Year 2010

VCE

Mathematical Methods CAS

Trial Examination 2

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Victorian Certificate of Education 2010

STUDENT NUMBER

Figures									Letter
Words									

MATHEMATICAL METHODS CAS

Trial Written Examination 2

Reading time: 15 minutes
Total writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	5	5	58
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved **graphics** calculator (memory DOES NOT need to be cleared) and, if desired, one scientific calculator.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer booklet of 32 pages with a detachable sheet of miscellaneous formulas at the end of this booklet.
- Answer sheet for multiple choice questions.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, and sign your name in the space provided to verify this.
- All written responses must be in English.

At the end of the examination

- Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION 1

Instructions for Section I

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
A correct answer scores 1 mark, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No mark will be given if more than one answer is completed for any question.

Question 1

If $f(x)=1+3g(x)$ for $0 \le x \le 4$ then $\int_0^4 \left(f(x)-g(x)\right)dx$ is equal to

A. $3\int_0^4 g(x)\,dx$

B. $x+2\int_0^4 g(x)\,dx$

C. $4-2\int_0^4 g(x)\,dx$

D. $4+2\int_0^4 g(x)\,dx$

E. $4+3\int_0^4 g(x)\,dx$

Question 2

Consider the graph of the function $f(x)=\left|\log_e|x+a|\right|$, where *a* is a real constant, which of the following is **false?**

A. The maximal domain is $R\backslash\{-a\}$

B. The graph crosses the *y*-axis at $y=\left|\log_e|a|\right|$

C. The graph crosses the *x*-axis at $x=\pm1-a$

D. Since $f(x)\ge 0$, the range is $R^+ \cup \{0\}$

E. The graph is continuous.

Question 3

Using a linear approximation, $\cos\left(29^0\right)$ is approximately equal to

A. 0.875

B. $\frac{1}{2}-\frac{\pi\sqrt{3}}{360}$

C. $\frac{1}{2}+\frac{\pi\sqrt{3}}{360}$

D. $\frac{\sqrt{3}}{2}-\frac{\pi}{360}$

E. $\frac{\sqrt{3}}{2}+\frac{\pi}{360}$

Question 4

If $f(x)=\frac{\sqrt{x^2+16}}{g(x)}$ and $g(3)=3$ and $g'(3)=-1$ then $f'(3)$ is equal to

A. $\frac{26}{5}$

B. $\frac{34}{45}$

C. $-\frac{16}{45}$

D. $-\frac{3}{5}$

E. $-\frac{24}{5}$

Question 5

A certain curve has its gradient given by $\dfrac{1}{2x-9}$. If the curve crosses the *x*-axis at $x=\dfrac{5}{2}$ then it

A. crosses the *y*-axis at $\log_e\left(\dfrac{9}{4}\right)$

B. crosses the *y*-axis at $\log_e(2)$

C. crosses the *y*-axis at $\log_e\left(\dfrac{3}{2}\right)$

D. crosses the *y*-axis at $\log_e(3)$

E. does not cross the *y*-axis.

Question 6

The point $(-1,1)$ is a stationary point on the graph of $y=x^5-bx+c$, where $b,\ c\in R$, then

A. $b=5$ and $c=3$

B. $b=5$ and $c=-2$

C. $b=5$ and $c=-3$

D. $b=-5$ and $c=3$

E. $b=-5$ and $c=7$

Question 7

A cube has its volume increasing at a constant rate of p cm^3/min. The rate at which each of the sides is increasing at in cm/min, when each side length is L cm, is equal to

A. $\frac{p}{3L^2}$

B. $3pL^2$

C. $\frac{p}{L^3}$

D. $\frac{pL^3}{t}$

E. $\frac{2L^2}{p}$

Question 8

The graph of $y = \log_e(x)$ is transformed into the graph of $y = -3\log_e\left(\frac{x}{2}\right)$ by

A. A dilation by a scale factor of -3 from the *y*-axis and a dilation by a scale factor of 2 from the *x*-axis.

B. A reflection in the *y*-axis, then a dilation by a scale factor of 3 parallel to the *y*-axis, followed by a dilation by a scale factor of $\frac{1}{2}$ parallel to the *x*-axis.

C. A reflection in the *y*-axis, then a dilation by a scale factor of 3 parallel to the *x*-axis, followed by a dilation by a scale factor of $\frac{1}{2}$ parallel to the *y*-axis.

D. A reflection in the *x*-axis, then a dilation by a scale factor of 3 parallel to the *y*-axis, followed by a dilation by a scale factor of $\frac{1}{2}$ parallel to the *x*-axis.

E. A reflection in the *x*-axis, then a dilation by a scale factor of 3 parallel to the *y*-axis, followed by a translation of $\log_e(8)$ units up and parallel to the *y*-axis.

