INSIGHT

Trial Exam Paper

2009

MATHEMATICAL METHODS

Written examination 2

STUDENT NAME:

QUESTION AND ANSWER BOOK

Reading time: 15 minutes
Writing time: 2 hours

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	4	4	58
		Total	80

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one approved **graphics** calculator (memory DOES NOT need to be cleared) and, if desired, one scientific calculator, one bound reference.
- Students are NOT permitted to bring the following items into the examination: blank sheets of paper and/or white out liquid/tape.

Materials provided

- The question and answer book of 21 pages, with a separate sheet of miscellaneous formulas.
- An answer sheet for multiple-choice questions.

Instructions

- Write your **name** in the box provided and on the answer sheet for multiple-choice questions.
- Remove the formula sheet during reading time.
- You must answer the questions in English.

At the end of the exam

- Place the answer sheet for multiple-choice questions inside the front cover of this question book.

Students are NOT permitted to bring mobile phones or any other electronic devices into the examination.

SECTION 1

Instructions for Section 1

Answer **all** questions in pencil on the multiple-choice answer sheet.
Select the response that is **correct** for the question.
A correct answer scores 1 mark, an incorrect answer scores 0.
Marks will not be deducted for incorrect answers.
If more than one answer is selected no marks will be awarded.

Question 1

The function $f:[0,2\pi]\rightarrow R,\ f(x)=1-3\sin(2x-\frac{\pi}{2})$ has a range and period respectively of

A. $[0,3]$ and 2

B. $[-1,3]$ and 2π

C. $[-2,4]$ and π

D. $[-3,3]$ and π

E. $[-2,4]$ and 2π

Question 2

The range of the function $f(x)=e^{|x|}-3$ is

A. $[-2,\infty)$

B. R^{+}

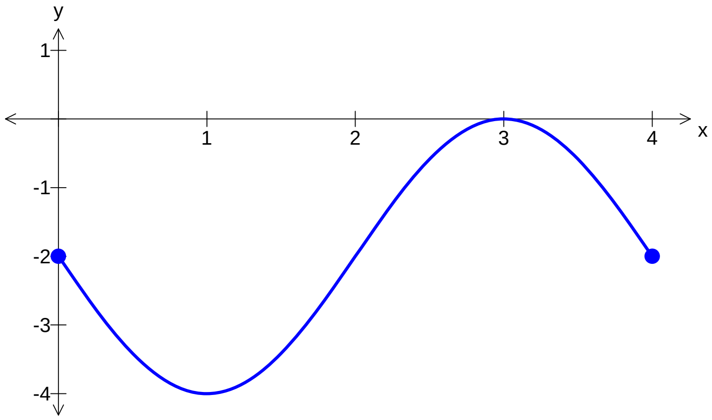
C. $(-2,0]$

D. $(-2,0)$

E. $(-3,\infty)$

Question 3

The diagram below shows one cycle of the graph of a circular function.



A possible equation for the function whose graph is shown is

A. $y = 2 - 2\sin(4x)$

B. $y = -2 - 2\sin(4x)$

C. $y = 4 - 2\sin(4\pi x)$

D. $y = -2 - 2\sin(\frac{\pi}{2}x)$

E. $y = -2 - 2\sin(\frac{1}{2}x)$

Question 4

One cycle of the graph of the function with the equation $y = \tan(ax)$ has successive vertical asymptotes at $x = \frac{3}{8}$ and $x = \frac{5}{8}$.

