Neap:

Trial Examination 2010

VCE Mathematical Methods (CAS) Units 3 & 4

Written Examination 2

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 2 hours

Student's Name: ______________________________

Teacher's Name: ______________________________

Structure of Booklet

Section	Number of questions	Number of questions to be answered	Number of marks
1	22	22	22
2	4	4	58
			Total 80

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator (memory DOES NOT need to be cleared) and, if desired, one scientific calculator. For approved computer based CAS, their full functionality may be used.

Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

Question and answer booklet of 20 pages with a detachable sheet of miscellaneous formulas in the centrefold.
Answer sheet for multiple-choice questions.

Instructions

Detach the formula sheet from the centre of this booklet during reading time.

Write **your name** and **teacher's name** in the space provided above on this page.

All written responses must be in English.

At the end of the examination

Place the answer sheet for multiple-choice questions inside the front cover of this booklet.

Students are NOT permitted to bring mobile phones and/or any other electronic communication devices into the examination room.

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2010 VCE Mathematical Methods (CAS) Units 3 & 4 Written Examination 2.

SECTION 1

Instructions for Section 1

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.

Question 1

The simultaneous linear equations,

$2x + py = 3q$

$3x + qy = 2p$

where p and q are real constants, have a unique solution provided, as follows:

A. $p = \frac{2q}{3}$

B. $p \neq \frac{2q}{3}$

C. $p = \frac{3q}{2}$

D. $p \neq \frac{3q}{2}$

E. $p = -\frac{3q}{2}$

Question 2

Which of the following defines a function f for which $f(-x) = -f(x)$?

A. $f(x) = x^2$

B. $f(x) = \frac{2}{x}$

C. $f(x) = \cos(x)$

D. $f(x) = \log_e(x^2)$

E. $f(x) = e^x$

Question 3

The maximal domain D of the function $f: D \to R$ with rule $f(x) = \log_e\left|1 - \frac{2}{x}\right|$ is

A. $(-\infty, 2)$

B. $R\backslash\{2\}$

C. $R\backslash\{0\}$

D. $R\backslash\{0, 2\}$

E. $R\backslash\{\pm 2\}$

Question 4

The general solution to the equation $\sqrt{3}\tan\left(\frac{x}{2}\right) + 1 = 0$ is

A. $2n\pi + \frac{\pi}{3},\ n \in Z$

B. $2n\pi - \frac{\pi}{3},\ n \in Z$

C. $2n\pi \pm \frac{\pi}{3},\ n \in Z$

D. $2n\pi + \frac{\pi}{6},\ n \in Z$

E. $2n\pi - \frac{\pi}{6},\ n \in Z$

Question 5

Three functions have rules as follows:

I. $y = 4x$

II. $y = \frac{x}{2}$

III. $y = \frac{x^2}{2}$

Let f be a function that satisfies the equation $f(2x + 2y) = 2f(x) + 2f(y)$ where x and y are any real numbers.

Then from the rules above, the following are possible rules for this function:

A. I., II. and III.

B. I. and III. only

C. II. and III. only

D. I. and II. only

E. I. only

Question 6

The continuous random variable X has a normal distribution with mean 17 and variance 4.

If the random variable Z has the standard normal distribution, then the probability that X is greater than 14 is equal to

A. $\Pr(Z < 1.5)$

B. $\Pr(Z < -1.5)$

C. $\Pr(Z < 0.75)$

D. $\Pr(Z < -0.75)$

E. $\Pr(Z > -0.75)$

Question 7

Let f and g be differentiable functions on R with the following properties:

$g(x) > 0$ for all real values of x and $f(0) = 1$.

Given that $h(x) = f(x) \times g(x)$ and $h'(x) = f(x) \times g'(x)$, then $f(x) =$

A. $f'(x)$

B. $g(x)$

C. e^x

D. 0

E. 1

Question 8

If $\frac{dy}{dx} = -\frac{1}{x}$, then the average rate of change of y with respect to x on the closed interval [1, 4] is

A. $\frac{1}{4}$

B. $-\frac{1}{2}\log_e 2$

C. $-\frac{2}{3}\log_e 2$

D. $-\frac{1}{3}\log_e \frac{1}{4}$

E. 2

Question 9

Which of the following gives the best approximation to the x-coordinate of the point at which the tangent to the graph of $f(x) = 2e^{4x^2}$ is parallel to the line $y - 3x = 1$?