Question 9

If a is a real constant, then the inverse of the function

$f: R\setminus\{-a\} \to R$, $f(x) = \dfrac{1}{x+a}$ is

A. $f^{-1}: R \to R$, $f^{-1}(x) = x + a$

B. $f^{-1}: R\setminus\{0\} \to R$, $f^{-1}(x) = x + a$

C. $f^{-1}: R\setminus\{0\} \to R$, $f^{-1}(x) = \dfrac{1-ax}{x}$

D. $f^{-1}: (-a,\infty) \to R$, $f^{-1}(x) = x + \dfrac{1}{a}$

E. $f^{-1}: (-a,\infty) \to R$, $f^{-1}(x) = \dfrac{x}{1+ax}$

Question 10

Consider the graph of the function $f: R \to R$, $f(x) = a(x-h)^n + k$,

where $a \in R\setminus\{0\}$, and n is an integer, which of the following is **false?**

A. If $n < 0$ the line $x = h$ is a vertical asymptote.

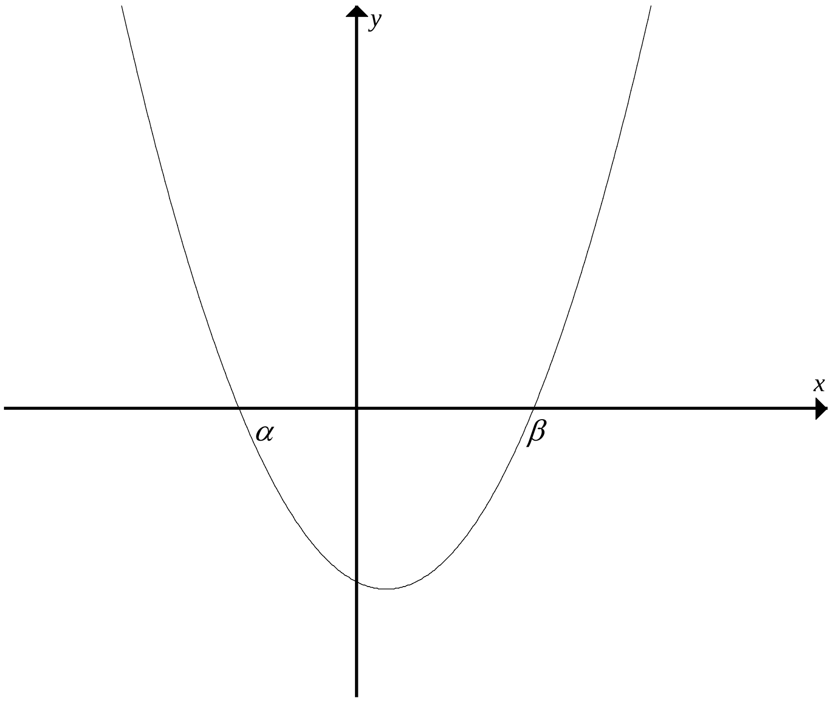
B. If $n < 0$ the line $y = k$ is a horizontal asymptote.

C. If $n > 0$ the gradient at the point $x = h$ is zero.

D. If $a < 0$ and $n > 0$ with n even, the point (h,k) is a local minimum.

E. If $n > 0$ with n odd, the point (h,k) is a stationary point of inflexion.

Question 11



A cubic graph is given by $y = f(x)$. The graph of the **gradient** function $f'(x)$ is shown above, this graph crosses the x-axis at $x = \alpha$ and $x = \beta$. The graph of $y = f(x)$ has

A. a local maximum at $x = \alpha$ and a local minimum at $x = \beta$

B. a local minimum at $x = \alpha$ and a local maximum at $x = \beta$

C. a local minimum at $x = 0$

C. a local maximum at $x = 0$

E. a stationary point of inflection at $x = 0$

Question 12

The average rate of change of the function with the rule $y = 2\cos(2x)$ over $0 \le x \le \frac{\pi}{8}$ is equal to

A. $\frac{8\left(\sqrt{2}-2\right)}{\pi}$

B. $\frac{\pi\left(\sqrt{2}-2\right)}{8}$

C. $\frac{4\sqrt{2}}{\pi}$

D. $-\frac{8\sqrt{2}}{\pi}$

E. $\frac{16\sqrt{2}}{\pi}$

Question 13

The speed *v*, in metres per second, of an object moving in a straight line is given by a function of time *t*, in seconds, where $v(t) = \frac{72}{(3t+2)^2}$ where $t \ge 0$.

Which of the following is true?

A. The initial speed of the object is 18 m/s and the distance travelled by the object in the first two seconds is 36 metres.

B. The initial speed of the object is 18 m/s and the distance travelled by the object in the first two seconds is 12 metres.

C. The initial speed of the object is 18 m/s and the distance travelled by the object in the first two seconds is 9 metres.

D. The initial speed of the object is 36 m/s and the distance travelled by the object in the first two seconds is 9 metres.

E. The initial speed of the object is 36 m/s and the distance travelled by the object in the first two seconds is 72 metres.