A possible value for a is

A. 2

B. 2π

C. 4

D. 4π

E. 8

Question 5

$\log_e\left(\dfrac{3}{e^{5x}}\right)$ is equal to

A. $\dfrac{3}{5x}$

B. $\dfrac{\log_e(3)}{5x}$

C. $\log_e(3) - \log_e(5x)$

D. $3 - \log_e(5x)$

E. $\log_e(3) - 5x$

Question 6

If $e^x = 1 + 6e^{-x}$, then x is equal to

A. $3, -2$

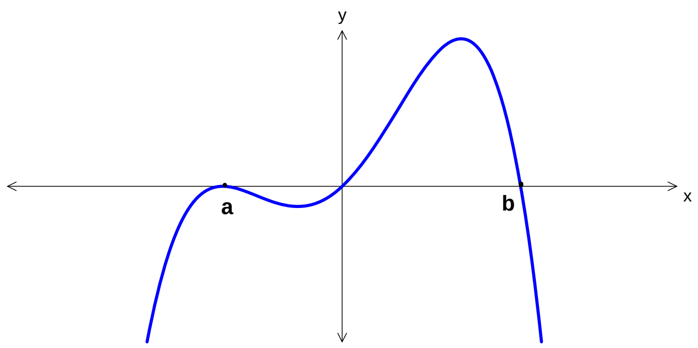
B. $\log_e 3$

C. $\log_e 3, -\log_e 2$

D. $-\log_e 3,\ \log_e 2$

E. $-3,\ 2$

Question 7



The graph shown could be that of a function f with the equation

A. $y = -x(x+a)^2(x-b)$

B. $y = -x(x+a)(x-b)$

C. $y = x(x-a)^2(x-b)$

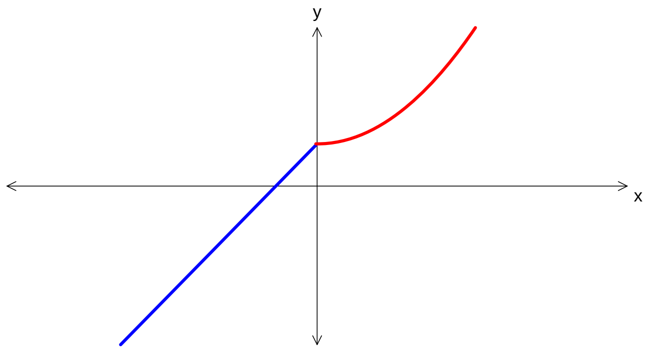
D. $y = -x(x+a)^2(b-x)$

E. $y = x(x-a)^2(b-x)$

SECTION 1 – continued

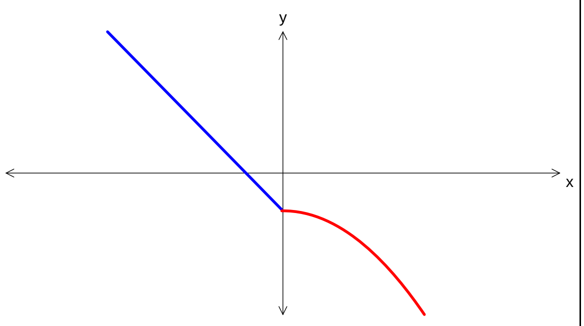
Question 8

The graph of the function with the equation $y = f(x)$ is shown below.

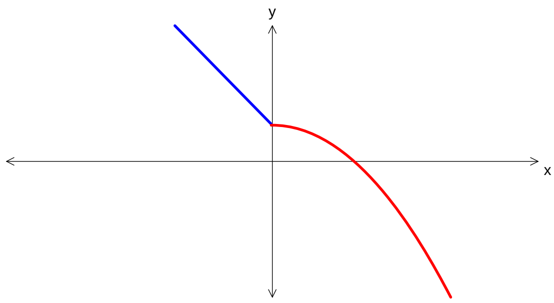


Which of the following is most likely to be the graph of the inverse function?

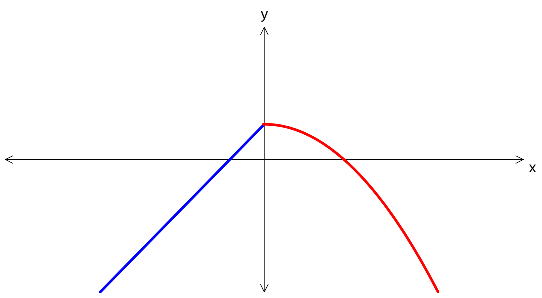
A.



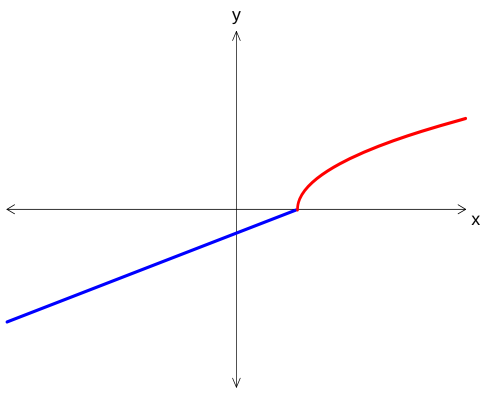
B.



