt{\sin^2(x) - \cos^2(x)} = 10\sqrt{1 - 2\cos^2(x)}$

Equivalent forms of the answer were acceptable. There was some poor algebra and use of brackets in this question; for example, $(\sqrt{200}\cos x)^2 = 200\cos x^2$ and $\sqrt{100\sin^2(x) - 100\cos^2(x)} = 10\sin(x) - 10\cos(x)$.

3d.

Marks	0	1	Average
%	78	22	0.2

$T = \frac{1}{3} \times \text{base} \times \text{height} = \frac{1}{3} \times 400\cos^2(x) \times \sqrt{100\sin^2(x) - 100\cos^2(x)}$

$= \frac{1}{3} \times 400\cos^2(x) \times 10\sqrt{1 - 2\cos^2(x)} = \frac{4000}{3}\sqrt{\cos^4(x) - 2\cos^6(x)}$

This was a 'show that' question and adequate working had to be shown. Students are expected to be familiar with the identity $\sin^2(x) + \cos^2(x) = 1$ and equivalent forms such as $\sin^2(x) = 1 - \cos^2(x)$. This could have been used in the previous question.

3e.

Marks	0	1	2	3	4	Average
%	15	37	20	11	17	1.3

$\frac{dT}{dx} = \frac{8000\cos(x)\sin(x)(3\cos^2(x) - 1)}{3\sqrt{(1 - 2\cos^2(x))}} = 0$, $x = \cos^{-1}\left(\frac{\sqrt{3}}{3}\right)$ or $x = 2\tan^{-1}\left(\frac{\sqrt{6} - \sqrt{2}}{2}\right)$, $T = \frac{4000\sqrt{3}}{27}$

Several approaches were possible to find the zeroes of the derivative. Some students obtained the derivative by hand and noted that this would be zero when $3\cos^2(x) - 1 = 0$ on the given domain. Others used technology, and some used a combination of by-hand working and technology. Some of the outputs from technology were long, but correct and acceptable expressions. Some students did not restrict the domain and gave a general solution for x. Students could use the graph of the function to check their solutions with respect to the domain $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$. Exact answers were required for x and T.

3f.

Marks	0	1	2	Average
%	79	8	12	0.4

Solve $T(x) = \frac{2000\sqrt{3}}{27}$ = 128.3000598 …; x = 0.81 or x = 1.23

This question could be answered independently of 3e. using a numerical/graphical approach to finding the maximum value, then solving for $T(x)$ equal to half of this value numerically. Some students did not give the answers correct to two decimal places or they gave one answer only. Some students incorrectly solved

$\frac{4000\sqrt{3}}{27} = \frac{4000}{6}\sqrt{\cos^4(x) - 2\cos^6(x)}$.

Question 4

4a.

Marks	0	1	2	3	4	Average
%	16	4	14	11	54	2.8

$f(x) = \frac{1}{27}(2x-1)^3(6-3x)+1$, $f'(x) = 0$, Stationary point of inflection at $x = \frac{1}{2}$,

Local maximum at $x = \frac{13}{8}$

This question was done quite well, with more than half of the cohort being awarded full marks. Many students did the first derivative test, which was unnecessary. Other students found the x-intercepts, which were not required. Students should be familiar with the terminology 'point of inflection'.

4b.

Marks	0	1	2	3	Average
%	44	11	7	38	1.4

$f(x) = \frac{1}{27}(ax-1)^3(b-3x)+1$, $f'(x) = 0$ hence $x = \frac{ab+1}{4a}$ or $x = \frac{1}{a}$

A number of students did not use the appropriate syntax to separate the coefficient a from the variable x when using technology and wrote $f'(x) = -\frac{(ax-1)^3}{9}$. Others solved the second bracket equal to zero, getting $x_{\max} = \frac{b}{3}$.

4c.

Marks	0	1	Average
%	53	47	0.5

$a = 0$

The denominator cannot equal zero. If a equals zero, f will be linear as can be readily seen from the corresponding graph. Some students incorrectly wrote $a \neq 0$ rather than identifying the value of a for which f has no stationary points.

4d.

Marks	0	1	2	Average
%	83	3	14	0.3

$\frac{1}{a} = \frac{ab+1}{4a}$, $a = \frac{3}{b}$ or since $f(x) = -\frac{1}{27b}(ax-1)^4+1$, $\frac{1}{a} = \frac{-b}{-3}$, $a = \frac{3}{b}$

Some students tried to work out a 'discriminant'.

4e.

Marks	0	1	Average
%	63	37	0.4

2

It was pleasing to see a number of students answer this question correctly even though they had not attempted the previous parts. Some students wrote 3 as f is a quartic polynomial function.

4f.

Marks	0	1	2	3	Average
%	82	12	3	3	0.3

$x = \frac{1}{a} = 1$, $a = 1$, $p = \frac{ab+1}{4a} = \frac{b+1}{4}$, $f(p) = p$, $p = 4$

or solve $f(p) = p$, $f'(1) = 1$ and $f'(p) = p$, $p \neq 1$, $p = 4$

This question was not answered well. A small number of students used the second method and solved the equations using technology.