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MATHS METHODS 3 & 4 TRIAL EXAMINATION 2 SOLUTIONS 2009

1. C	9. B	17. E
2. A	10. D	18. A
3. D	11. B	19. D
4. E	12. D	20. B
5. A	13. C	21. E
6. D	14. B	22. E
7. A	15. C	
8. A	16. A	

Section 1 – Multiple-choice solutions

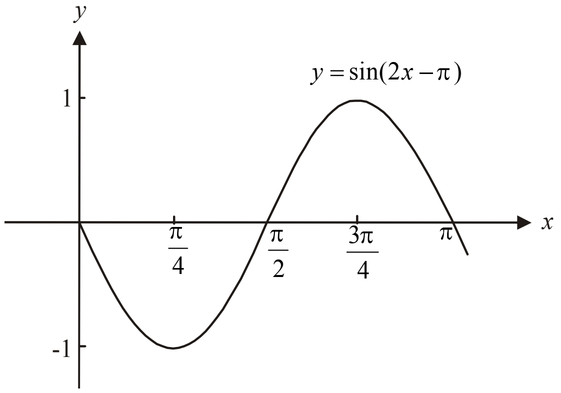
Question 1

$$
\begin{aligned}
E(X) &= \text{mean of } X \\
&= -2\times 0\cdot 3 + -1\times 0\cdot 2 + 0\times 0\cdot 4 + 1\times 0\cdot 1 \\
&= -0\cdot 7
\end{aligned}
$$

The answer is C.

Question 2

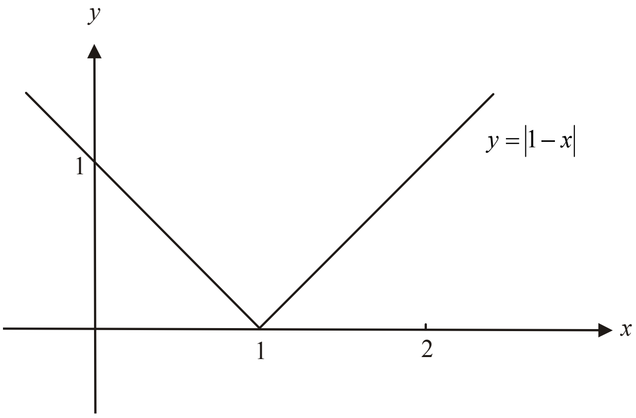
f^{-1} exists if f is a $1:1$ function.
Sketch the graph of $y=\sin(2x-\pi)$ for $x\geq 0$.



If $a=\frac{\pi}{4}$, then f is a $1:1$ function and f^{-1} exists.
The answer is A.

Question 3

Sketch the function $y = |1 - x|$.



The rate of change is the gradient of the function. At $x = 2$, the gradient $= 1$.
The answer is D.

Question 4

$y = \sin(e^{2x})$
This is a composite function so use the chain rule.

Method 1 – fast way
$$y = \sin(e^{2x})$$
$$\frac{dy}{dx} = 2e^{2x}\cos(e^{2x})$$
The answer is E.

Method 2
$$y = \sin(e^{2x}) \quad \text{Let } u = e^{2x}$$
$$= \sin(u) \qquad \frac{du}{dx} = 2e^{2x}$$
$$\frac{dy}{du} = \cos(u)$$
$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} \quad \text{(Chain rule)}$$
$$= \cos(u) \cdot 2e^{2x}$$
$$= 2e^{2x}\cos(e^{2x})$$
The answer is E.

Question 5

This is a binominal distribution with $n = 8$ and $p = 0.1$.

$$\begin{aligned}\Pr(X \geq 2) &= 1 - \Pr(X < 2)\\ &= 1 - \{\Pr(X = 0) + \Pr(X = 1)\}\\ &= 1 - \{{}^8C_0 (0 \cdot 1)^0 (0 \cdot 9)^8 + {}^8C_1 (0 \cdot 1)^1 (0 \cdot 9)^7\}\\ &= 1 - (0 \cdot 430467... + 0 \cdot 382637...)\\ &= 0 \cdot 186896...\\ &= 0 \cdot 1869 \text{ (to four decimal places)}\end{aligned}$$

The answer is A.