Question 14

If $f(x) = \cos(2x)$ and $g(x) = x^2$, then $\frac{d}{dx}\left[g\left(f(x)\right)\right]$ is equal to

A. $-2f'(x)f(x)$

B. $2f'(x)f(x)$

C. $-4x\,f'\left(g(x)\right)$

D. $f'(x)g\left(f(x)\right)$

E. $-2x\,g\left(f'(x)\right)$

Question 15

The transformation $T : R^2 \to R^2$, which maps the curve with equation $y = \frac{1}{x}$ to the curve with equation $y = \frac{4}{8-2x} - 2$, could have the rule

A. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 4 \\ 1 \end{bmatrix}$

B. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 2 \\ -2 \end{bmatrix}$

C. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & -\frac{1}{2} \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -4 \\ -1 \end{bmatrix}$

D. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -\frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

E. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 4 \\ -2 \end{bmatrix}$

Question 16

The graphs of $y = x^2 + mx + m$ and $y = 2mx + 2m$ where m is a real constant, will intersect at more than one point if

A. $m \in (-\infty, -4) \cup (0, \infty)$

B. $m \in (-4, 0)$

C. $m \in (0, \infty) \cup (4, \infty)$

D. $m \in (0, 4)$

E. $m \in \{-4, 0, 4\}$

Question 17

A discrete random variable has a binomial distribution. The expression $1 - \left(0.75^7 + 7(0.25)(0.75)^6\right)$ represents the probability of

A. exactly one success in seven trials each with probability of success equal to 0.75.

B. at least one success in seven trials each with probability of success equal to 0.75.

C. at least one success in seven trials each with probability of success equal to 0.25.

D. more than one success in seven trials each with probability of success equal to 0.25.

E. more than one success in seven trials each with probability of success equal to 0.75.

Question 18

For two events A and B, $\Pr(A)=\frac{8}{15}$, $\Pr(B)=\frac{1}{3}$ and $\Pr(A\cap B)=\frac{1}{5}$

Which of the following statements is **true?**

A. A and B are independent.

B. A and B are mutually exclusive.

C. $\Pr(A'\cup B')=\frac{1}{5}$

D. $\Pr(A\cup B)=\frac{2}{3}$

E. $\Pr(A'\cap B')=\frac{2}{3}$

Question 19

Consider the probability density function defined by

$f(x)=\frac{1}{\pi(1+x^2)}$ for $x\in R$. Then

A. the mode is zero and the mean does not exist.

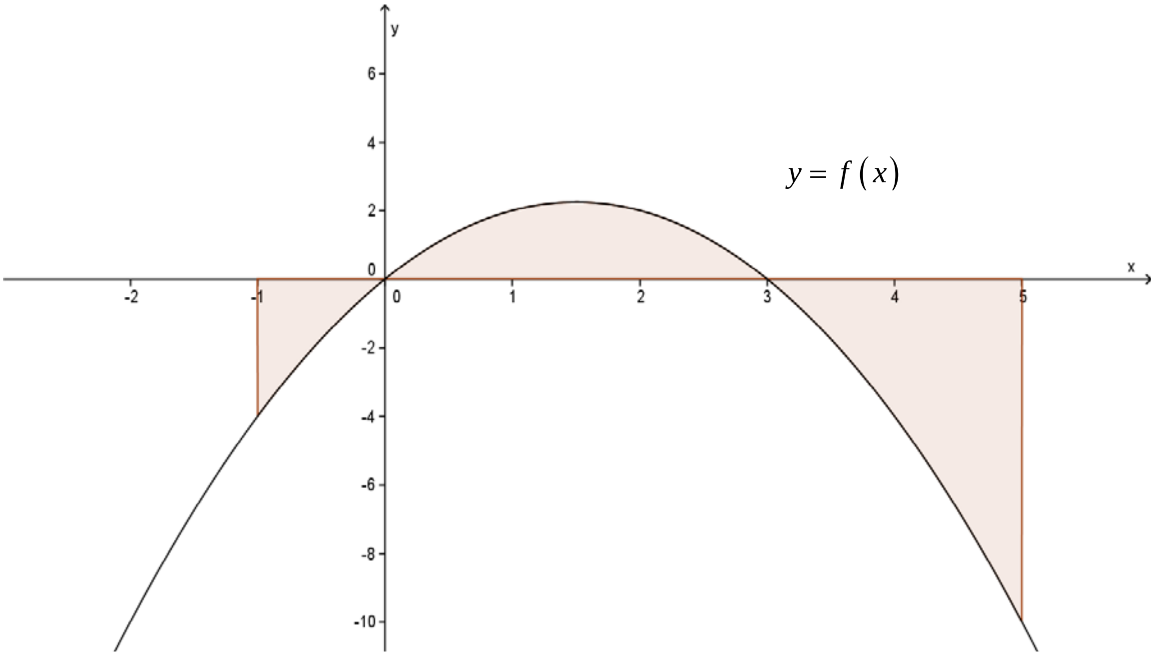
B. the mode is zero and the mean is zero.