A. 0.014

B. 0.168

C. 0.276

D. 0.318

E. 0.342

Question 10

X is a discrete random variable representing the number of passengers on a ferry crossing from one side of a river to the other.

Given that $E(X) = 15$ and $E(X^2) = 250$, which one of the following intervals contains an approximate 95% distribution in which the number of passengers on a particular trip would lie?

A. [0, 30]
B. [5, 25]
C. [10, 20]
D. [0, 65]
E. [15, 30]

Question 11

The life of a certain battery, in hours, can be modelled by the random variable X with the probability density function:

$$f(x) = \begin{cases} \dfrac{k}{x^3} & \text{if } x > 50 \\ 0 & x \le 50 \end{cases}$$

According to this model, the median life of a battery, in hours, is closest to

A. 50
B. 70
C. 71
D. 75
E. 100

Question 12

The graph of f where $f(x) = x^{-\frac{3}{2}}$, undergoes a dilation of 4 parallel to the x-axis.

The same result could be achieved had f undergone a dilation of factor

A. $\frac{1}{8}$ from the y-axis.
B. 8 from the y-axis.
C. $\frac{1}{8}$ from the x-axis.
D. 8 from the x-axis.
E. $\frac{1}{16}$ from the x-axis.

Question 13

A manufacturer of football jumpers for schools makes them in two different sizes: large and medium. They are delivered in a box containing 20 jumpers of which 40% of the jumpers are large and 60% are medium.

Assuming that each box contains a random selection of jumpers, the probability that there are more than 17 medium-sized jumpers is given by

A. ${}^{20}C_{18}(0.6)^{18}(0.4)^{2} + {}^{20}C_{19}(0.6)^{19}(0.4) + 0.6^{20}$

B. ${}^{20}C_{17}(0.6)^{17}(0.4)^{3} + {}^{20}C_{18}(0.6)^{18}(0.4)^{2} + {}^{20}C_{19}(0.6)^{19}(0.4) + 0.6^{20}$

C. ${}^{20}C_{18}(0.4)^{18}(0.6)^{2} + {}^{20}C_{19}(0.4)^{19}(0.6) + 0.4^{20}$

D. ${}^{20}C_{17}(0.4)^{17}(0.6)^{3} + {}^{20}C_{18}(0.4)^{18}(0.6)^{2} + {}^{20}C_{19}(0.4)^{19}(0.6) + 0.4^{20}$

E. $1 - {}^{20}C_{18}(0.6)^{18}(0.4)^{2} - {}^{20}C_{19}(0.6)^{19}(0.4) - 0.6^{20}$

Question 14

For $f(x) = |x^2 - 4|$, the derivative $f'(x)$ is defined by

A. $f'(x) = 2x,\ x \in R\backslash\{-2, 2\}$

B. $f'(x) = |2x|,\ x \in R\backslash\{-2, 2\}$

C. $f'(x) = \begin{cases} -2x & x \in R\backslash[-2, 2] \\ 2x & x \in (-2, 2) \end{cases}$

D. $f'(x) = \begin{cases} 2x & x \in R\backslash[-2, 2] \\ -2x & x \in (-2, 2) \end{cases}$

E. $f'(x) = \begin{cases} 2x & x \in R\backslash(-2, 2) \\ -2x & x \in (-2, 2) \end{cases}$

Question 15

For $y = \log_e\sqrt{\dfrac{1}{f(x)}}$, $\dfrac{dy}{dx}$ is equal to

A. $-\dfrac{f'(x)}{2f(x)}$

B. $\dfrac{f'(x)}{2\sqrt{f(x)}}$

C. $-\dfrac{f'(x)}{2(f(x))^{\frac{3}{2}}}$

D. $-\dfrac{2f'(x)}{f(x)}$

E. $\log_e\dfrac{f'(x)}{(f(x))^2}$

Question 16

The inverse of the function $f: (-\infty, -2] \rightarrow R$, $f(x) = x^2 + 4x$ is