Question 6

$$\int \left(\frac{2}{x+1} + \sin(2x) \right) dx$$

$$= 2\log_e |x+1| - \frac{1}{2}\cos(2x) + c$$

The answer is D.

Question 7

$$\begin{aligned}\int_0^4 (2 - 5f(x))dx &= \int_0^4 2dx - 5\int_0^4 f(x)dx\\ &= [2x]_0^4 - 5 \times 3\\ &= 8 - 0 - 15\\ &= -7\end{aligned}$$

The answer is A.

Question 8

Sketch the function.

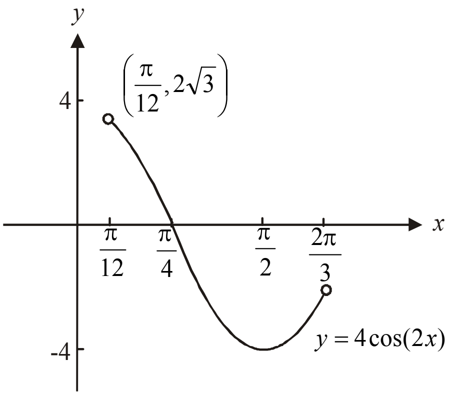
At $x = \frac{\pi}{12}$,

$$4\cos(2x)$$

$$= 4\cos\left(\frac{\pi}{6}\right)$$

$$= 4 \times \frac{\sqrt{3}}{2}$$

$$= 2\sqrt{3}$$



From the diagram,

$r_f = [-4, 2\sqrt{3})$

The answer is A.

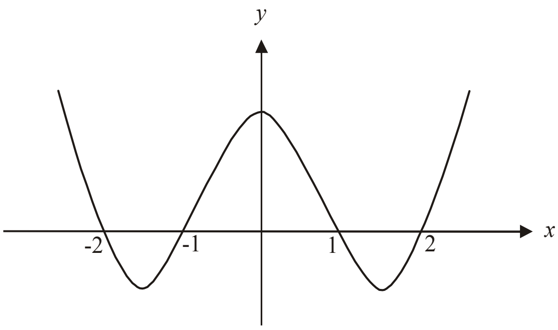
Question 9

$$\Pr(X<5\,|\,X<10)=\frac{\Pr(X<5\cap X<10)}{\Pr(X<10)}$$
$$=\frac{\Pr(X<5)}{0\cdot 5}$$
$$=\frac{0\cdot 00621}{0\cdot 5}$$
$$=0\cdot 01242$$

The answer is B.

Question 10

$$y=x^4-5x^2+4$$
$$=(x^2-4)(x^2-1)$$
$$=(x-2)(x+2)(x-1)(x+1)$$



The graph is symmetrical about the *y*-axis.
Use your calculator to find the minimum turning points.
They occur at $(-1\cdot 58114, -2\cdot 25)$ and, by symmetry at $(1\cdot 58114, -2\cdot 25)$.
The gradient is positive for $x\in(-1\cdot 5811...,0)\cup(1\cdot 5811...,\infty)$
The closest answer is D.

Question 11

$$\text{Average rate of change}=\frac{f(2)-f(1)}{2-1}$$
$$=\log_e(5)-\log_e(3)$$
$$=\log_e\left(\frac{5}{3}\right)$$

The answer is B.

Question 12

Option A is incorrect because h is discontinuous at $x = 1$ and $x = 3$.
Option B is incorrect because h does not exist at $x = 3$.
Option C is incorrect because $h(x) \le 0$ for $x \in [-3, 0]$.
Option D is correct because $h(x)$ exists at $x = 1$.
Option E is incorrect because $h'(1)$ does not exist. This is because the limits for $h'(x)$ from the left and right hand side of $x = 1$ are not equal.
The answer is D.

