2009 VCAA Mathematical Methods Exam 1 Solutions

Q1a The product rule: $\frac{dy}{dx}=1+\log_e x$

Q1b The quotient rule: $f'(x)=\frac{-(2x+2)\sin x-2\cos x}{(2x+2)^2}$

$=\frac{-(x+1)\sin x-\cos x}{2(x+1)^2}$

$f'(\pi)=\frac{-(\pi+1)\sin\pi-\cos\pi}{2(\pi+1)^2}=\frac{1}{2(\pi+1)^2}$

Q2a $\int\frac{1}{1-2x}dx=-\frac{1}{2}\log_e|1-2x|$

Q2b $\int_1^4\left(\sqrt{x}+1\right)dx=\left[\frac{2x^{\frac{3}{2}}}{3}+x\right]_1^4=\frac{23}{3}$

Q3 Equation of $f(x)$: $y=\frac{3}{x}-4$

Equation of inverse: $x=\frac{3}{y}-4$, i.e. $y=\frac{3}{x+4}$

$f^{-1}:R\backslash\{-4\}\to R$ where $f^{-1}(x)=\frac{3}{x+4}$

Q4 $\tan(2x)=\sqrt{3}$, $x\in\left(-\frac{\pi}{4},\frac{\pi}{4}\right)\cup\left(\frac{\pi}{4},\frac{3\pi}{4}\right)$,

$2x=\frac{\pi}{3}$, $\frac{4\pi}{3}$

$x=\frac{\pi}{6}$, $\frac{2\pi}{3}$

Q5a $\frac{1}{4}\times\frac{1}{3}=\frac{1}{12}$

Q5b $\Pr(1,4)+\Pr(2,3)+\Pr(3,2)+\Pr(4,1)=\frac{1}{12}+\frac{1}{12}+\frac{1}{12}+\frac{1}{12}=\frac{1}{3}$

Q5c Conditional probability $=\frac{\frac{1}{12}}{\frac{1}{3}}=\frac{1}{4}$

Q6 $\frac{dV}{dt}=10$, $V=2\pi r^2$, $\frac{dV}{dr}=4\pi r$

$\frac{dV}{dt}=\frac{dV}{dr}\times\frac{dr}{dt}$

$\frac{dr}{dt}=\frac{\frac{dV}{dt}}{\frac{dV}{dr}}=\frac{10}{4\pi r}$

When $r=30$, $\frac{dr}{dt}=\frac{1}{12\pi}$ mm per min

Q7a $\Pr(X>1\mid X\le 3)=\frac{\Pr(X>1\cap X\le 3)}{\Pr(X\le 3)}$

$=\frac{0.4+0.2}{0.1+0.2+0.4+0.2}=\frac{2}{3}$

Q7b $\overline{X}=0\times0.1+1\times0.2+2\times0.4+3\times0.2+4\times0.1=2$

$Var(x)=0^2\times0.1+1^2\times0.2+2^2\times0.4+3^2\times0.2+4^2\times0.1-2^2=1.2$

Q8 $f(x)=e^x+k$, $k\in R$, $f'(x)=e^x$

Gradient of tangent at $x=a$ is $f'(a)=e^a$

Equation of tangent: $y=e^a x$ since it passes through $(0,0)$

At $x=a$, $f(a)=e^a+k=e^a a$

$\therefore k=e^a(a-1)$

Q9 For $2\log_e x-\log_e(x+3)=\log_e\frac{1}{2}$ to be defined, $x>0$

$\log_e\frac{x^2}{x+3}=\log_e\frac{1}{2}$, $\frac{x^2}{x+3}=\frac{1}{2}$, $(2x-3)(x+1)=0$

Since $x>0$, $\therefore x+1\ne 0$

$\therefore 2x-3=0$, $x=\frac{3}{2}$

Q10a $f(x+h)\approx f(x)+hf'(x)$

Let $f(x)=\sqrt[3]{x}=x^{\frac{1}{3}}$, $f'(x)=\frac{1}{3x^{\frac{2}{3}}}$

Let $x=8$, $\sqrt[3]{8.06}\approx\sqrt[3]{8}+0.06\times\frac{1}{3\times8^{\frac{2}{3}}}=2+0.005=2.005$

Q10b $f'(x)=\frac{1}{3x^{\frac{2}{3}}}$ is a decreasing function for $x>0$. As x increases from a point, say at $x=a>0$, the tangent at $x=a$ is always higher than $f(x)$.

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