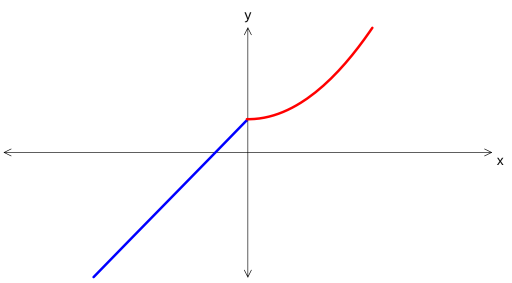
C.



D.

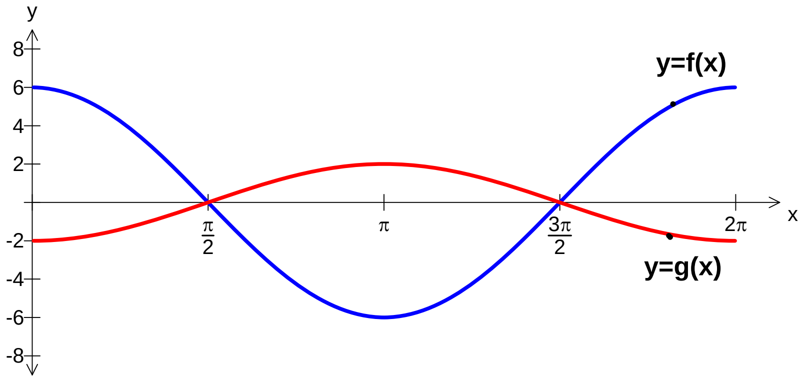


E.



Question 9

The diagram below shows the graphs of two circular functions, *f* and *g*.



The graph of the function with the equation $y = f(x)$ is transformed into the graph of the function with the equation $y = g(x)$ by

A. a dilation by a scale factor of $\frac{1}{3}$ from the *y*- axis followed by a reflection in the *x*-axis.

B. a dilation by a scale factor of 3 from the *y*- axis followed by a reflection in the *x*-axis.

C. a dilation by a scale factor of 3 from the *y*- axis followed by a reflection in the *y*-axis.

D. a dilation by a scale factor of $\frac{1}{3}$ from the *x*- axis followed by a reflection in the *x*-axis.

E. a dilation by a scale factor of 3 from the *x*- axis followed by a reflection in the *y*-axis.

Question 10

The function defined by $f : A \to R,\ f(x) = e^{(x-b)^2}$, $b \in R$, will have an inverse function for all values of *b*, if its domain *A* is

A. R

B. $R \setminus \{b\}$

C. $[b, \infty)$

D. R^+

E. $[-b, \infty)$

Question 11

If $f(x) = e^{2x}$ and $g(x) = \frac{1}{\sqrt{x}}$ then $g(f(1))$ is

A. $\frac{1}{e}$

B. $-e$

C. $\sqrt{e}$

D. 1

E. –1

Question 12

If $y = \log_e(\sin(3x))$ then $\frac{dy}{dx}$ is equal to

A. $\frac{3\cos(3x)}{\sin(3x)}$

B. $\frac{1}{\sin(3x)}$

C. $\frac{1}{3\cos(3x)}$

D. $3\tan(3x)$

E. $-3\tan(3x)$

Question 13

The equation of the normal to the curve with equation $y = x\cos(x)$, at the point on the curve with x-coordinate 2π is

A. $y = \frac{1}{x\sin(x)}$

B. $y = -x$

C. $y = x$

D. $y = -x + 4\pi$

E. $y = x - 4\pi$

Question 14

If $y = 2e(e^x - 1)$ then the rate of change of y with respect to x when $x = 0$ is

A. $2e^2$

B. 0

C. $2e - 1$

D. $2e^2 - 2e$

E. $2e$

Question 15

Using the approximation formula, $f(x+h) \approx f(x) + hf'(x)$ where $f(x) = x^3$ with $x = 4$, an approximate value for 4.3^3 is given by

A. $f(4) + 0.3f'(4)$

B. $f(4) + 0.0027f'(4)$

C. $f(64) + 0.3f'(64)$

D. $f(64) + 0.0027f'(64)$

E. $f'(4.3)$

Question 16

If $f'(x) = 5e^{2x}$ and c is a real constant, then $f(x)$ is equal to

A. $\frac{5}{2}e^{2x} + c$

B. $10e^{2x} + c$

C. $\frac{5}{2}e^{3x} + c$

D. $\frac{5}{2}e^{x^2} + c$

E. $5e^{x^2} + c$

Question 17

Let p be a function defined on the interval $[3, 5]$ and q a function such that $q'(x) = p(x)$, for all $x \in [3, 5]$