Question 13

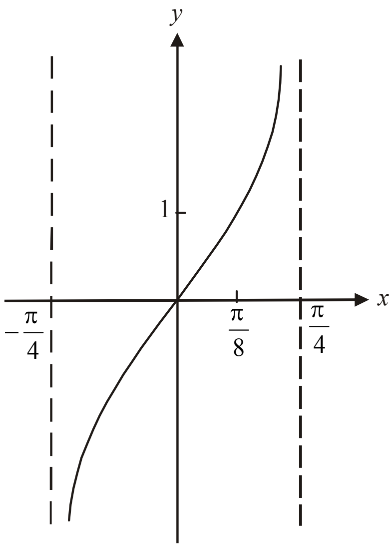
$$e^{2x} - 2e^{x} = 0$$
$$e^{x}(e^{x} - 2) = 0$$
$$e^{x} = 0 \quad \text{or} \quad e^{x} - 2 = 0$$
For $e^{x} = 0$, there is no real solution.
For $e^{x} = 2$
$$x = \log_e(2)$$
The answer is C.

Question 14

$$\frac{dV}{dt} = 5$$
$$V = \frac{4}{3}\pi r^3 \text{ (from formula sheet)}$$
$$\frac{dV}{dr} = 4\pi r^2$$
$$\frac{dr}{dt} = \frac{dr}{dV} \cdot \frac{dV}{dt}$$
$$= \frac{1}{4\pi r^2} \cdot 5$$
$$= \frac{5}{4\pi r^2}$$
The answer is B.

Question 15

Do a quick sketch.



A dilation by a factor of 3 from the x-axis changes the rule $y = \tan(2x)$ to become

$$\frac{y}{3} = \tan(2x)$$

$$y = 3\tan(2x)$$

A reflection in the x-axis changes the rule to

$$-y = 3\tan(2x)$$

$$y = -3\tan(2x)$$

Note that the domain is not affected by these two transformations.
The answer is C.

Question 16

Since we have a probability density function,

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

$$\int_0^1 \frac{x^2}{k}dx = 1$$

$$\frac{1}{k}\left[\frac{x^3}{3}\right]_0^1 = 1$$

$$\left[\frac{x^3}{3}\right]_0^1 = k$$

$$\frac{1}{3} - 0 = k$$

$$k = \frac{1}{3}$$

The answer is A.

Question 17

$$f(x) = \frac{h(x)}{\log_e(2x)}, x > 0$$

$$f'(x) = \frac{\log_e(2x)h'(x) - \frac{1}{x}h(x)}{(\log_e(2x))^2}$$ (Quotient rule)

The answer is E.

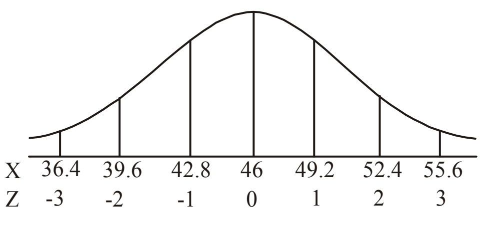
Question 18

Let w = minimum weight required.

$\Pr(X < w) = 0 \cdot 1$

$w = 41 \cdot 899$

The answer is A.



Question 19

$$\int_0^{\frac{\pi}{8}} (\cos(2x) - \sec^2(2x))dx$$

$$= \left[\frac{1}{2}\sin(2x) - \frac{1}{2}\tan(2x)\right]_0^{\frac{\pi}{8}}$$

$$= \frac{1}{2}\left\{\left(\sin\left(\frac{\pi}{4}\right) - \tan\left(\frac{\pi}{4}\right)\right) - ((\sin(0) - \tan(0))\right\}$$

$$= \frac{1}{2}\left\{\left(\frac{1}{\sqrt{2}} - 1\right) - 0\right\}$$

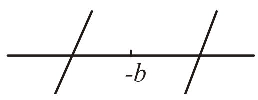
$$= \frac{1}{2}\left(\frac{1}{\sqrt{2}} - 1\right)$$

The answer is D.

Question 20

For a stationary point of inflection to occur $f'(x) = 0$. This only occurs at $x = -b$ and at $x = d$.