C. the mode is $\frac{1}{\pi}$ and the median is zero.

D. the mode is $\frac{1}{\pi}$ and the mean is zero.

E. the mode is $\frac{1}{\pi}$ and the mean does not exist.

Question 20



The total area of the shaded regions in the diagram above is given by

A. $\int_{-1}^{5} f(x)\,dx$

B. $\int_{0}^{-1} f(x)\,dx + \int_{0}^{3} f(x)\,dx + \int_{5}^{3} f(x)\,dx$

C. $\int_{-1}^{0} f(x)\,dx + \int_{0}^{3} f(x)\,dx + \int_{3}^{5} f(x)\,dx$

D. $\left| \int_{-1}^{0} f(x)\,dx + \int_{0}^{3} f(x)\,dx - \int_{3}^{5} f(x)\,dx \right|$

E. $\left| \int_{-1}^{0} f(x)\,dx + \int_{0}^{3} f(x)\,dx + \int_{3}^{5} f(x)\,dx \right|$

Question 21

A normal distribution curve is defined by $f(x)=\frac{1}{\sqrt{10\pi}}e^{-\frac{(x-5)^2}{10}}$ for $x \in R$, then

A. The mean is 5 and the standard deviation is 5

B. The mean is 5 and the standard deviation is $\sqrt{5}$

C. The mean is 5 and the standard deviation is $2\sqrt{5}$

D. The mean is -5 and the standard deviation is 5

E. The mean is $\sqrt{5}$ and the standard deviation is $\sqrt{5}$

Question 22

Given the discrete probability distribution defined by
$\Pr(X=x)=c(4-x)$ for $x=0,1,2$ Then

A. The mode and the median are both 0

B. The mode and the median are both 1.

C. The mode is 0 and the median is 1.

D. The mode is 1 and the median is 2.

E. The mode is 2 and the median is 1.

END OF SECTION 1

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

a. Consider the function $f : R \to R$, $f(x) = ax^3 - 6ax + 12$ where $a > 0$

i. Show that $f(x)$ has stationary points at $x = \pm\sqrt{2}$

1 mark

ii. Find values of *a*, for which the graph of $y = f(x)$ crosses the *x*-axis at three distinct points.

3 marks

iii. Explain why the graph of $y = f(x)$ has exactly one zero when $0 < a < \frac{3\sqrt{2}}{2}$

1 mark

iv. If $0 < a < \frac{3\sqrt{2}}{2}$ show that $f(-1) > 0$ and explain why the graph of $y = f(x)$ does not have a zero in the interval $-1 \leq x \leq 1$.

2 marks

b. Consider the function $g:\left(\frac{-\pi}{2},\frac{\pi}{2}\right)\to R$, $g(x)=12\tan(x)-\frac{5}{\cos(x)}+\cos(x)$

i. Show that $g'(x)=\frac{f(\sin(x))}{\cos^2(x)}$ for an appropriate value of *a*.