$\int_3^5 p(x)\,dx$ is equal to

A. $q(x) + c$

B. $p(5) - p(3)$

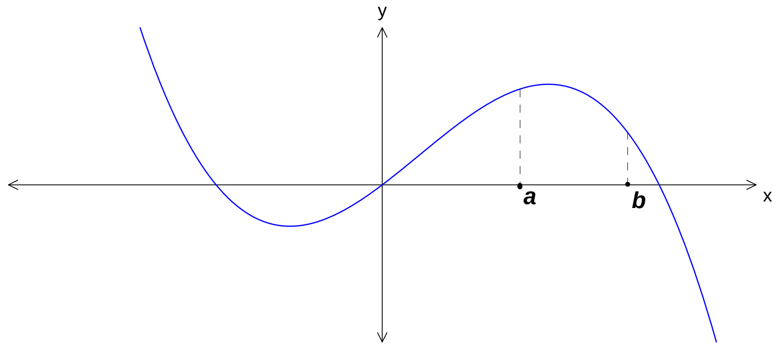
C. $q(5) - q(3)$

D. $q'(5) - q'(3)$

E. $p(x) + c$

Question 18

The graph of the function with equation $y = f(x)$ is shown below.



Let g be a function such that $g'(x) = f(x)$.

On the interval (a, b), the graph of g will have a

A. maximum turning point.
B. minimum turning point.
C. negative gradient.
D. positive gradient.
E. stationary point of inflection.

Question 19

The number of defective batteries in a box of batteries ready for sale is a random variable with a binomial distribution with mean 8.1 and standard deviation 0.9.

If a battery is drawn at random from the box, the probability that it is not defective is

A. 0.1
B. 0.9
C. 0.09
D. 0.3
E. 0.7

Question 20

Let X be a normally distributed random variable with mean μ and standard deviation σ.
Which one of the following is not always true?

A. $\Pr(X > \mu) = 0.5$

B. $\Pr(X > a) = 1 - \Pr(X < a)$

C. $\Pr(\mu - \sigma < X < \mu + \sigma) \approx 0.68$

D. $\Pr(\mu - 2\sigma < X < \mu + 2\sigma) \approx 0.95$

E. $\Pr(a < X < b) = \Pr(X > b) - \Pr(X > a)$

Question 21

The random variable X has the following probability distribution.

x	1	2	3
$\Pr(X=x)$	a	b	0.1

If the mean of X is 1.3, then the values of a and b respectively are—

A. 0.5, 0.4

B. 0.8, 0.1

C. 0.1, 0.8

D. 0.3, 0.7

E. 0.4, 0.5

Question 22

Juicy Giant orange juice is packed in small glass bottles labelled as containing 250 *ml*. The packing process produces bottles that are normally distributed with a standard deviation of 3 *ml*. In order to guarantee that only 1% of bottles are under-volume, the actual mean volume, in *ml*, would be required to be closest to

A. 256

B. 257

C. 258

D. 250.01

E. 247

END OF SECTION 1

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.
A decimal answer will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working must be shown.
Where an instruction to **use calculus** is stated for a question, you must show an appropriate derivative or antiderivative.
Unless otherwise stated diagrams are not drawn to scale.

Question 1

Grace shops each day at one of two supermarkets, Costless and Spendway. Her choice of supermarket each day depends only on which supermarket she has shopped at on the previous day. If she shops at Costless one day then the chance of her shopping at Costless the next day is $\frac{3}{5}$ while if she shops at Spendway one day then the chance of her shopping at Spendway the next day is $\frac{2}{3}$.

a. If she shops at Spendway one day, what is the chance she shops at Costless the next day?

1 mark

b. Suppose she shops at Costless one Friday.

i. What is the exact probability she shops at Costless the next Sunday?

3 marks

SECTION 2 – Question 1 – continued
TURN OVER

ii. What is the exact probability she shops at Costless on exactly two of the next three days?

2 marks

The time, t in minutes, she spends in the supermarket is independent of the type of supermarket she shops at and is a random variable with probability density function

$$f(t) = \begin{cases} at(60-t) & if \;\; 0 \le t \le 60 \\ 0 & otherwise \end{cases}$$

c. Show that $a = \dfrac{1}{36000}$.

2 marks

d. What is the probability, correct to 3 decimal places, that Grace spends longer than 45 minutes shopping in the supermarket?

2 marks

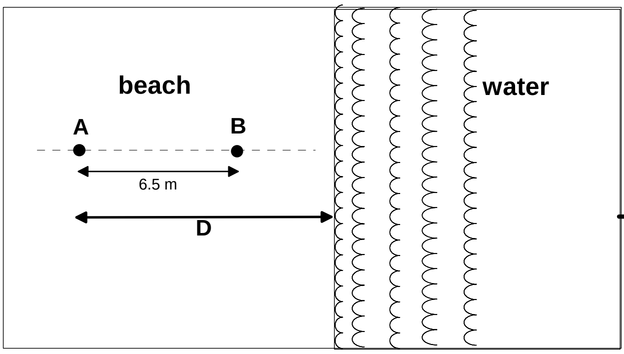
e. It is known that on one particular day Grace spends longer than 45 minutes shopping in the supermarket. What is the probability, correct to 3 decimal places, that she spends less than 50 minutes shopping in the supermarket?

2 marks
Total 12 marks

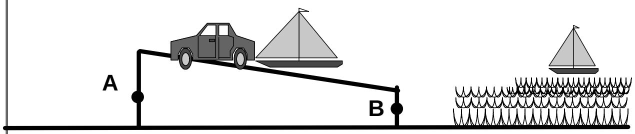
Question 2

Keith likes to holiday at Point Roadknight beach where the waves roll on to the beach at regular intervals.

The diagrams below show the beach with two markers, *A* and *B* at the start and end of the boat ramp, where the base of marker *A* is 6.5 metres further up the beach than the base of marker *B*. The line *AB* is perpendicular to the water's edge.



View from above

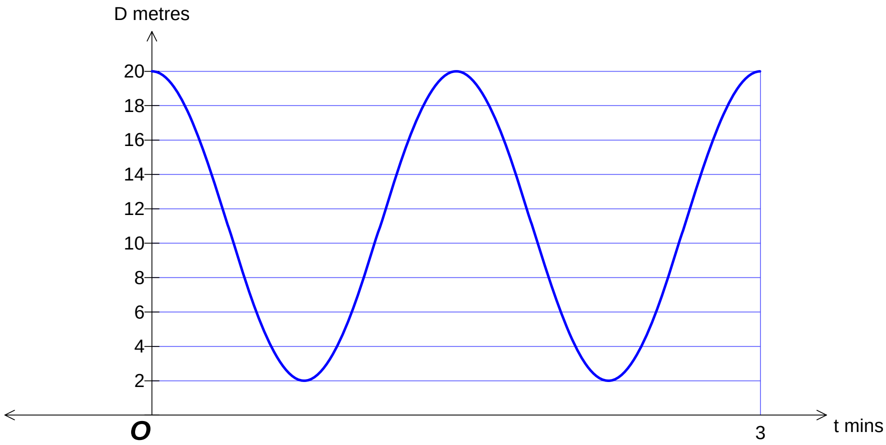


side view

Keith records the horizontal distance of the water's edge from the top marker *A*. He calculates that on one particular day the distance *D* metres of the water's edge from the top marker *A* is a function of time *t* (in minutes from when he starts to observe the waves). It can be modelled exactly by the equation

$$D = a\cos(bt) + c$$

where a, b and c are positive constants. The graph of *D* as a function of time *t* is shown below:



a. State the maximum and minimum distances of the water's edge from marker *A* on this day.

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2 marks

b. Find the number of waves that hit the beach in one hour on this day.

2 marks

c. Find the values of *a*, *b* and *c*.

3 marks

d. **i.** Write down an equation, the solution of which gives the times when the marker at *B* is in the water on this day.

1 mark

ii. Find the exact percentage of time that the marker at B is in the water in one cycle.

3 marks

Due to winds, tides and currents, on some days the waves come further up the beach and are closer together. Keith observes that on such a day the distance of the water's edge from marker A can be described by the equation

$D = (8 + S)\cos\left(\frac{8\pi t}{3}\right) + 11$ where S metres is the seasonal tidal factor which varies with the factors described above. S is normally distributed with a mean of 2 and a standard deviation of 0.3.

e. On a particular day the waves just reach the top of the boat ramp at marker A. Find the value of S on this day.

---

2 marks

f. Marcus decides to go fishing from the boat ramp at Point Roadknight beach, however he does not know the value of S. How many metres up the boat ramp from marker B should he stand so that he has an 80% chance of not getting wet from the waves? Answer correct to two decimal places.

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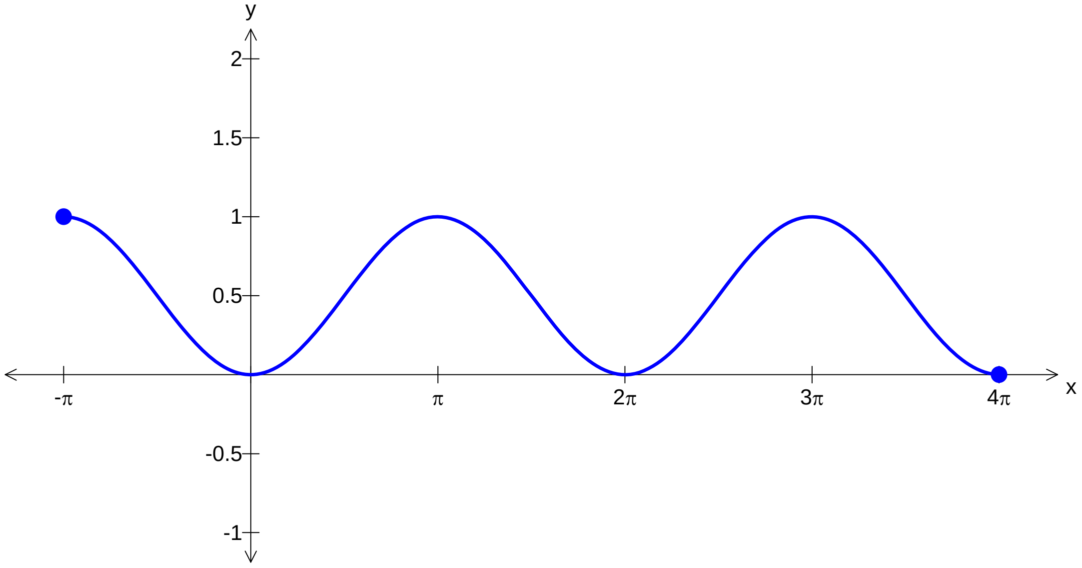
3 marks
Total 16 marks

SECTION 2 – continued

Question 3

The graph of $f:[-\pi, 4\pi] \to R,\ f(x) = \sin^2\left(\frac{x}{2}\right)$ is shown below.



a. **i.** Use calculus to find $f'(\frac{3\pi}{2})$ and $f'(\frac{5\pi}{2})$.

3 marks

ii. Find the exact equations of the **normals** to the curve at the points where $x = \frac{3\pi}{2}$ and $x = \frac{5\pi}{2}$.

4 marks

b. The graph of $y = f(x)$ is transformed to give the graph of $y = f(x) + b$. Find the value of b, correct to 3 decimal places, such that the graph of the normal at $x = \frac{3\pi}{2}$ and the graph of the normal at $x = \frac{5\pi}{2}$ intersect on the *x*-axis.

3 marks

$$\text{Let } g : [-\pi, 4\pi] \to R,\ g(x) = \sin^2\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right) + 1.$$

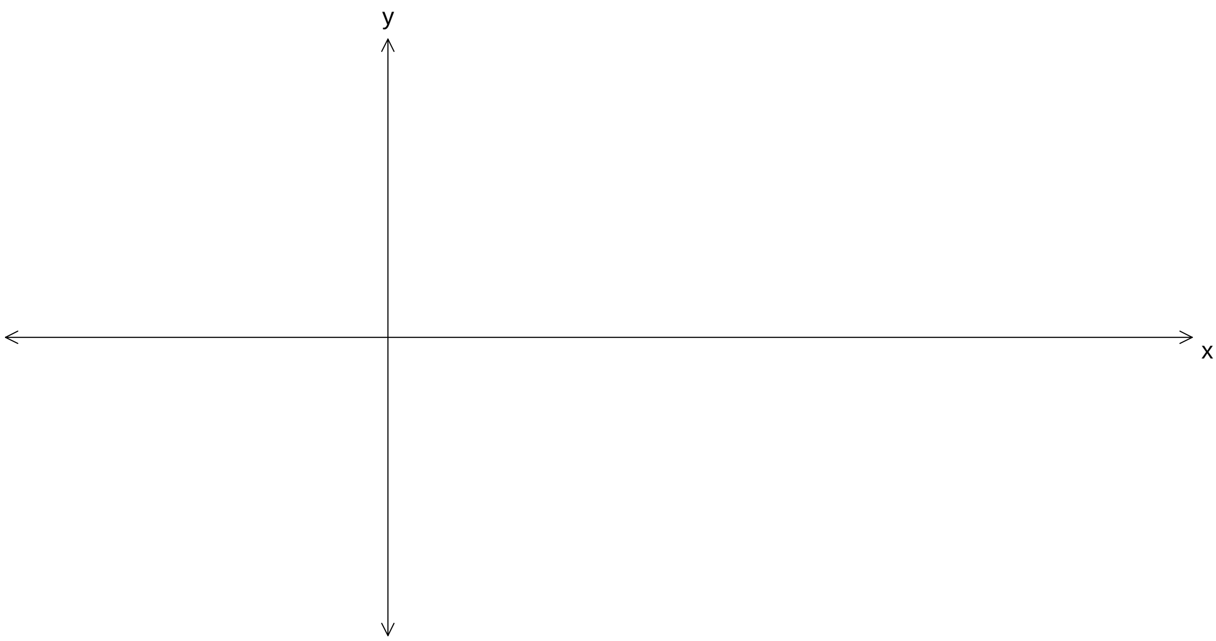
c. **i.** Use calculus to find $g'(x)$.

2 marks

ii. Solve the equation $g'(x) = 0$ for $x \in [-\pi, 4\pi]$. Give exact values.

3 marks

iii. Sketch the graph of $y = g(x)$ on the axes below, labelling intercepts, stationary points and endpoints with exact coordinates.



2 marks
Total 17 marks

Question 4

Let $f : R \to R,\ f(x) = 9e^{-x} - 5$.

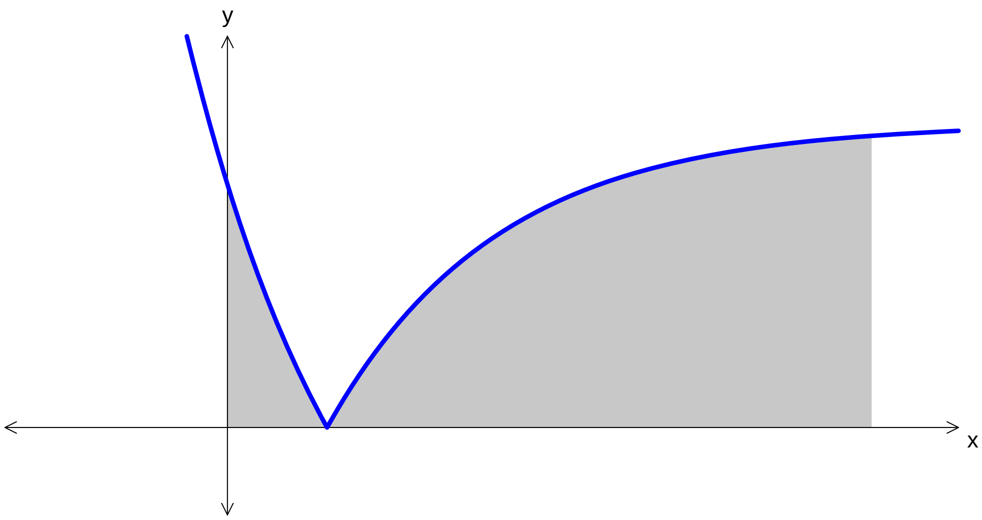
a. **i.** State the range of the function, f.

1 mark

ii. Find the rule and the domain of the inverse function, f^{-1}.

3 marks

A panel of a stained glass window has a section of lead outline described by the graph with the equation $g(x) = |9e^{-x} - 5|$ and is shown below. The shaded area is red-coloured glass.



b. Find the exact value of the x-intercept.

2 marks

c. The red-coloured glass is bounded by the area under the graph, the x-axis, the y-axis and the line $x = 4$. Find the exact area of the red-coloured glass.

4 marks

d. Find the value of a, correct to 3 decimal places, such that the area of the red-coloured glass enclosed by the graph , the x-axis, the y-axis and the line $x = a$ is equal to 35 square units.

3 marks
Total 13 marks

END OF QUESTION AND ANSWER BOOK