A. $f^{-1}: (-\infty, -2] \rightarrow R$, $f^{-1}(x) = -2 + \sqrt{x+4}$

B. $f^{-1}: [-2, \infty) \rightarrow R$, $f^{-1}(x) = -2 + \sqrt{x+4}$

C. $f^{-1}: [-4, \infty) \rightarrow R$, $f^{-1}(x) = -2 + \sqrt{x+4}$

D. $f^{-1}: [-4, \infty) \rightarrow R$, $f^{-1}(x) = -2 - \sqrt{x+4}$

E. $f^{-1}: (-\infty, -4] \rightarrow R$, $f^{-1}(x) = -2 - \sqrt{x+4}$

Question 17

A particular computer gambling game generates a set of random numbers whose value can be modelled by the discrete random variable X, and whose probability function is given by:

$\Pr(X = x) = kx!$ $x = 0, 1, 2, 3, 4,$ where k is a constant.

Two independent values of X are generated for the game.

What is the probability those two numbers are the same?

A. $\frac{1}{10}$

B. $\frac{539}{1156}$

C. $\frac{3}{10}$

D. $\frac{617}{1156}$

E. $\frac{309}{578}$

Question 18

Which of the following gives the average value for the function $f(x) = \cos(x)$ on the interval [–2, 4]?

A. $\frac{\sin 4 - \sin 2}{6}$

B. $\frac{\sin 4 - \sin 2}{2}$

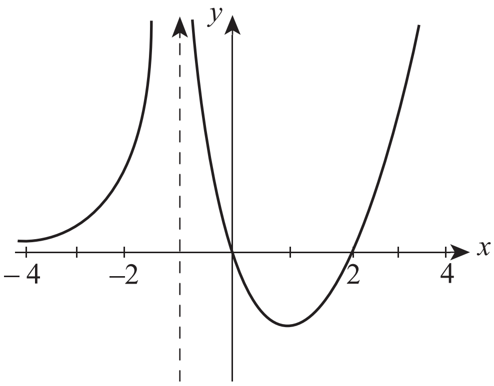
C. $\frac{\sin 2 - \sin 4}{2}$

D. $\frac{\sin 2 + \sin 4}{2}$

E. $\frac{\sin 2 + \sin 4}{6}$

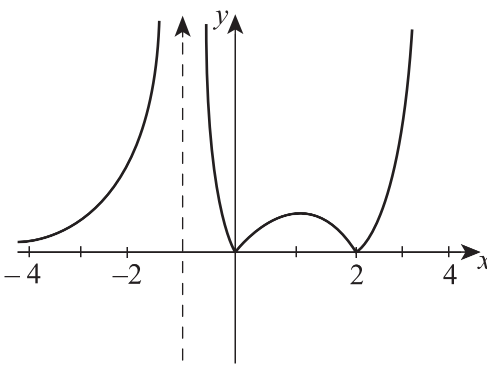
Question 19

The graph of a function f with domain $R\backslash\{-1\}$ is shown.

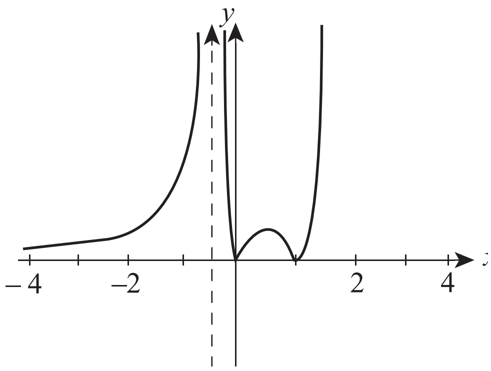


The graph of $y = \left| f\left(\frac{x}{2}\right) \right|$ is best represented by:

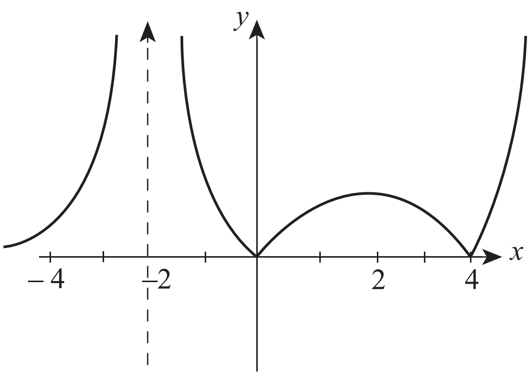
A.