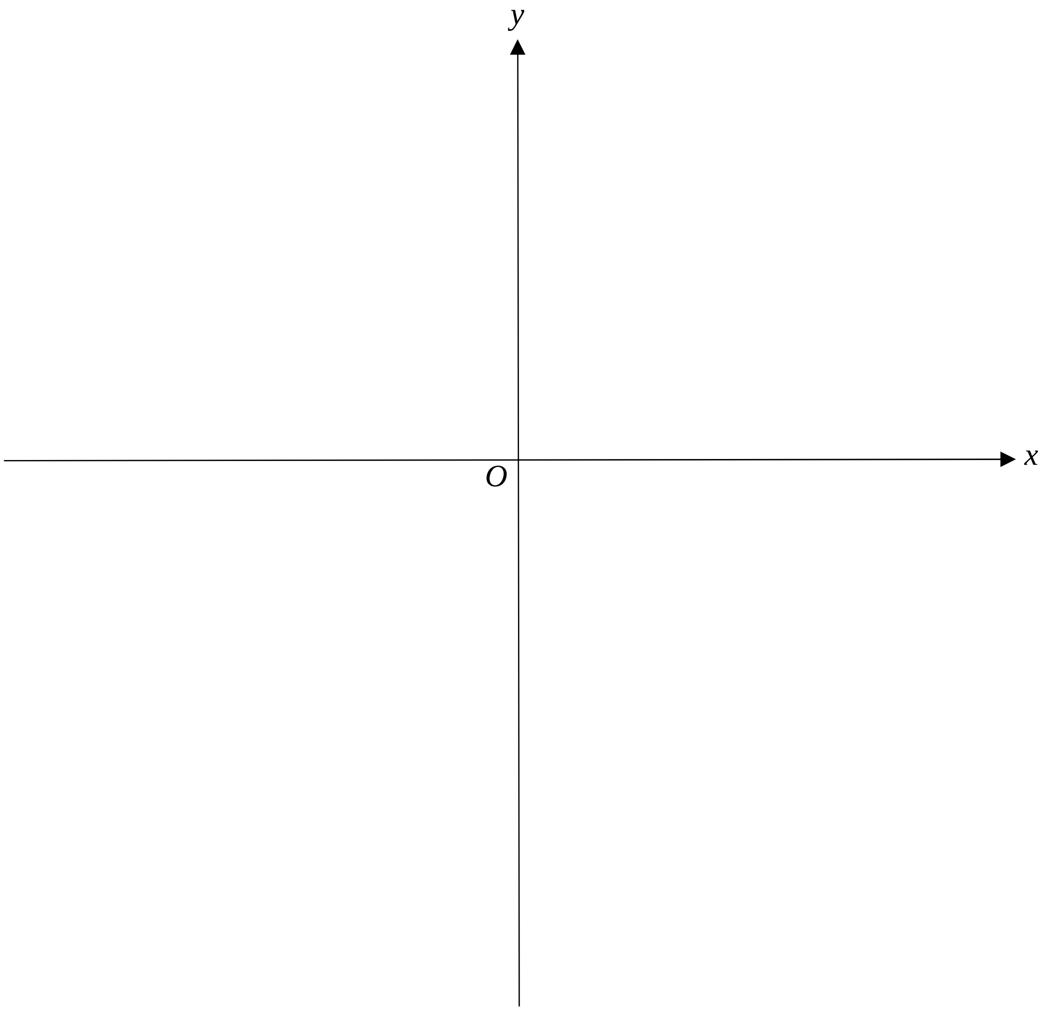
2 marks

ii. Explain why $g(x)$ has an inverse function.

2 marks

iii. Sketch the graph of $y = g(x)$ on the axes below.

Label any asymptotes with their equations and give the exact value of the y-intercept and the x-intercept correct to two decimal places.



2 marks

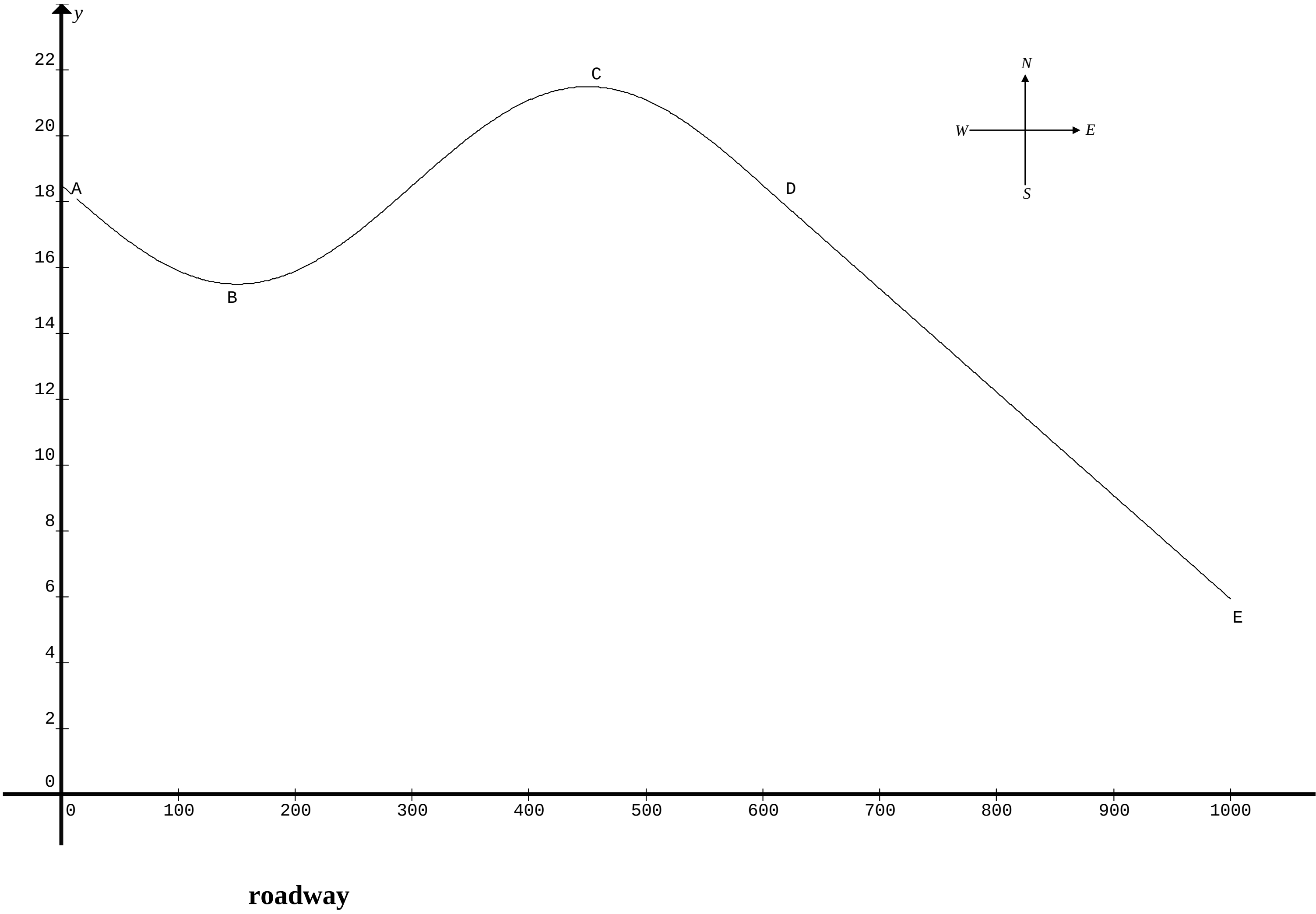
Total 13 marks

Question 2

The centre of a bike track *ABCDE* runs in a west-east direction as shown in the diagram below. An origin *O* and a Cartesian coordinate system is displayed, with all measurements in metres. To the south of the *x*-axis is a roadway and the centre of the bike track can be modelled by a curve of the form

$$f(x) = \begin{cases} a + b\sin(nx) & \text{for } 0 \le x \le 600 \\ mx + c & \text{for } 600 \le x \le 1000 \end{cases}$$

