2010 Mathematical Methods (CAS) GA 2: Examination 1

GENERAL COMMENTS

The number of students who sat for the 2010 examination was 15 603. Eleven per cent of students scored 90% or more of the available marks, compared with 15% in 2009. There were many very good responses and it was pleasing to see that a number of students obtained full marks.

This was the first year that material specific to this study as well as material in common with the former Mathematical Methods study was included in examination 1, for example Question 6. Students who chose the matrix approach to this question were more successful than those who attempted it by starting with a descriptive approach to transformations.

It was encouraging to see that the majority of students could successfully apply the product rule for differentiation in Questions 1a. and 9a. However, students' application of the chain rule in Question 1b. and anti-differentiation in Questions 2b. and 9b. was poor.

The correct representation of an integral with dx, for example $\int f(x)dx$, continues to be an issue for some students and incorrect notation, such as the omission of the dx, will be penalised.

A number of students provided answers without any working to multiple-mark questions and consequently could not be awarded marks for method. The instruction at the beginning of the paper is clear, 'In questions where more than one mark is available, appropriate working **must** be shown'. Students should ensure that they do not complete their working on the formula sheet, but in the space provided on the examination paper. This was of particular concern in Question 8, where students guessed the correct answer but needed to show their working. They also needed to show that there was no other solution.

Students' skills in geometry; algebra and graphing in relation to quadratic equations; indices; fraction operations; surds; transposition; mensuration and simplification were of concern for assessors.

Care must be taken not to factorise unnecessarily. Many students who correctly applied the product rule in Question 1a. then went on to incorrectly factorise the expression (which was not required). Factorisation should only be considered if the question specifically requires it or if the student is seeking solutions to an algebraic equation. On the other hand, many students who could produce the correct quadratic equation in Question 8 could not factorise the expression, nor use the quadratic formula with which they should be familiar. Some students who used the quadratic formula correctly, did not recognise that $484=22^2$.

It was evident that students had the most difficulty with Question 11. This involved mathematics in the form of using 'similar triangles' to get a relationship between the height and radius of a cylinder within a cone. Many students tried unsuccessfully to make links between the volumes or surface areas. However, students needed to use their response to part a in part b, and their response in part b in part c.

SPECIFIC INFORMATION

Question 1a.

Marks	**0**	**1**	**2**	**Average**
%	12	9	79	**1.7**

$3x^2e^{2x} + 2x^3e^{2x}$

This was a differentiation question involving the use of the product rule. Some students who applied the rule correctly then factorised incorrectly.

Question 1b.

Marks	**0**	**1**	**2**	**Average**
%	35	5	60	**1.3**

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$f'(x) = \frac{1}{x^2+1} \times 2x$

$f'(2) = \frac{4}{5} = 0.8$

Many students had difficulty applying the chain rule. These students often did not multiply by the derivative of $x^2 + 1$.

Question 2a.

Marks	0	1	Average
%	24	76	0.8

$\frac{1}{2}\sin(2x+1) + c,$ where c is a real constant (but not necessary)

This question was well done by the majority of students.

Question 2b.

Marks	0	1	2	3	Average
%	20	43	7	30	1.5

$\int_2^3 \frac{1}{1-x}\,dx = \left[-\log_e(|1-x|)\right]_2^3$

$= -\log_e(2) + \log_e(1)$

$= -\log_e(2) = \log_e(\frac{1}{2}), \quad \therefore p = \frac{1}{2}$

Many students did not include '-' in front of the log.

There were a number of incorrect responses that had $\log_e(|-2|) - \log_e(|-1|) = \log_e\left(\left|\frac{-2}{-1}\right|\right) = \log_e(2)$.

Question 3a.

Marks	0	1	Average
%	36	64	0.7

$\frac{1}{\left(\frac{1}{x^2}\right)^2} = x^4$

Students did not need to specify the domain; however, students should always consider the domain if it is different from the maximal domain. Students were expected to provide an algebraic simplified response.

Question 3b.

Marks	0	1	Average
%	46	54	0.6

2

The solution depended on the domain, but students were penalised only if they also included -2.

Question 4a.

Marks	0	1	2	Average
%	6	30	64	1.6

Amplitude = 4, period = 6π

Most students had the correct response for the amplitude; however, it was disappointing to see the number of students who had a period of 6.

Question 4b.

