**Victorian Certificate of Education
2010**

STUDENT NUMBER

Letter

Figures

Words

MATHEMATICAL METHODS (CAS)
Written examination 1

Friday 5 November 2010

Reading time: 9.00 am to 9.15 am (15 minutes)
Writing time: 9.15 am to 10.15 am (1 hour)

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	Number of questions to be answered	Number of marks
11	11	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

Materials supplied
- Question and answer book of 11 pages, with a detachable sheet of miscellaneous formulas in the centrefold.
- Working space is provided throughout the book.

Instructions
- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

This page is blank

Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

a. Differentiate $x^3 e^{2x}$ with respect to x.

2 marks

b. For $f(x) = \log_e(x^2 + 1)$, find $f'(2)$.

2 marks

Question 2

a. Find an antiderivative of $\cos(2x + 1)$ with respect to x.

1 mark

b. Find p given that $\displaystyle\int_{2}^{3} \frac{1}{1-x}\, dx = \log_e(p)$.

3 marks

Question 3

Let $f: R^+ \to R$ where $f(x) = \dfrac{1}{x^2}$.

a. Find $g(x) = f(f(x))$.

1 mark

b. Evaluate $g^{-1}(16)$, where g^{-1} is the inverse function of g.

1 mark

Question 4

a.   Write down the amplitude and period of the function

$$f: R \rightarrow R, f(x) = 4 \sin\left(\frac{x + \pi}{3}\right).$$

_____

_____

_____

_____

                                                                                                          2 marks

b.   Solve the equation $\sqrt{3} \sin(x) = \cos(x)$ for $x \in [-\pi, \pi]$.

_____

_____

_____

_____

                                                                                                          2 marks

Question 5

Let X be a normally distributed random variable with mean 5 and variance 9 and let Z be the random variable with the standard normal distribution.

a.   Find $\Pr(X > 5)$.

_____

_____

                                                                                                          1 mark

b.   Find b such that $\Pr(X > 7) = \Pr(Z < b)$.

_____

_____

_____

_____

                                                                                                          2 marks

TURN OVER

Question 6

The transformation $T: R^2 \rightarrow R^2$ is defined by

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -1 \\ 4 \end{bmatrix}.$$

The image of the curve $y = 2x^2 + 1$ under the transformation T has equation $y = ax^2 + bx + c$.

Find the values of a, b and c.

3 marks

Question 7

The continuous random variable X has a distribution with probability density function given by

$$f(x) = \begin{cases} ax(5-x) & \text{if } 0 \le x \le 5 \\ 0 & \text{if } x < 0 \text{ or if } x > 5 \end{cases}$$

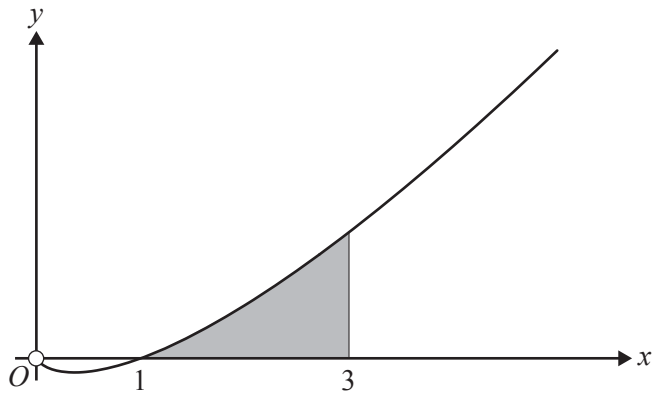where a is a positive constant.

a. Find the value of a.

3 marks

b. Express $\Pr(X < 3)$ as a definite integral. (Do **not** evaluate the definite integral.)

1 mark

Question 8

The discrete random variable X has the probability distribution

x	-1	0	1	2
$\Pr(X = x)$	p^2	p^2	$\dfrac{p}{4}$	$\dfrac{4p+1}{8}$

Find the value of p.

3 marks

Question 9

Part of the graph of $f: R^+ \to R, f(x) = x \log_e(x)$ is shown below.

a. Find the derivative of $x^2 \log_e(x)$.

_____

_____

_____

1 mark

b. Use your answer to **part a.** to find the area of the shaded region in the form $a \log_e(b) + c$ where a, b and c are non-zero real constants.

_____

_____

_____

_____

_____

_____

_____

_____

_____

3 marks

Question 10

The line $y = ax - 1$ is a tangent to the curve $y = x^{\frac{1}{2}} + d$ at the point $(9, c)$ where a, c and d are real constants. Find the values of a, c and d.