The centre of the bike track is smoothly joined at the point *D*, 600 metres east of the origin, by the sine curve *ABCD* and the straight line segment *DE*. The points *A* and *D* are both 18.5 metres north of the roadway, and the point *B* is 15.5 metres north of the roadway.



a.i. Show and explain why

$a = 18.5$, $b = -3$, $n = \dfrac{\pi}{300}$, $m = -\dfrac{\pi}{100}$ and $c = 6\pi + 18.5$

5 marks

ii. Find the distance *DE* as measured along the centre of the bike track.
Given an exact answer.

1 mark

b. For a curve $y = f(x)$ the length of the curve *s*, from $x_1 \le x \le x_2$ is given by

$$s = \int_{x_1}^{x_2} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx .$$

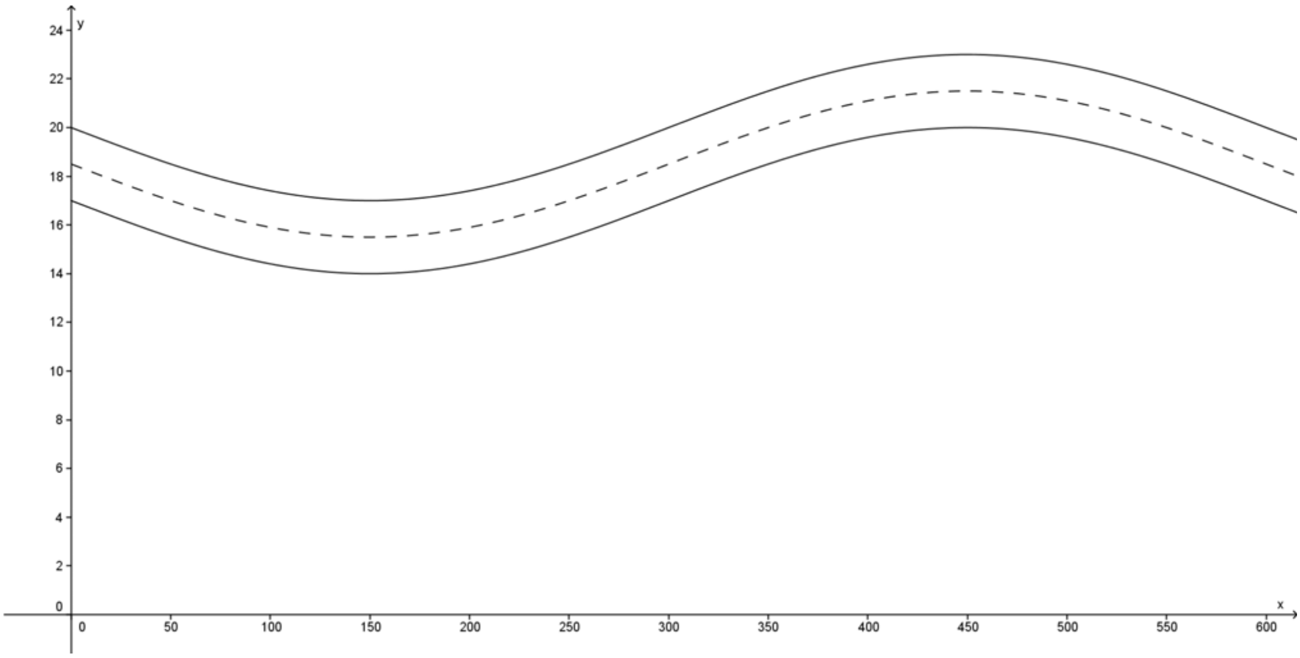
i. Using this, write down in terms of two definite integrals, an expression which gives the total length of the centre of the bike track from *A* to *E*.

2 marks

ii. Hence or otherwise, find the total length of the centre of the bike track, giving your answer in metres correct to two decimal places.

1 mark

c. In reality the bike track is 3 metres wide throughout and is modelled by functions f_n and f_s, which give the north and south edges of the bike track, with the centre of the bike track dotted in the middle, as shown in the diagram below.



i. Write down the functions f_n and f_s.

2 marks

ii. The total bike track, that is the area between the functions f_n and f_s, is to be asphalted, find the area in square metres of asphalt required.

1 mark

iii. The area north of the roadway to the south edge of the bike track is to grassed, find the total area of the grassed region giving an exact answer in square metres.