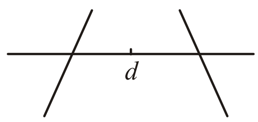
To the left of $x = -b, f'(x) > 0$ and to the right of $x = -b,\ f'(x) > 0$ so we have a stationary point of inflection at $x = -b$.



Note that to the left of $x = d,\ f'(x) > 0$ and to the right of $x = d, f'(x) < 0$

So we have a local maximum at $x = d$.

The answer is B.



Question 21

$f(x) = \frac{1}{\sqrt{x-1}}$, has a maximal domain.
That maximal domain is given by

$$x - 1 > 0$$
$$x > 1$$

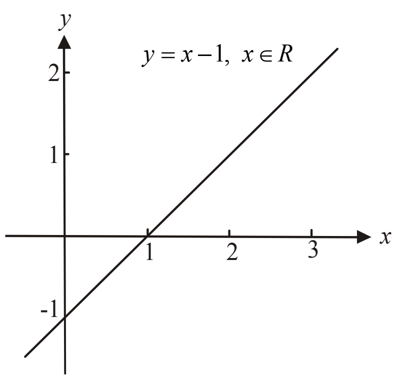
So $d_f = (1, \infty)$.

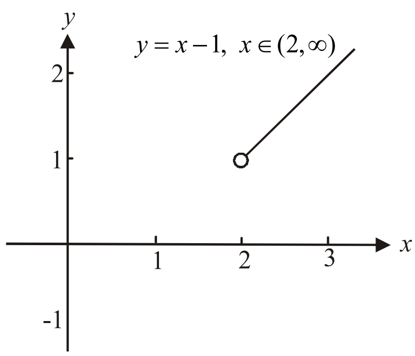
$f(g(x))$ exists iff $r_g \subseteq d_f$

So we require $r_g \subseteq (1, \infty)$

Method 1

The graph of $y = x - 1$ for $x \in R$ is shown in the diagram on the left below.
In order to restrict the range to $(1, \infty)$, we are going to have to restrict the domain to $x \in (2, \infty)$ as shown on the graph on the right below.





Since $r_g \subseteq (1, \infty)$, $d_g \subseteq (2, \infty)$.
So $a \neq -2, -1, 0$ or 1
So $a = 2$ is the only possible answer.
The answer is E.

Method 2

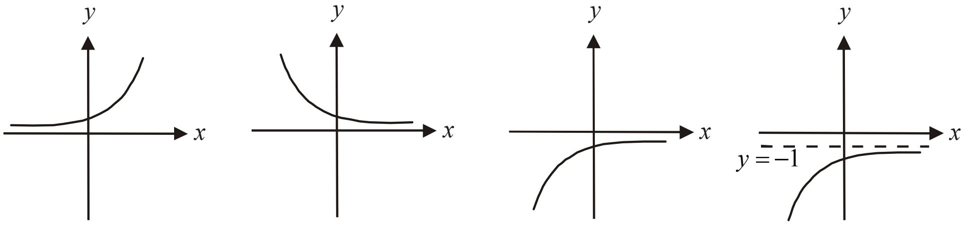
We require $r_g \subseteq (1, \infty)$

$$r_g = (a - 1, \infty)$$
$$a - 1 \geq 1$$
$$a \geq 2$$

So $a = 2$ is the only possible answer.

The answer is E.

Question 22



$y = e^{x}$ $y = e^{-x}$ $y = -e^{-x}$ $y = -e^{-x} - 1$

The only possible graph is E.
The answer is E.

SECTION 2

Question 1

a. The graph of $y = f(x)$ passes through $(0,0)$

so, $0 = e^{a\times 0} - b$

$b = e^0$

$b = 1$ as required

(1 mark)

b. $f(x) = e^{ax} - 1$

$f'(x) = ae^{ax}$ **(1 mark)**

$f'(1) = ae^a$

Since $f'(1) = 2e^2$ (given)

then $a = 2$

(1 mark)

c. i. Note that the equation of the asymptote of $y = e^{2x} - 1$ is $y = -1$.