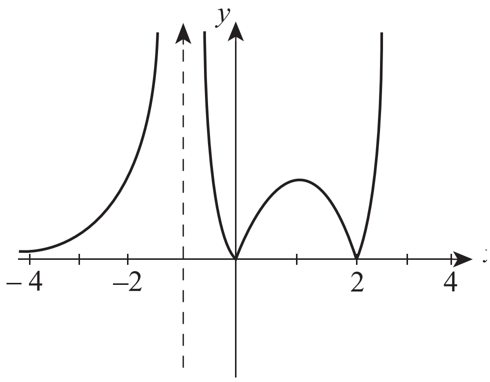
B.



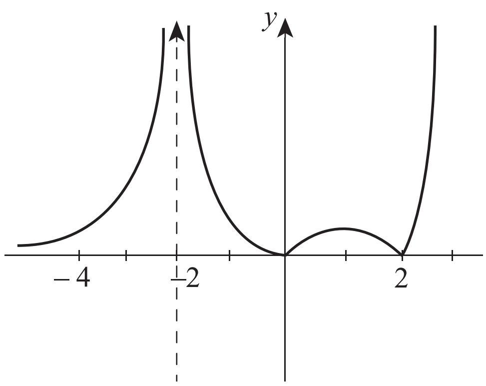
C.



D.



E.



Question 20

The number of real solutions to the equation $e^{2x}(x^2+1)\sqrt{(x^2-1)}\log_e(x^2)=0$ is

A. 0
B. 1
C. 2
D. 3
E. 4

Question 21

For the function $g : R \to R,\ g(x) = (x-4)^2(1-x)$, the subset of R for which the gradient of g is positive is

A. (1, 4)
B. $R\backslash[1, 4]$
C. (2, 4)
D. $R\backslash[2, 4]$
E. $R\backslash[-2, 4]$

Question 22

If $f'(x)$ and $g'(x)$ are defined for all values of x and are related in such a way that $f'(x) > g'(x)$ for all real values of x, then the graphs of $y = f(x)$ and $y = g(x)$

A. intersect exactly once.
B. do not intersect.
C. intersect no more than once.
D. could intersect more than once.
E. have a common tangent at their points of intersection.

END OF SECTION 1

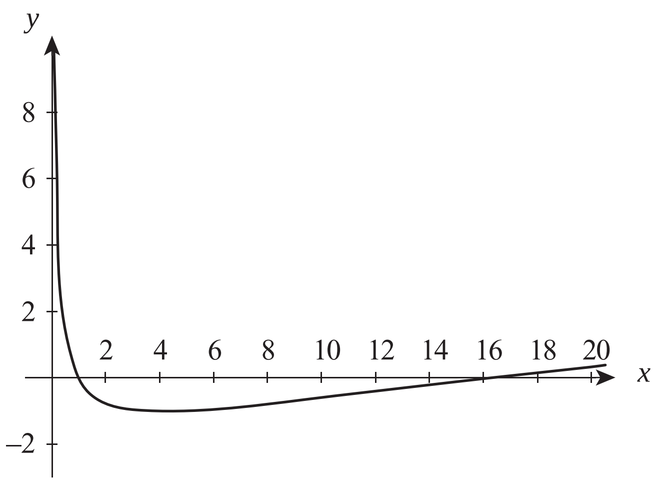
SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Question 1

Let $f : R^+ \to R,\ f(x) = \dfrac{4}{\sqrt{x}} + \sqrt{x} - 5$.



a. State the interval for which the graph of f is strictly increasing.

2 marks

b. Determine the absolute minimum value of f.

1 mark

c. Find the gradient of the tangent to the graph of f at the point where $x = 16$, and hence find the equation of the tangent at this point.

2 marks

d. Let $g : R^+ \to R$, $g(x) = |f(x)|$.

On the set of axes on page 10, sketch the graph of $y = g(x)$, clearly indicating the coordinates of any local maxima and local minima.

3 marks

e. Determine the gradient function $g'(x)$ for the function g.