2 marks

iv. A tree is to be planted at a point *T*, this point is on the north edge of the bike track and furthest from the roadway, write down the coordinates of *T*.

1 mark
Total 15 marks

Question 3

a. The time for a quarter of a football match has been found to be normally distributed with a mean of 27 minutes, with a standard deviation of 2 minutes.

i. Find the probability, that a quarter of a football match goes longer than 30 minutes, if it known that it went for at least 25 minutes. Give your answer correct to four decimal places.

1 mark

ii. A statistician claims that 75% of all quarters of football go for longer than T minutes. Find the value of T, give your answer in minutes and seconds.

2 marks

iii. Find the probability that in a match which consists of four quarters, at least one quarter goes for longer than 30 minutes. Assume that all quarters are independent and give your answer correct to four decimal places.

2 marks

b. A footballer is in and out of the team, and the probability that he plays one week depends on how he played the week before. If he plays one week, the probability the he will play again in the team the following week is 0.75, while if he does not play in the team one week, the probability that he will play in the team the following week is 0.4. If he just played in the team one week, find the probability correct to four decimal places that he

i. plays in the team for the next three weeks.

1 mark

ii. plays in the team for exactly two of the next three weeks.

2 marks

iii. In the long run what percentage of the matches will he play for the season? Give your answer as a percentage, correct to two decimal places.

2 marks

c. The total distance s that a player runs for a match is a continuous random variable with a probability density function given by

$$f(s)=\begin{cases} k\sqrt{s} & \text{for } 0\le s\le 4 \\ a\cos\left(\dfrac{\pi(s-4)}{12}\right) & \text{for } 4\le s\le 10 \\ 0 & \text{elsewhere} \end{cases}$$

where a and k are constants.

i. Write down two linear simultaneous equations for a and k.

3 marks

ii. Show that $a=\dfrac{3\pi}{4(2\pi+9)}$

1 mark

iii. Sketch the probability density function on the axis below. Label the maximum with its coordinates, correct to three decimal places.

2 marks

iv. Find the exact probability that a player runs for less than 6 km, during a match.

2 marks

d. There are eight matches on each weekend of a football season. Alan takes part in a competition in which he earns one point if he picks more than half of the wining teams for a weekend, and zero points otherwise. The probability that Alan correctly picks the winning team in any match is $\frac{2}{3}$. Assuming that no match is drawn, find, giving all answers correct to four decimal places that

i. Alan earns one point for any given weekend.

1 mark

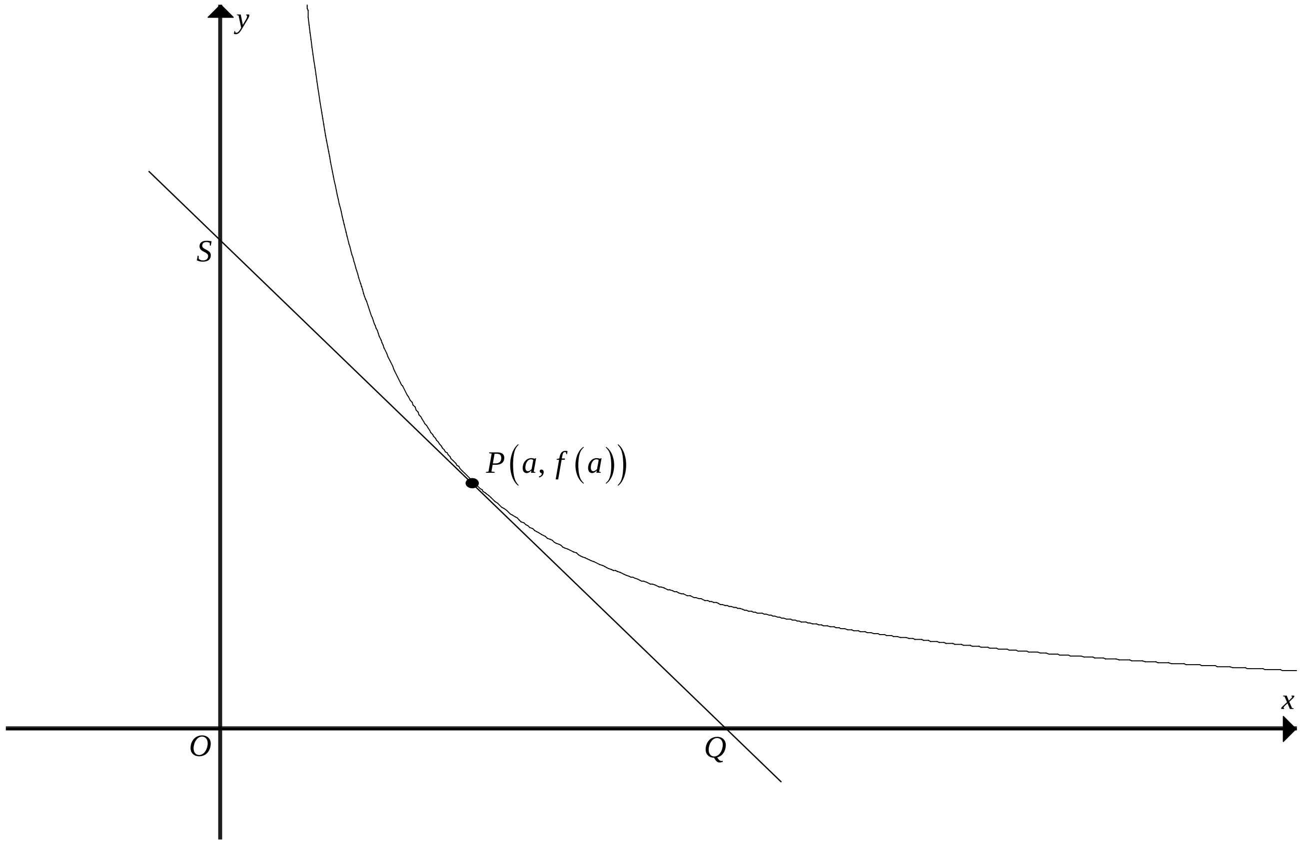
ii. Alan earns at least 16 points in a twenty-two week season.