Marks	0	1	2	Average
%	27	34	39	1.1

$$\frac{\sin(x)}{\cos(x)} = \tan(x) = \frac{1}{\sqrt{3}}$$

$$x = \frac{\pi}{6}, -\frac{5\pi}{6}$$

This question was very poorly done and most students who identified the equation using tan(x) were not able to solve it.

Question 5a.

Marks	0	1	Average
%	28	72	0.7

0.5

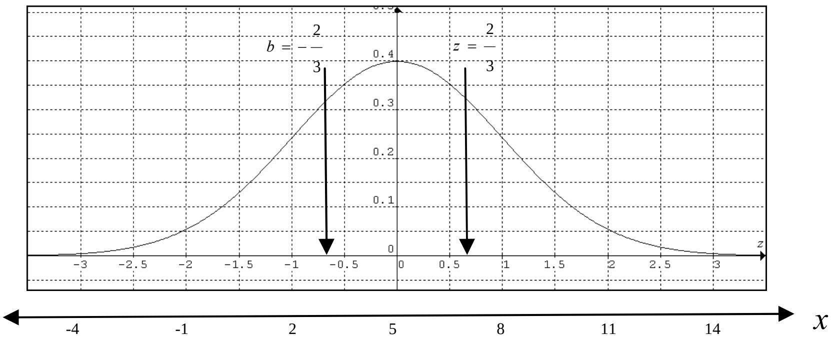
Some students tried to consider discrete distributions.

Question 5b.

Marks	0	1	2	Average
%	46	23	31	0.9

If $X \sim N(5,9)$ then $\sigma^2 = 9$ and $\sigma = 3$,

$$z = \frac{x-\mu}{\sigma} = \frac{7-5}{3} = \frac{2}{3}$$



$$\Pr(X > 7) = \Pr(Z > \frac{2}{3}) = \Pr(Z < -\frac{2}{3}),\ b = -\frac{2}{3}$$

Most students realised that transformation to the standard normal distribution was needed but many substituted the variance instead of the standard deviation. The symmetry properties of the normal distribution eluded many.

Question 6

Marks	0	1	2	3	Average
%	43	16	17	23	1.2

$$T\begin{bmatrix} x \\ y \end{bmatrix} or \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 3x-1 \\ 2y+4 \end{bmatrix} \quad \text{so } x' = 3x-1 \Rightarrow x = \frac{x'+1}{3} \text{ and } y' = 2y+4 \Rightarrow y = \frac{y'-4}{2}$$

$$y = 2x^2 + 1 \text{ becomes } \frac{y'-4}{2} = 2\left(\frac{x'+1}{3}\right)^2 + 1$$

$$y' = \frac{4}{9}(x')^2 + \frac{8}{9}x' + 6\frac{4}{9} \quad \therefore\ a = \frac{4}{9}\ b = \frac{8}{9}\ c = \frac{58}{9}$$

Three different approaches were taken to this question. Students who attempted to define the transformations (dilation of factor 3 from the *y*-axis; dilation of factor 2 from the *x*-axis; translation 1 unit left and translation 4 units up) often misinterpreted both the dilations and the horizontal translation, and then did not rearrange to construct the new rule.

Students who chose to determine the inverse matrix often made errors in their inverse and/or the multiplication of the matrices.

Question 7a.

Marks	0	1	2	3	Average
%	23	14	13	49	1.9

$$a\int_0^5 (5x - x^2)\,dx = 1$$

$$a[\frac{5x^2}{2} - \frac{x^3}{3}]_0^5 = a[\frac{125}{2} - \frac{125}{3}] = a\frac{125}{6} = 1$$

$$\therefore a = \frac{6}{125}$$

Some students differentiated instead of anti-differentiated and others had difficulty subtracting the two fractions with different denominators. Some students still did not include the *dx* as part of the integral.

Question 7b.

Marks	0	1	Average
%	37	63	0.7

$\Pr(X < 3) = \int_{-\infty}^3 f(x)dx$ or $\int_0^3 f(x)dx$

or $\int_0^3 ax(5-x)dx$ or $\int_0^3 \frac{6}{125}x(5-x)dx$ were acceptable, but not $\int_{-\infty}^3 ax(5-x)dx$, since this part of the rule of *f* does not apply for $x < 0$.

This question was generally well done, but a surprising number of students used 2 as their upper terminal value, treating the distribution to be of discrete integers. A number of students continued to work out the numeric value incorrectly and thus lost the mark they would have been awarded for setting up a correct definite integral.

