Mathematical Association of Victoria
Trial Exam 2010

MATHEMATICAL METHODS (CAS)

STUDENT NAME ____

Written Examination 1

Reading time: 15 minutes
Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
9	**9**	**40**

Note

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

Materials supplied

- Question and answer book of 10 pages, with a detachable sheet of miscellaneous formulas at the back
- Working space is provided throughout the book.

Instructions

- Detach the formula sheet from the back of this book during reading time.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Question 1

a. If $\Pr(A) = 0.3$ and $\Pr(B) = 0.6$ and events A and B are independent, find $\Pr(A' \mid B)$.

2 marks

b. If there are two black socks and four white socks in a drawer and two socks are drawn without replacement, what is the probability that they are both the same colour?

2 marks

Question 2

a. For $f(x) = (\sin(2x) + 1)^2$, evaluate $f'\left(\frac{\pi}{2}\right)$.

3 marks

b. Let $y=(x^2-2x)e^x$.

i. Find $\dfrac{dy}{dx}$.

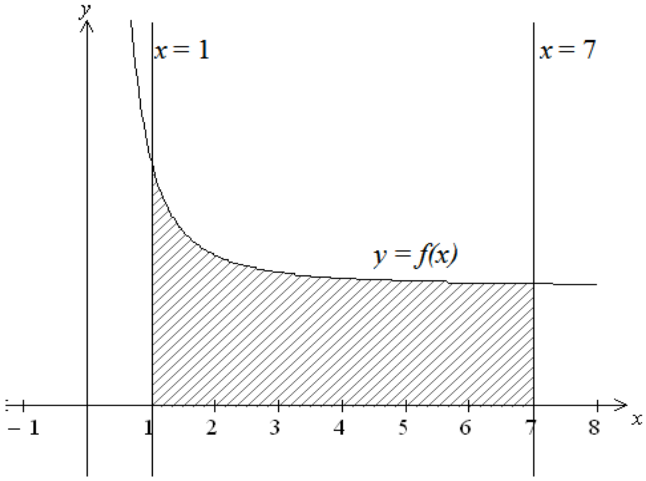
ii. **Hence** find an antiderivative of x^2e^x.

2 + 2 = 4 marks

Question 3

Let $f:(0,\infty)\rightarrow R,\ f(x)=1+\frac{1}{x^2}$

a. Consider the shaded region, bounded by the graph of *f*, the *x*-axis and the lines with equations $x=1$ and $x=7$.



i. Show that the area of the shaded region is $\frac{48}{7}$.

ii. Hence find the average value of *f* over the interval [1, 7].

2 + 1 = 3 marks

b. Find the **domain** and **rule** of the inverse function, f^{-1}.

3 marks

Question 4

A cylindrical fuel storage tank has a **radius** of 4 metres. Fuel enters the tank at a constant rate of 32 m^3/hour. At what rate is the height of fuel rising in the tank, in m/hour?

3 marks

Question 5

A binomial random variable, X has a mean of 3 and a **variance** of $\frac{3}{4}$. Find $\Pr(X = 2)$.

3 marks

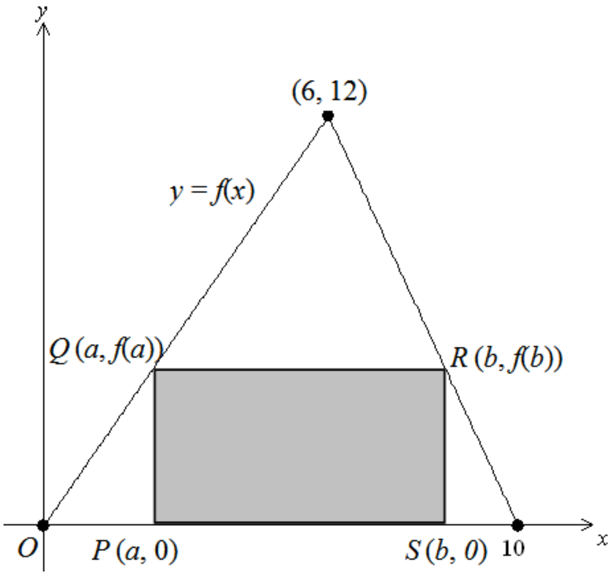
Working Space

Question 6

The graph of f is shown, where

$$f(x)=\begin{cases} 2x & \text{if } 0 \le x \le 6 \\ 30-3x & \text{if } 6 < x \le 10 \end{cases}$$

Consider the inscribed rectangle shown, with vertices $P(a,0)$, $Q(a,f(a))$, $R(b,f(b))$ and $S(b,0)$, where $0 \le a \le 6$ and $6 < b \le 10$.



a. Find an expression for b in terms of a.

1 mark

b. Show that the area of the rectangle, A, is given by

$A = 20a - \dfrac{10a^2}{3}$.