Method 1

The graph of $y = f(x)$ has undergone a reflection in the x-axis followed by a translation of 1 unit down. **(1 mark)** – reflection **(1 mark)** - translation

Method 2

The graph of $y = f(x)$ has undergone a translation of 1 unit up followed by a reflection in the x-axis. **(1 mark)** – reflection **(1 mark)** – translation

ii. Method 1

After the reflection in the x-axis, the rule $y = e^{2x} - 1$ becomes $-y = e^{2x} - 1$ so $y = 1 - e^{2x}$. **(1 mark)**

After the translation of 1 unit down, $y = 1 - e^{2x}$ becomes

$y = 1 - e^{2x} - 1 = -e^{2x}$. So $g(x) = -e^{2x}$ as required. **(1 mark)**

Method 2

After the translation of 1 unit up, the rule $y = e^{2x} - 1$ becomes $y = e^{2x} - 1 + 1$

so $y = e^{2x}$. **(1 mark)**

After the reflection in the x-axis, $y = e^{2x}$ becomes $-y = e^{2x}$ so $y = -e^{2x}$.

So $g(x) = -e^{2x}$ as required. **(1 mark)**

d. The graphs of $y = f(x)$ and $y = g(x)$ intersect when

$f(x) = g(x)$

$e^{2x} - 1 = -e^{2x}$ **(1 mark)**

$2e^{2x} = 1$

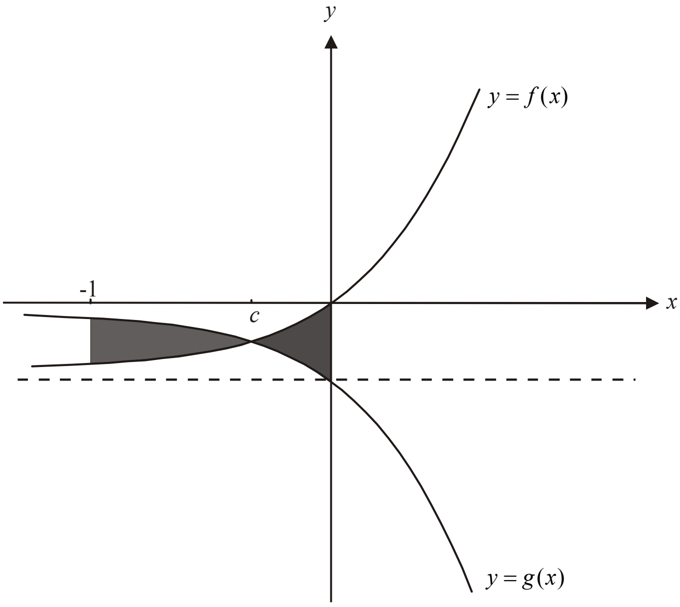
$e^{2x} = \frac{1}{2}$

$\log_e\left(\frac{1}{2}\right) = 2x$

$x = \frac{1}{2}\log_e(2^{-1})$

So $c = -\frac{1}{2}\log_e(2)$ **(1 mark)**

e.



Area required $= \int_{-1}^{c} (g(x) - f(x))dx + \int_{c}^{0} (f(x) - g(x))dx$

OR $\int_{-1}^{-\frac{1}{2}\log_e(2)} (g(x) - f(x))dx + \int_{-\frac{1}{2}\log_e(2)}^{0} (f(x) - g(x))dx$

(1 mark) for first integrand and terminals **(1 mark)** – for second integrand and terminals

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f.

$f(x) + e^x - 1 = 0$

$e^{2x} - 1 + e^x - 1 = 0$

$e^{2x} + e^x - 2 = 0$

Let $a = e^x$

$a^2 + a - 2 = 0$

$(a + 2)(a - 1) = 0$

Sub back

$(e^x + 2)(e^x - 1) = 0$ **(1 mark)**

For $e^x + 2 = 0$

$e^x = -2$

no real solutions exist.

For $e^x - 1 = 0$

$e^x = 1$

$x = 0$

(1 mark)

Total 13 marks

Question 2

a. **i.** Pr(rides on next 5 days)

$= 0 \cdot 6^5$

$= 0 \cdot 0778$ (correct to 4 decimal places)

(1 mark)

ii. This is a binomial distribution with $n = 5, x = 2$ and $p = 0 \cdot 6$.