2 marks

f. Write an integral expression, in terms of $f(x)$, that would find the area enclosed by the graph of $y = g(x)$ and the x-axis, and calculate this area.

2 marks

g. The point $(a, 0)$, where $0 < a < 16$, is such that the area enclosed by the graph of $y = g(x)$, the x-axis and the line $x = a$ is half the area found in **Question 1 f**.

Write an equation using a suitable integral that could be used to find a and also find the value of a.

2 marks

h. Let $h : R \to R,\ h(x) = x^2$.

Find the rule of $g(h(x))$ and find the value(s) of x for which it has an absolute minimum.

3 marks
Total 17 marks

Question 2

The flow of traffic at a particular intersection is being investigated to determine whether traffic lights should be installed at the intersection. Data has been collected over a 35 minute period and the number of cars per minute passing through the intersection follows the mathematical model:

$$N(t) = 68 + k\sin(at) \quad 0 \le t \le 35$$

where $N(t)$ measures the **rate** at which cars flow through the intersection in cars per minute, t is in minutes, and k and a are positive constants.

a. **i.** Given that the minimum flow of traffic is observed as 62 cars per minute, state the value of k.

ii. If this minimum flow first occurs at the 22 minute mark, show that the value of a is equal to $\frac{3\pi}{44}$.

1 + 2 = 3 marks

b. Over what interval is the traffic flow increasing during the observed period?

2 marks

c. To the nearest whole number, how many cars pass through the intersection over the 35 minute period?

2 marks

d. Calculate the average value of the traffic flow over the time interval $5 \le t \le 15$, giving your answer correct to 1 decimal place.

2 marks

e. **i.** State what the expression $\frac{N(15)-N(5)}{10}$ represents in regard to the traffic flow.

____

ii. When the expression in **Question 2 e i.** is evaluated by hand, the result is:

$$\frac{3}{5}\left(\sin\left(\frac{45\pi}{44}\right)-\sin\left(\frac{15\pi}{44}\right)\right)$$

When the expression in **Question 2 e i.** is evaluated by a CAS, the result is:

$$-\frac{3}{5}\left(\sin\left(\frac{\pi}{44}\right)+\cos\left(\frac{7\pi}{44}\right)\right)$$

Explain, using algebra, why these two expressions are equivalent.

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____

____

iii. Evaluate your expression in **Question 2 e ii.** correct to 1 decimal place and give its correct unit.

____

1 + 2 + 1 = 4 marks

Total 13 marks

Question 3

Tempranillo is a style of wine very popular in Spain. A particular winery produces bottles which are exported to Australia in two sizes: standard and large.

For each size, the content of which is in litres, a randomly chosen bottle is normally distributed with mean and standard deviation either given or to be determined. See the table below.

Bottle Size	**Mean**	**Standard deviation**
standard	0.760	0.008
large	μ	σ

a. Calculate the probability that a randomly chosen standard bottle contains less than 0.750 litres, giving your answer correct to 4 decimal places.

1 mark

b. Of the large bottles, 8% contain more than 1.023 litres and 4% contain less than 0.994 litres.

i. Show that the mean and standard deviation of the large bottles can be found by solving the pair of equations: $\mu + 1.4051\sigma = 1.023$ and $\mu - 1.7507\sigma = 0.994$.

ii. Hence evaluate the mean and standard deviation for the large bottles, expressing your answer correct to 3 decimal places in each case.

3 + 1 = 4 marks

c. Find the probability that a box of 12 randomly chosen standard bottles contains at least 3 bottles whose contents are less than 0.750 litres. Answer correct to 3 decimal places.

2 marks

d. A wine store stocks both the standard and large bottles. The store has many regular customers who make consistent weekly orders of bottles of the Tempranillo. Data shows that 75% of customers who order the standard bottle will stay with it for their subsequent order, while 25% will change to order large bottles. However, 65% of customers who order large bottles will change their preference to order the standard bottles next time with only 35% of customers reordering the large bottles.

i. Write down a transition matrix that can be used to represent this information.

ii. At the end of a particular week, 60% of sales were for the large bottles and 40% for the standard bottles.