_____

_____

_____

_____

_____

_____

_____

_____

4 marks

Question 11

A cylinder fits exactly in a right circular cone so that the base of the cone and one end of the cylinder are in the same plane as shown in the diagram below. The height of the cone is 5 cm and the radius of the cone is 2 cm.

The radius of the cylinder is r cm and the height of the cylinder is h cm.
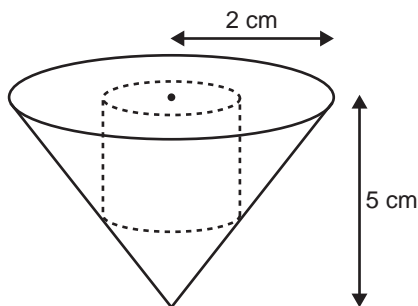
For the cylinder inscribed in the cone as shown above

a.   find h in terms of r

_____

_____

_____

_____

2 marks

The total surface area, S cm^2, of a cylinder of height h cm and radius r cm is given by the formula

$$S = 2\pi rh + 2\pi r^2.$$

b. find S in terms of r

_____

_____

_____

_____

_____

1 mark

c. find the value of r for which S is a maximum.

_____

_____

_____

_____

_____

2 marks

END OF QUESTION AND ANSWER BOOK

MATHEMATICAL METHODS (CAS)

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

This page is blank

Mathematical Methods (CAS)
Formulas

Mensuration

area of a trapezium: $\dfrac{1}{2}(a+b)h$ volume of a pyramid: $\dfrac{1}{3}Ah$

curved surface area of a cylinder: $2\pi rh$ volume of a sphere: $\dfrac{4}{3}\pi r^3$

volume of a cylinder: $\pi r^2 h$ area of a triangle: $\dfrac{1}{2}bc\sin A$

volume of a cone: $\dfrac{1}{3}\pi r^2 h$

Calculus

$$\frac{d}{dx}\left(x^n\right)=nx^{n-1} \qquad \int x^n dx = \frac{1}{n+1}x^{n+1}+c, n\neq -1$$

$$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax} \qquad \int e^{ax} dx = \frac{1}{a}e^{ax}+c$$

$$\frac{d}{dx}\left(\log_e(x)\right)=\frac{1}{x} \qquad \int \frac{1}{x} dx = \log_e|x|+c$$

$$\frac{d}{dx}\left(\sin(ax)\right)=a\,\cos(ax) \qquad \int \sin(ax) dx = -\frac{1}{a}\cos(ax)+c$$

$$\frac{d}{dx}\left(\cos(ax)\right)=-a\,\sin(ax) \qquad \int \cos(ax) dx = \frac{1}{a}\sin(ax)+c$$

$$\frac{d}{dx}\left(\tan(ax)\right)=\frac{a}{\cos^2(ax)}=a\,\sec^2(ax)$$

product rule: $\dfrac{d}{dx}(uv)=u\dfrac{dv}{dx}+v\dfrac{du}{dx}$ quotient rule: $\dfrac{d}{dx}\left(\dfrac{u}{v}\right)=\dfrac{v\dfrac{du}{dx}-u\dfrac{dv}{dx}}{v^2}$

chain rule: $\dfrac{dy}{dx}=\dfrac{dy}{du}\dfrac{du}{dx}$ approximation: $f(x+h)\approx f(x)+hf'(x)$

Probability

$\Pr(A)=1-\Pr(A')$ $\Pr(A\cup B)=\Pr(A)+\Pr(B)-\Pr(A\cap B)$

$\Pr(A|B)=\dfrac{\Pr(A\cap B)}{\Pr(B)}$ transition matrices: $S_n=T^n\times S_0$

mean: $\mu=\text{E}(X)$ variance: $\text{var}(X)=\sigma^2=\text{E}((X-\mu)^2)=\text{E}(X^2)-\mu^2$

	probability distribution	mean	variance
discrete	$\Pr(X=x)=p(x)$	$\mu=\sum x\,p(x)$	$\sigma^2=\sum(x-\mu)^2\,p(x)$
continuous	$\Pr(a<X<b)=\displaystyle\int_a^b f(x)dx$	$\mu=\displaystyle\int_{-\infty}^{\infty}x\,f(x)dx$	$\sigma^2=\displaystyle\int_{-\infty}^{\infty}(x-\mu)^2 f(x)dx$

END OF FORMULA SHEET