Pr(rides 2 out of next 5 days)

$= \Pr(X = 2)$

$= {}^5C_2 (0 \cdot 6)^2 (0 \cdot 4)^3$

(1 mark)

$= 0 \cdot 2304$

(1 mark)

b. **i.** $\Pr(wwww) = 0 \cdot 4^4$

$= 0 \cdot 0256$

(1 mark)

ii. $\Pr(wcc) + \Pr(cwc) + \Pr(ccw)$

(1 mark)

$= 0 \cdot 4 \times 0 \cdot 6 \times 0 \cdot 7 + 0 \cdot 6 \times 0 \cdot 3 \times 0 \cdot 6 + 0 \cdot 6 \times 0 \cdot 7 \times 0 \cdot 3$

(1 mark)

$= 0 \cdot 402$

(1 mark)

iii. Method 1

Pr(walked on at least 1 of next 3)

$= 1 - $ Pr(walked zero times out of next 3)

$= 1 - \Pr(ccc)$

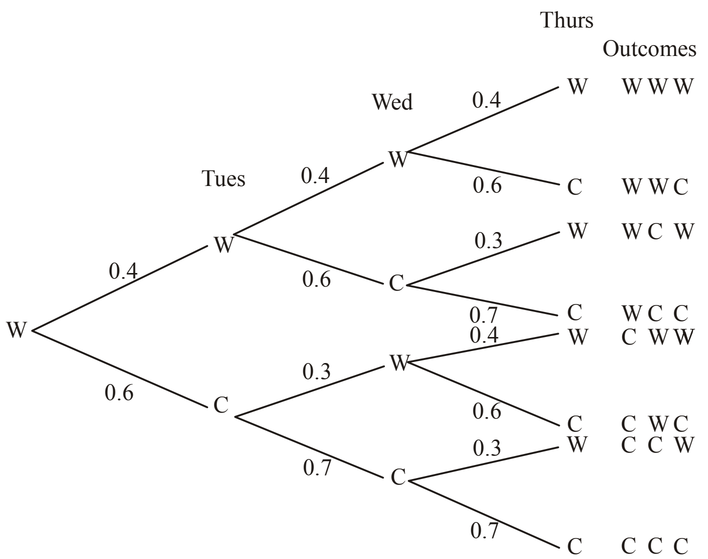
(1 mark)

$= 1 - (0 \cdot 6 \times 0 \cdot 7 \times 0 \cdot 7)$

$= 0 \cdot 706$

(1 mark)

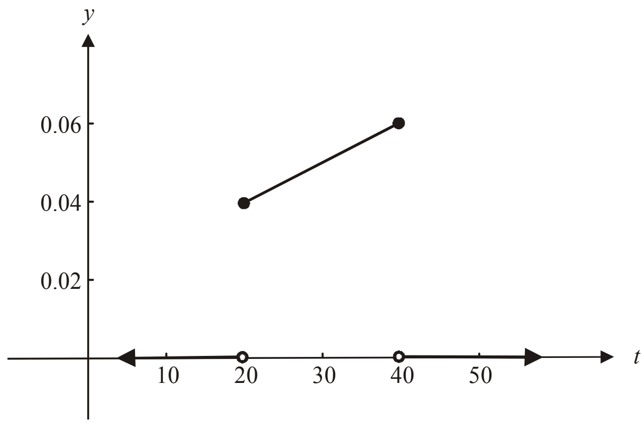
Method 2 - use a tree diagram



Pr(walked on at least 1 of next 3)
$= 1 - \Pr(ccc)$ **(1 mark)**
$= 1 - (0\cdot 6 \times 0\cdot 7 \times 0\cdot 7)$
$= 0\cdot 706$

(1 mark)

c. **i.**



(1 mark) correct linear function and included endpoints for $20 \le t \le 40$
(1 mark) correct marking of function along *t*-axis

ii. From the graph, we see that the mode is 40; that is, the value of t with the highest probability; that is, the highest value of $f(t)$.

