2009 Mathematical Methods 3/4 Trial Exam 1 Solutions
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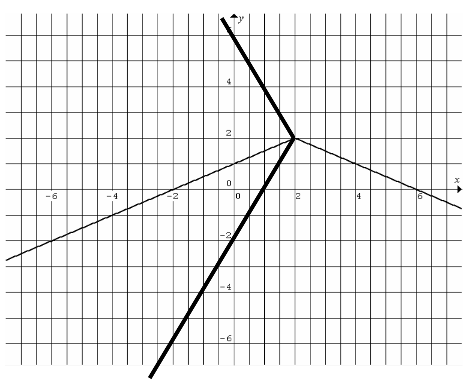
1a. The x^5 term of $P(x)$ is ${}^nC_5\left(1^{n-5}\right)(-x)^5 = -{}^nC_5 x^5$. The coefficient is $-{}^nC_5$.

1b. $(1-x)^n = P(x)$, $\frac{d}{dx}P(x) = -n(1-x)^{n-1}$,

$f(x) = \frac{P(x)}{\frac{d}{dx}P(x)} = \frac{(1-x)^n}{-n(1-x)^{n-1}} = \frac{1-x}{-n}$, $\therefore f(2) = \frac{1-2}{-n} = \frac{1}{n}$

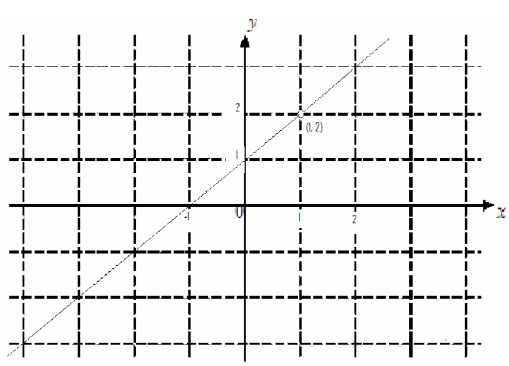
2a. The graph of $g(x)$ is the reflection in the x-axis of the graph of $\frac{1}{2}|x|$, followed by horizontal and vertical translations of 2 units each. The equation of $g(x)$ is $y = -\frac{1}{2}|x-2| + 2$.

2b.



3. $uv = \frac{x^4-1}{x^3-x^2+x-1} = \frac{\left(x^2-1\right)\left(x^2+1\right)}{x^2(x-1)+1(x-1)}$

$= \frac{(x+1)(x-1)\left(x^2+1\right)}{(x-1)\left(x^2+1\right)} = x+1$ and $x \neq 1$.



4. $\cos\left(\frac{2x}{3}\right) = \sqrt{3}\sin\left(\frac{2x}{3}\right)$, $\frac{\sin\left(\frac{2x}{3}\right)}{\cos\left(\frac{2x}{3}\right)} = \frac{1}{\sqrt{3}}$, $\therefore \tan\left(\frac{2x}{3}\right) = \frac{1}{\sqrt{3}}$,

and given $-\frac{3\pi}{2} \le x \le \frac{3\pi}{2}$, $\therefore -\pi \le \frac{2x}{3} \le \pi$.

Hence $\frac{2x}{3} = -\frac{5\pi}{6}$, $\frac{\pi}{6}$. $\therefore x = -\frac{5\pi}{4}$, $\frac{\pi}{4}$.

5. $f(x+h) \approx f(x) + hf'(x)$, $\frac{f(x+h)-f(x)}{h} \approx f'(x)$,

$\frac{-0.01-0.28}{p-2.7} \approx -2.9$, $\therefore p \approx 2.8$.

6a. $f(x) = \frac{1}{\sqrt{2x-1}}$, $g(x) = e^{-x}$.

$\therefore f(g(x)) = \frac{1}{\sqrt{2g(x)-1}} = \frac{1}{\sqrt{2e^{-x}-1}}$, $\therefore 2e^{-x}-1 > 0$.

Hence $e^{-x} > \frac{1}{2}$, $e^x < 2$, $x < \log_e 2$.

The domain is $(-\infty, \log_e 2)$.