2 marks

c. Find the value of a for which the area of the rectangle is a maximum. Hence find the maximum area of the rectangle.

2 marks

Question 7

The transformation $T: R^2 \rightarrow R^2$ with rule

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -\pi \\ 3 \end{bmatrix}$$

maps the curve with equation $y = \cos(x)$ onto the curve with equation $y = h(x)$. Find an expression for $h(x)$.

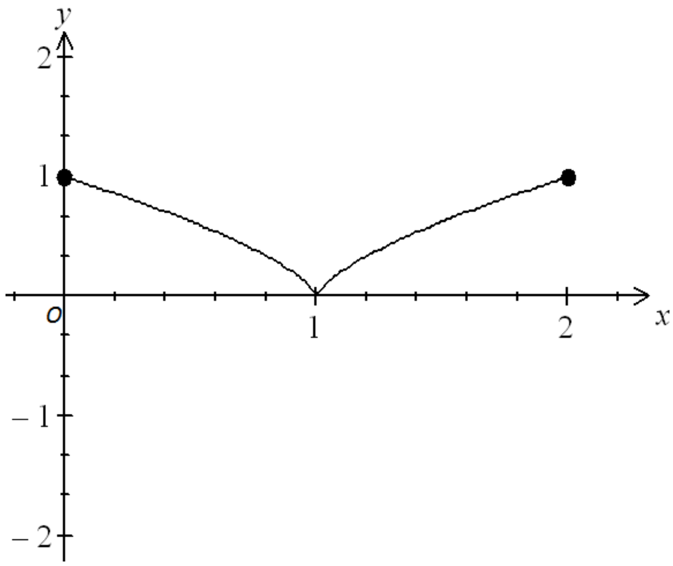
3 marks

Question 8

Let $f:[0,2]\to R$, where $f(x)=(x-1)^{\frac{2}{3}}$

a. Find f', the derivative of f

b. The graph of f is shown on the set of axes below. Sketch the graph of f' on the same set of axes, labelling any asymptote with its equation and labelling any endpoints with their coordinates.



3 marks

Question 9

A Markov chain has a transition matrix $T=\begin{bmatrix}0.2 & 0.9\\0.8 & 0.1\end{bmatrix}$. In the long term, the steady state matrix will be $\begin{bmatrix}x\\1-x\end{bmatrix}$ where $0\le x\le 1$. Find the value of x.

2 marks

Question 10

Let $g : R^+ \to R,\ g(x) = x - \log_e(x)$.

The equation of a **tangent** to the graph of g is $y = -\dfrac{x}{2} + k$.

Find the value of k.

4 marks

END OF QUESTION AND ANSWER BOOK

Mathematical Methods (CAS) Formula Sheet

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$	volume of a pyramid:	$\frac{1}{3}Ah$
curved surface area of a cylinder:	$2\pi rh$	volume of a sphere:	$\frac{4}{3}\pi r^3$
volume of a cylinder:	$\pi r^2 h$	area of a triangle:	$\frac{1}{2}bc\sin A$
volume of a cone:	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$

$\int x^n dx=\frac{1}{n+1}x^{n+1}+c, n\neq -1$

$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$

$\int e^{ax}dx=\frac{1}{a}e^{ax}+c$

$\frac{d}{dx}(\log_e(x))=\frac{1}{x}$

$\int \frac{1}{x}dx=\log_e|x|+c$

$\frac{d}{dx}(\sin(ax))=a\ \cos(ax)$

$\int \sin(ax)dx=-\frac{1}{a}\cos(ax)+c$

$\frac{d}{dx}(\cos(ax))=-a\ \sin(ax)$

$\int \cos(ax)dx=\frac{1}{a}\sin(ax)+c$

$\frac{d}{dx}(\tan(ax))=\frac{a}{\cos^2(ax)}=a\ \sec^2(ax)$

product rule: $\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$

approximation: $f(x+h)\approx f(x)+hf'(x)$

Probability

$\Pr(A)=1-\Pr(A')$

$\Pr(A\cup B)=\Pr(A)+\Pr(B)-\Pr(A\cap B)$

$\Pr(A|B)=\frac{\Pr(A\cap B)}{\Pr(B)}$

transition matrices: $S_n=T^n\times S_0$

mean: $\mu=\mathrm{E}(X)$

variance: $\mathrm{var}(X)=\sigma^2=\mathrm{E}((X-\mu)^2)=\mathrm{E}(X^2)-\mu^2$

probability distribution		mean	variance
discrete	$\Pr(X=x)=p(x)$	$\mu=\sum x\,p(x)$	$\sigma^2=\sum (x-\mu)^2\,p(x)$
continuous	$\Pr(a<X<b)=\int_a^b f(x)dx$	$\mu=\int_{-\infty}^{\infty} x\,f(x)dx$	$\sigma^2=\int_{-\infty}^{\infty}(x-\mu)^2 f(x)dx$

END OF FORMULA SHEET