(1 mark)

iii. Let $m =$ median

$$\int_{20}^{m} f(t)\,dt = 0\cdot 5$$ **(1 mark)**

$$\int_{20}^{m} \frac{1}{1000}(t+20)dt = 0\cdot 5$$

$$\frac{1}{1000}\left[\frac{t^2}{2} + 20t\right]_{20}^{m} = 0\cdot 5$$ **(1 mark)**

$$\left(\frac{m^2}{2} + 20m\right) - (200 + 400) = 500$$

$$\frac{m^2}{2} + 20m - 1100 = 0$$ **(1 mark)**

$$\left.\begin{array}{l} m = -70\cdot 9902... \\ \text{or } m = 30\cdot 9902... \end{array}\right\}$$

Since $20 \le m \le 40,\ m = 31$ minutes (to the nearest minute).

(1 mark) correct answer
Total 16 marks

Question 3

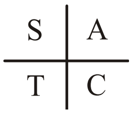
a. $f(x)=\cos(2x)$

$f'(x)=-2\sin(2x)$

(1 mark)

b. The gradient of the tangent is – 1 when

$$f'(x)=-1$$
$$-2\sin(2x)=-1$$
$$\sin(2x)=\frac{1}{2} \qquad x\in\left[0,\frac{\pi}{2}\right]$$



$$2x=\frac{\pi}{6},\frac{5\pi}{6} \qquad 2x\in[0,\pi]$$
$$x=\frac{\pi}{12},\frac{5\pi}{12}$$

$$f\left(\frac{\pi}{12}\right)=\cos\left(\frac{\pi}{6}\right) \qquad f\left(\frac{5\pi}{12}\right)=\cos\left(\frac{5\pi}{6}\right)$$

(1 mark)

$$=\frac{\sqrt{3}}{2} \qquad =-\frac{\sqrt{3}}{2}$$

Required points are $\left(\frac{\pi}{12},\frac{\sqrt{3}}{2}\right)$ and $\left(\frac{5\pi}{12},-\frac{\sqrt{3}}{2}\right)$.

(1 mark)

c. i. Tangent passes through $\left(\frac{\pi+\sqrt{3}}{6},0\right)$ and $\left(0,\frac{\sqrt{3}\pi+3}{6}\right)$.

Method 1

$$m=\frac{y_2-y_1}{x_2-x_1}=\left(\frac{\sqrt{3}\pi+3}{6}-0\right)\div\left(0-\frac{\pi+\sqrt{3}}{6}\right)$$
$$=-\left(\frac{\sqrt{3}\left(\pi+\sqrt{3}\right)}{\pi+\sqrt{3}}\right)$$
$$=-\sqrt{3} \quad \text{as required}$$

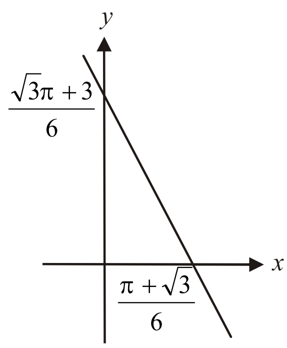
(1 mark)

Method 2

Note that the gradient must be negative since the tangent is sloping up to the left.

Gradient of tangent is given by

$$\frac{\text{rise}}{\text{run}}$$
$$=-\left(\frac{\sqrt{3}\pi+3}{6}\div\frac{\pi+\sqrt{3}}{6}\right)$$
$$=-\left(\frac{\sqrt{3}\pi+3}{6}\times\frac{6}{\pi+\sqrt{3}}\right)$$
$$=-\left(\frac{\sqrt{3}\left(\pi+\sqrt{3}\right)}{\pi+\sqrt{3}}\right)$$
$$=-\sqrt{3} \quad \text{as required}$$



(1 mark)

ii. $f'(x) = -2\sin(2x) = -\sqrt{3}$ $\quad x \in \left[0, \frac{\pi}{2}\right]$ **(1 mark)**