Find the percentage of each type of bottle sold after a further three weeks.

iii. Show that in the long term, exactly $\frac{5}{18}$ of the orders are placed for the large bottles.

1 + 3 + 2 = 6 marks

e. The standard and large bottles of Tempranillo are used to pour wine by the glass in a restaurant. Each glass is poured to 125 mL and if the last glass from the bottle is short of 125 mL, the wine is disposed as the restaurant will not mix from different bottles.

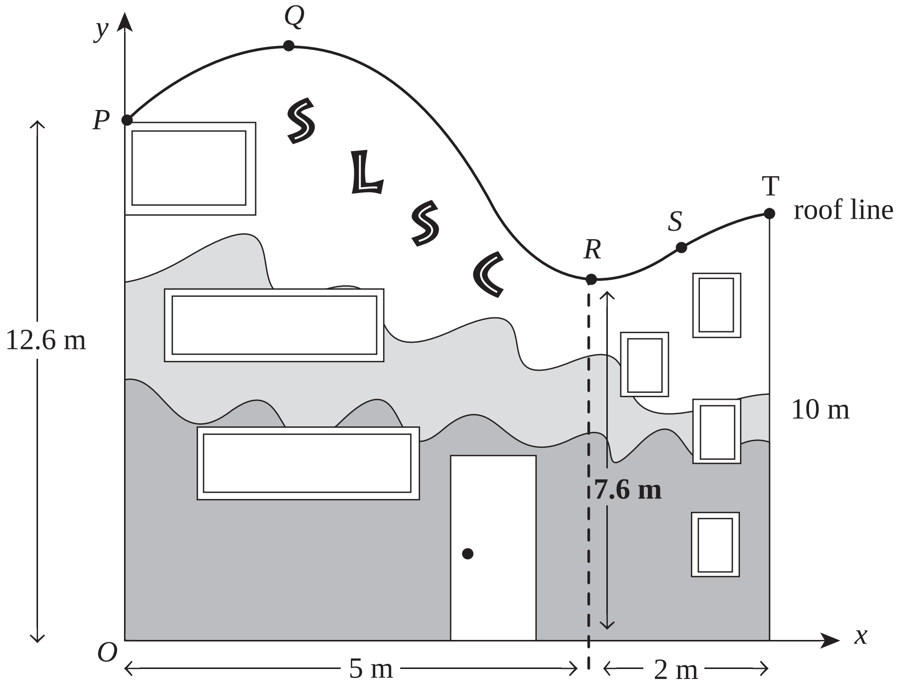
By considering 100 bottles of each size, determine whether a greater proportion of wastage is likely to occur from a standard or large bottle.

2 marks

Total 15 marks

Question 4

An architect is designing a new clubhouse for a local Surf Life Saving Club (SLSC). The roof is to be designed to look like a smooth wave. A diagram of the side profile of the clubhouse is shown below with some key measurements and points noted. Relative to the point *O*, the point *P* has coordinates (0, 12.6), the point *Q* is a local maximum, the point *R* is a local minimum and the roof is to be horizontal at the point *T*, whose coordinates are (7, 10).



The architect's first attempt to design a smooth roof is to use a hybrid function *f*, where *f(x)* meters measures the height of the roof above the ground and *x* meters measures the horizontal distance from *O*.

$$f(x) = \begin{cases} 0.2x^3 - 1.8x^2 + 3x + 12.6, & x \in [0, 6] \\ ax^2 + bx + c, & x \in (6, 7] \end{cases}$$

a. Verify that the function *f* meets the requirements of the roof line at the point *R* as shown on the diagram above.

3 marks

b. Determine the *y*-coordinate of the point *S*, given that the *x*-coordinate at *S* is 6.

1 mark

c. The function f is continuous at S.

Show that the rule for the section ST of the roof is given by $y = -x^2 + 14x - 39$.

3 marks

d. Show that the derivative function for the model is not defined at $x = 6$, i.e. that the function is not smooth at the point S.

3 marks

e. The architect decides to modify the hybrid function by changing the parabolic part of the function to a cubic with a stationary point of inflection at the point *T*.

Determine the equation of this cubic and whether or not this makes the model smooth at the point *S*.

3 marks

Total 13 marks

END OF QUESTION AND ANSWER BOOKLET