Question 8

Marks	0	1	2	3	Average
%	17	28	24	32	1.7

$$p^2 + p^2 + \frac{p}{4} + \frac{4p+1}{8} = 1$$

$$\Rightarrow 2p^2 + \frac{6p+1}{8} = 1$$

$$16p^2 + 6p + 1 = 8$$

$$16p^2 + 6p - 7 = 0$$

$$\Rightarrow (8p+7)(2p - 1)=0$$

or using the quadratic formula

$$p = \frac{-6 \pm \sqrt{36 - 4\times16\times-7}}{32}$$

$$\Rightarrow p = \frac{-6 \pm \sqrt{484}}{32} = \frac{-6 \pm 22}{32} = \frac{-28}{32} \text{ or } \frac{16}{32}$$

$$p > 0 \ \therefore p = \frac{1}{2} \ or \ 0.5$$

Many students had difficulty with this question. Although most students understood that the probabilities should add to 1, a number of students incorrectly worked with E(*X*) =1 instead.

Some students struggled to rearrange to make the quadratic equal zero in order to find solutions. Use of the quadratic formula, with which students should be familiar, was poor.

Where the quadratic formula was used correctly, some students could not identify p = 0.5 as the solution.

Question 9a.

Marks	0	1	Average
%	23	77	0.8

$$2x\log_e(x)+x^2\times\frac{1}{x}=2x\log_e(x)+x$$

This question was completed correctly by the majority of students.

Question 9b.

Marks	0	1	2	3	Average
%	53	17	9	20	1

$$\int_1^3 (2x\log_e(x)+x)dx=\left[x^2\log_e(x)\right]_1^3$$

$$2\int_1^3 x\log_e(x)dx+\left[\frac{x^2}{2}\right]_1^3=\left[x^2\log_e(x)\right]_1^3$$

$$\int_1^3 x\log_e(x)dx=\frac{1}{2}\left(\left[x^2\log_e(x)\right]_1^3-\left[\frac{x^2}{2}\right]_1^3\right)$$

$$=\frac{9}{2}\log_e(3)-2$$

Few students who got the correct answer for Question 9a. could complete this question. The most common error was not dividing everything through by 2.

Question 10

Marks	0	1	2	3	4	Average
%	28	17	12	8	35	2.1

Gradient of curve $\frac{dy}{dx}=\frac{1}{2}x^{-\frac{1}{2}}$

At $x = 9$ $\frac{dy}{dx}=\frac{1}{2}9^{-\frac{1}{2}}=\frac{1}{2}\times\frac{1}{3}=\frac{1}{6}=a$

Also at $x = 9$ $y = c$ so tangent $c = 9a - 1$ and curve $c = 3 + d$

$\Rightarrow c=9a-1=9\times\frac{1}{6}-1=1\frac{1}{2}-1=\frac{1}{2}$ and $d=c-3=-2\frac{1}{2}=\frac{-5}{2}$

$a=\frac{1}{6},c=\frac{1}{2},d=\frac{-5}{2}$

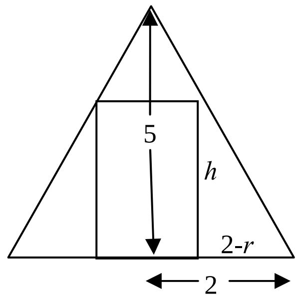
Most students understood that they needed to find the gradient of the curve and tangent at the same point. However, some thought that they were dealing with the 'normal' rather than the tangent, and many others thought that the gradient of the tangent was 'ax-1' rather than 'a'. A number of students equated the y-value to the gradient at x = 9. Another common error was to substitute x = 1 instead of x = 9.

Question 11a.

Marks	0	1	2	Average
%	68	21	11	0.5

By similar triangles: $\frac{h}{5} = \frac{2-r}{2}$

so $h = \frac{10-5r}{2}$ or $h = 5 - \frac{5r}{2}$



Students found this question difficult. Few students realised that similar figures were needed. A common incorrect formulation was $\frac{h}{5} = \frac{r}{2} \Rightarrow h = \frac{5r}{2}$.

Question 11b.

Marks	0	1	Average
%	53	47	0.5

$S = 10\pi r - 3\pi r^2$

Students needed to use a reasonable answer for $h(r)$ in Question11a. and make a correct substitution to be awarded the mark for this question.

Question 11c.

Marks	0	1	2	Average
%	64	26	10	0.5

$\frac{dS}{dr} = 10\pi - 6\pi r = 0$

$r = \frac{5}{3} = 1\frac{2}{3}$

Students needed to use their response to 11b. to determine the value of r that would give the maximum surface area.