1 mark
Total 20 marks

Question 4

The diagram below shows part of the graph of the function $f:(0,\infty)\to R$ where $f(x)=\dfrac{1}{x}$.

a. Let $P(a, f(a))$ where $a>0$ be a point on the graph of $y=\dfrac{1}{x}$.



i. Find in terms of *a*, the equation of the tangent to the curve at the point *P*.

2 marks

ii. The tangent to the curve at P, crosses the x-axis at Q and the y-axis at S, as shown in the diagram above. Write down the coordinates of the points Q and S.

2 marks

iii. Find the area of the triangle OQS and hence show that it is independent of a.

1 mark

b. Let n be a positive integer.

i. By considering the graph of $y=\frac{1}{x}$ show and explain why $\frac{1}{n+1} < \int_{n}^{n+1} \frac{dx}{x} < \frac{1}{n}$

2 marks

ii. Show that $\left(1+\frac{1}{n}\right)^{n} < e < \left(1+\frac{1}{n}\right)^{n+1}$

3 marks
Total 10 marks

END OF EXAMINATION

EXTRA WORKING SPACE

MATHEMATICAL METHODS CAS

Written examination 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Mathematical Methods and CAS Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$	Volume of a pyramid:	$\frac{1}{3}Ah$
curved surface area of a cylinder:	$2\pi rh$	volume of a sphere:	$\frac{4}{3}\pi r^3$
volume of a cylinder:	$\pi r^2 h$	area of triangle:	$\frac{1}{2}bc\sin(A)$
volume of a cone:	$\frac{1}{3}\pi r^2 h$		

Calculus

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + c\ ,\ n \neq -1$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$$

$$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$$

$$\int \frac{1}{x} dx = \log_e |x| + c$$

$$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$$

$$\int \sin(ax)\, dx = -\frac{1}{a}\cos(ax) + c$$

$$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$$

$$\int \cos(ax)\, dx = \frac{1}{a}\sin(ax) + c$$

$$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\sec^2(ax)$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

approximation: $f(x+h) \approx f(x) + h\,f'(x)$

Probability

$\Pr(A) = 1 - \Pr(A')$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A/B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

mean: $\mu = E(X)$

variance: $\text{var}(X) = \sigma^2 = E\left((X-\mu)^2\right) = E(X^2) - \mu^2$

probability distribution		mean	variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x\, p(x)$	$\sigma^2 = \sum (x-\mu)^2\, p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)\,dx$	$\mu = \int_{-\infty}^{\infty} x\, f(x)\,dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2\, f(x)\,dx$

ANSWER SHEET

STUDENT NUMBER

									Letter
Figures									
Words									

SIGNATURE ____

SECTION 1

1	**A**	**B**	**C**	**D**	**E**
2	**A**	**B**	**C**	**D**	**E**
3	**A**	**B**	**C**	**D**	**E**
4	**A**	**B**	**C**	**D**	**E**
5	**A**	**B**	**C**	**D**	**E**
6	**A**	**B**	**C**	**D**	**E**
7	**A**	**B**	**C**	**D**	**E**
8	**A**	**B**	**C**	**D**	**E**
9	**A**	**B**	**C**	**D**	**E**
10	**A**	**B**	**C**	**D**	**E**
11	**A**	**B**	**C**	**D**	**E**
12	**A**	**B**	**C**	**D**	**E**
13	**A**	**B**	**C**	**D**	**E**
14	**A**	**B**	**C**	**D**	**E**
15	**A**	**B**	**C**	**D**	**E**
16	**A**	**B**	**C**	**D**	**E**
17	**A**	**B**	**C**	**D**	**E**
18	**A**	**B**	**C**	**D**	**E**
19	**A**	**B**	**C**	**D**	**E**
20	**A**	**B**	**C**	**D**	**E**
21	**A**	**B**	**C**	**D**	**E**
22	**A**	**B**	**C**	**D**	**E**