6b. $g(f(x)) = e^{-\frac{1}{\sqrt{2x-1}}}$, $\therefore 2x-1 > 0$, $x > \frac{1}{2}$.

$g(f(x))$ is a increasing function.

As $x \to \frac{1}{2}$, $g(f(x)) \to 0^+$.

As $x \to +\infty$, $g(f(x)) \to 1^-$.

The range is $(0,1)$.

6c. $f(x) = \frac{1}{\sqrt{2x-1}}$, $(f(x))^{-1} = \sqrt{2x-1}$,

$g\left((f(x))^{-1}\right) = e^{-\sqrt{2x-1}}$, $\left(g\left((f(x))^{-1}\right)\right)^{-1} = e^{\sqrt{2x-1}}$.

$\frac{d}{dx}\left(g\left((f(x))^{-1}\right)\right)^{-1} = \frac{d}{dx}e^{\sqrt{2x-1}} = e^{\sqrt{2x-1}} \times \frac{1}{\sqrt{2x-1}} = \frac{e^{\sqrt{2x-1}}}{\sqrt{2x-1}}$.

7. $\int(\log_e 2x)dx - \log_e(2x)^e = \int ef(x)dx$,

$\int ef(x)dx - \int(\log_e 2x)dx = -\log_e(2x)^e$,

$\int(ef(x) - \log_e 2x)dx = -e\log_e(2x)$,

$ef(x) - \log_e 2x = \frac{d}{dx}(-e\log_e(2x))$,

$ef(x) - \log_e 2x = -\frac{e}{x}$,

$\therefore ef(e) - \log_e 2e = -\frac{e}{e}$,

$ef(e) - \log_e 2 - 1 = -1$.

$\therefore f(e) = \frac{\log_e 2}{e}$.

8a. $\Pr(X \le a) + \Pr(X \ge b) = 1 - \Pr(a < x < b) = 1 - 0.95 = 0.05$.

8b. $p = 0.5$, $Bi(n, 0.5)$ is symmetric about the mean μ.

$\sigma = \sqrt{np(1-p)} = \sqrt{0.25n} = 0.5\sqrt{n}$.

For smallest value of $b - a$, $a = \mu - 2\sigma$ and $b = \mu + 2\sigma$.

$\therefore b - a = 4\sigma = 2\sqrt{n}$.

9a. $\Pr(X < a \mid X > b) = \frac{\Pr(X < a \cap X > b)}{\Pr(X > b)} = \frac{\Pr(b < X < a)}{\Pr(X > b)}$

$= \frac{\Pr(X > b) - \Pr(X > a)}{\Pr(X > b)}$

$= \frac{0.2 - 0.1}{0.2} = 0.5$.

9b. $\Pr(X < a) = 0.9$ and $\Pr(X > b) = 0.2$, $\therefore b < a$.

If $X > a$, then X cannot be $< b$.

Hence $\Pr(X < b \mid X > a) = 0$.

10a. For $0 \le x \le \pi$, $f'(x) = \frac{1}{\pi}(\sin(x) + x\cos x)$ and

$f'(m_o) = 0$. $\therefore \frac{1}{\pi}(\sin(m_o) + m_o\cos(m_o)) = 0$.

Hence $\sin(m_o) + m_o\cos(m_o) = 0$.

10b. Given $\frac{d}{dx}(x\cos x) = \cos x - x\sin x$,

$\therefore x\sin x = \cos x - \frac{d}{dx}(x\cos x)$.

Since $\int_0^{m_e} \frac{1}{\pi} x\sin x dx = \frac{1}{2}$,

$\therefore \int_0^{m_e}\left(\cos x - \frac{d}{dx}(x\cos x)\right)dx = \frac{\pi}{2}$,

$\therefore \int_0^{m_e}\cos x dx - \int_0^{m_e}\left(\frac{d}{dx}(x\cos x)\right)dx = \frac{\pi}{2}$,

$\therefore [\sin x]_0^{m_e} - [x\cos x]_0^{m_e} = \frac{\pi}{2}$.

Hence $\sin(m_e) - m_e\cos(m_e) = \frac{\pi}{2}$.

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