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Student Name……

MATHEMATICAL METHODS (CAS) UNITS 3 & 4

TRIAL EXAMINATION 1

2010

Reading Time: 15 minutes
Writing time: 1 hour

Instructions to students

This exam consists of 11 questions.
All questions should be answered in the spaces provided.
There is a total of 40 marks available.
The marks allocated to each of the questions are indicated throughout.
Students may **not** bring any calculators or notes into the exam.
Where an exact answer is required a decimal approximation will not be accepted.
Where more than one mark is allocated to a question, appropriate working must be shown.
Diagrams in this trial exam are not drawn to scale.
A formula sheet can be found on page 11 of this exam.

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Question 1

a. If $y = e^{3x}(x^2 - 1)$, find $\frac{dy}{dx}$.

2 marks

b. Let $f(x) = \log_e(\cos(x))$. Find $f'(\pi)$.

2 marks

Question 2

a. Find $\int \left(\sqrt{x} + e^{2x}\right) dx$.

1 mark

b. If $f'(x) = \sin(3x)$ and $f(\pi) = \frac{4}{3}$, find $f(x)$.

2 marks

Question 3

If $f(x) = \log_e(x),\ x > 0$, show that $f(u) - 2f\left(\frac{1}{v}\right) = f(uv^2)$.

2 marks

Question 4

Two fair die are thrown simultaneously. One dice is red and the other is black.

a. Find the probability that the same number comes up on each dice.

1 mark

b. Find the probability that the number that comes up on the red dice is less than the number that comes up on the black dice.

1 mark

c. The two dice are thrown simultaneously four times. Find the probability that odd numbers on both die occur at least once.

2 marks

Question 5

Let $g: R \to R,\ g(x) = e^{x+1} - 2$.

Find g^{-1}, the inverse function of g.

3 marks

Question 6

Consider the system of simultaneous linear equations given by

$$mx + y = 2$$
$$2x + (m-1)y = m$$

Find the value(s) of m for which there is no solution.

3 marks

Question 7

The probability distribution of the random variable X is given in the table below.

X	2	3	4	5	6
$\Pr(X = x)$	0.2	0.4	0.1	0.2	0.1

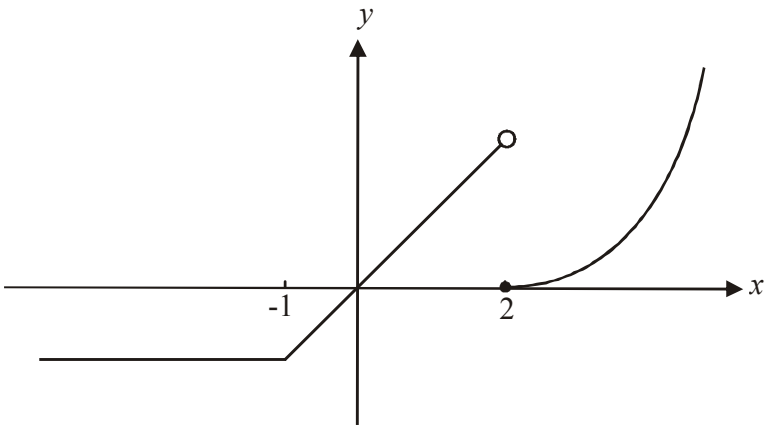
a. What is the median of X?

b. Find $\Pr(X \geq 3 \mid X < 6)$.

$1 + 2 = 3$ marks

Question 8

The graph of the function $y = f(x),\ x \in R$, is shown on the diagram below.



a. Sketch, on the same set of axes, the graph of the function $y = f'(x)$.

3 marks

b. What is the domain of the function $y = f'(x)$?

_____

_____

1 mark

Question 9

Find the general solution for x, of the equation $\sin(2x) = \frac{1}{\sqrt{2}}, x \in R$.

3 marks

Question 10

The velocity v, in metres per second, of a particle moving in a straight line at time t seconds, is given by $v(t) = 1 + \frac{10}{t+2}$, $t \geq 0$.

a. Sketch the graph of the velocity-time function on the set of axes below. Label clearly any endpoints or asymptotes.

3 marks

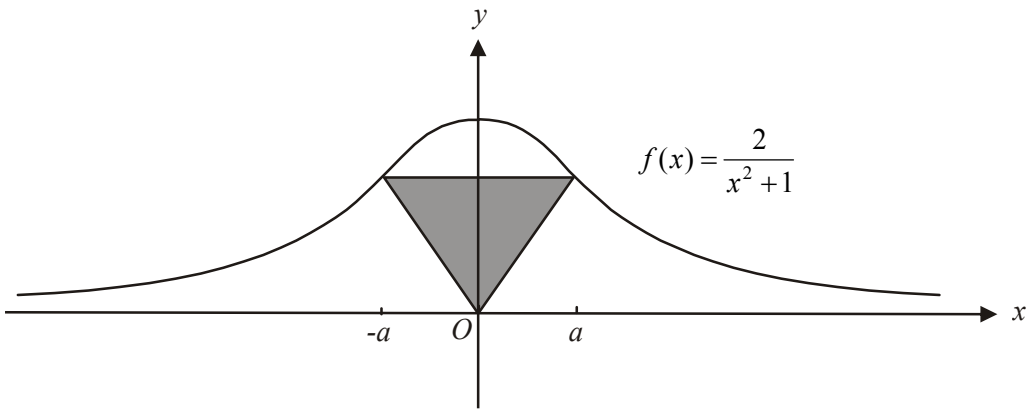
b. Find the value(s) of t for which the velocity of the particle is less than 3m/s.

1 mark

c. Find the distance travelled by the particle in the first second.

2 marks

Question 11



The diagram above shows a shaded isosceles triangle. One of the vertices of this triangle is at the origin $O(0,0)$ and the other two are on the graph of the function $f(x)=\frac{2}{x^2+1}$ at the points where $x=-a$ and $x=a$ where $a>0$.

a. Find an expression for the area A, of the triangle, in terms of a.

1 mark

b. Hence find the maximum area of the triangle and the value of a for which this occurs. Justify that it is a maximum area.

4 marks

Mathematical Methods CAS Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$	volume of a pyramid:	$\frac{1}{3}Ah$
curved surface area of a cylinder:	$2\pi rh$	volume of a sphere:	$\frac{4}{3}\pi r^3$
volume of a cylinder:	$\pi r^2 h$	area of a triangle:	$\frac{1}{2}bc\sin A$
volume of a cone:	$\frac{1}{3}\pi r^2 h$		

Calculus

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1} \qquad \int x^n dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax} \qquad \int e^{ax} dx = \frac{1}{a}e^{ax} + c$$

$$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x} \qquad \int \frac{1}{x} dx = \log_e|x| + c$$

$$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax) \qquad \int \sin(ax)dx = -\frac{1}{a}\cos(ax) + c$$

$$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax) \qquad \int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + c$$

$$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\sec^2(ax)$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$ quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$ approximation: $f(x+h) \approx f(x) + hf'(x)$

Transition Matrices

$S_n = T^n \times S_0$

Probability

$\Pr(A) = 1 - \Pr(A')$ $\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

mean: $\mu = E(X)$ variance: $\text{var}(X) = \sigma^2 = E((X-\mu)^2) = E(X^2) - \mu^2$

probability distribution		mean	variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \Sigma x\, p(x)$	$\sigma^2 = \Sigma (x-\mu)^2 p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)dx$	$\mu = \int_{-\infty}^{\infty} x f(x)dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx$