INSIGHT

Trial Exam Paper

2009

MATHEMATICAL METHODS/ MATHEMATICAL METHODS (CAS)

Written examination 1

STUDENT NAME:

QUESTION AND ANSWER BOOK

Reading time: 15 minutes
Writing time: 1 hour

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
11	11	40

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring notes of any kind, sheets of paper, white out liquid/tape or a calculator into the examination.

Materials provided

- The question and answer book of 11 pages, with a separate sheet of miscellaneous formulas.
- Working space is provided throughout the question book.

Instructions

- Write your **name** in the box provided.
- Remove the formula sheet during reading time.
- You must answer the questions in English.

Students are NOT permitted to bring mobile phones or any other electronic devices into the examination.

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Instructions

Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working must be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

Solve the following for x.

a. $2\log_9(x-1)+\log_9 3=1$

2 marks

b. $e^{2x}-5e^{x}+4=0$

3 marks
Total 2 + 3 = 5 marks

Question 2

Given $f:[0,\pi] \to R,\ f(x) = |2\cos(2x)+1|$, find

a. the values of x for which $f(x) = 0$.

2 marks

b. the exact value of $f'(x)$ when $x = \frac{\pi}{6}$.

2 marks

c. the interval over which the rate of change is negative.

1 mark

2 + 2 + 1 = 5 marks

Question 3

The graph of the function with the rule $y = x^{\frac{2}{3}}$ is transformed as follows:

- A dilation by a factor of 2 from the y-axis
- A reflection in the x-axis
- A translation of + 4 units parallel to the x-axis
- A translation of + 1 units parallel to the y-axis

a. Write down the equation of the rule of the transformed function.

_____

_____

_____

1 mark

b. State the domain and range of the transformed function.

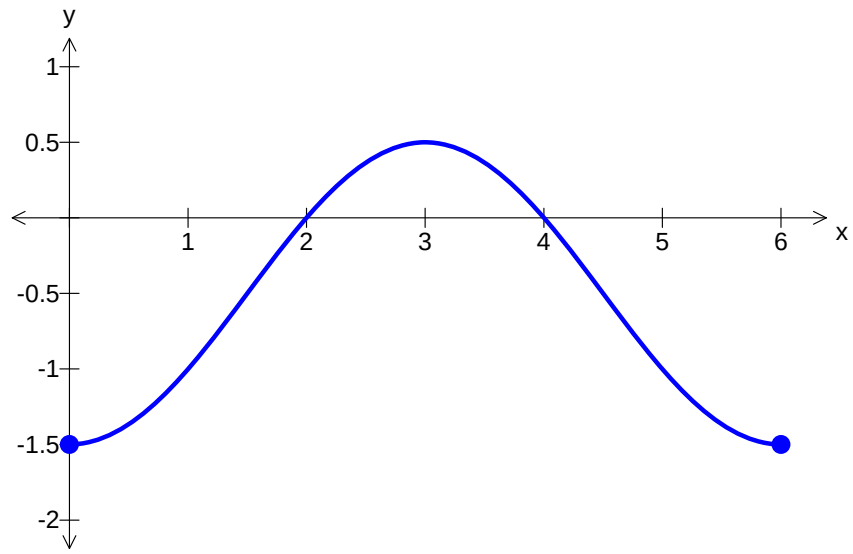
_____

_____

2 marks
Total 1 + 2 = 3 marks

Question 4

The diagram below shows one cycle of the graph of a circular function.



a. State the amplitude of the function.

_____

1 mark

b. State the equation of the function.

_____

_____

_____

_____

2 marks
Total 1 + 2 = 3 marks

Question 5

Let $\int_2^a (e^{2x-4})\,dx = \frac{1}{2}$. Find the exact value of a, where $a > 2$.

3 marks

Question 6

The random variable X is normally distributed with mean 50 and standard deviation 5.

The random variable Z is normally distributed with mean 0 and standard deviation 1.

If $\Pr(Z < -2) = 0.0228$, find

a. $\Pr(X < 40)$

1 mark

b. $\Pr(X < 60 \mid X > 50)$

2 marks

Total 1 + 2 = 3 marks

TURN OVER

Question 7

The life of a battery, in hours, can be modelled by the random variable X with probability density function $f(x) = \begin{cases} \frac{c}{x^2} & if \ \ x > 20 \\ 0 & if \ \ x \leq 20 \end{cases}$

a. Find the value of *c*.

_____

_____

_____

_____

_____

2 marks

b. Find the median life of a battery according to this model.

_____

_____

_____

_____

2 marks
Total 2 + 2 = 4 marks

Question 8

The Roosters and the Swans are rival handball teams. When they play against each other the probability of winning is dependent upon the result of their previous match. If the Roosters have won the previous match, then the probability that they will win the next match is 0.6. If the Swans have won the previous match, then the probability that they will win the next match is 0.7.

These two teams are about to start a "best of three" finals series which is played until either team has won 2 matches. The probability that the Roosters win the first match is 0.6.

a. Draw a tree diagram to show the possible outcomes.

1 mark

b. What is the probability that the Swans win the first two games?

_____

_____

_____

1 mark

c. Find the probability that the Swans win the finals series, i.e. win two games.

_____

_____

_____

_____

_____

_____

2 marks
Total 1 + 1 + 2 = 4 marks

TURN OVER

Question 9

Find the coordinates of the points on the curve $y = 4x^2 + 5$ at which the tangents drawn to the curve pass through the point (0, 4).

4 marks

Question 10

a. Find $\frac{d}{dx}(\log_e(3x^2 + 1))$.

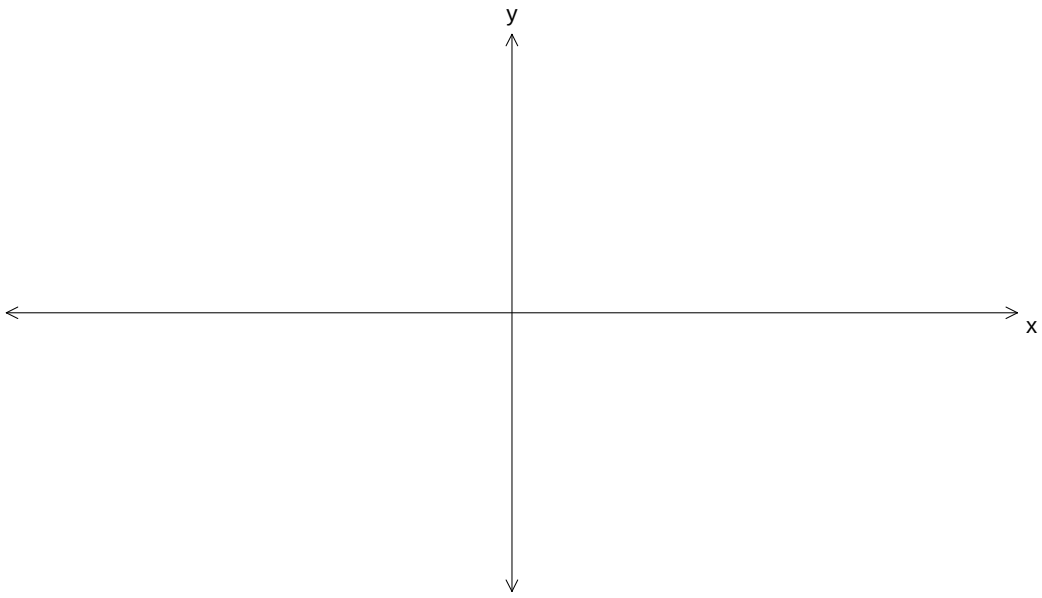
2 marks

b. Hence find $\int_0^4 \frac{x}{3x^2 + 1}\, dx$.

2 marks
Total 2 + 2 = 4 marks

Question 11

On the axes below, sketch the graph of the function with the rule $f(x) = \log_e(|x| - 2)$. Label any asymptotes with the equation and intercepts as coordinates.



2 marks

END OF QUESTION AND ANSWER BOOK