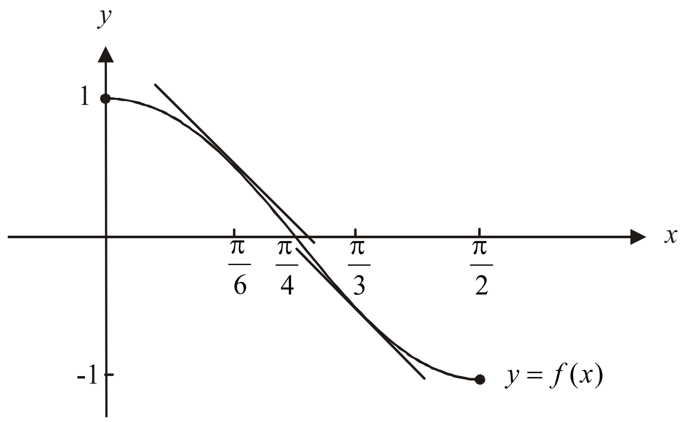
$$\sin(2x) = \frac{\sqrt{3}}{2} \qquad 2x \in [0, \pi]$$

$$2x = \frac{\pi}{3}, \frac{2\pi}{3}$$

$$x = \frac{\pi}{6}, \frac{\pi}{3}$$

(1 mark)

iii.



The x-intercept of the tangent we require is $\frac{\pi + \sqrt{3}}{6} = 0.8122...$

Now $\frac{\pi}{4} = 0.7853...$

From the diagram above, we can see that the tangent we require must pass through the point where $x = \frac{\pi}{6}$. The other possible tangent which passes through the point where $x = \frac{\pi}{3}$, has an x-intercept which is less than $\frac{\pi}{4}$ and therefore less than $\frac{\pi + \sqrt{3}}{6}$. **(1 mark)**

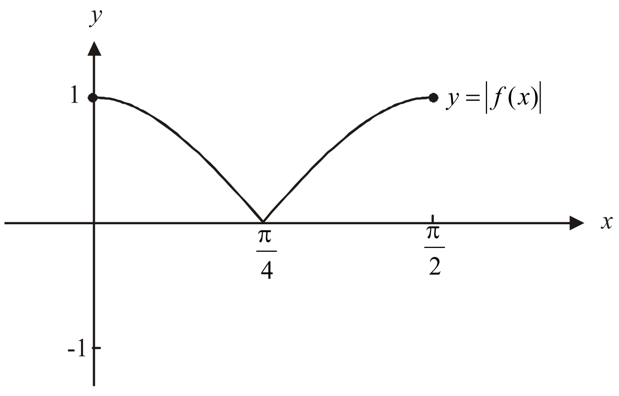
$$f(x) = \cos(2x)$$

$$f\left(\frac{\pi}{6}\right) = \cos\left(\frac{\pi}{3}\right)$$

$$= \frac{1}{2}$$

Point of tangency is $\left(\frac{\pi}{6}, \frac{1}{2}\right)$. **(1 mark)**

d. Do a quick sketch of the graph of $y = |f(x)|$.



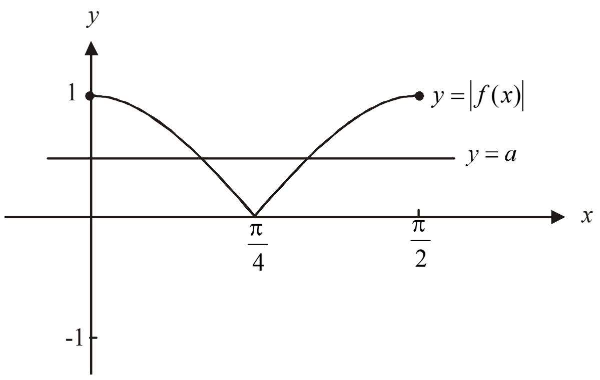
The minimum value of $|f(x)| = 0$. **(1 mark)**

The maximum value of $|f(x)| = 1$. **(1 mark)**

e. $|f(x)| - a = 0$

$|f(x)| = a$

The graphs of $y = |f(x)|$ and $y = a$ are shown below.



i. For exactly one solution we see that $a = 0$.

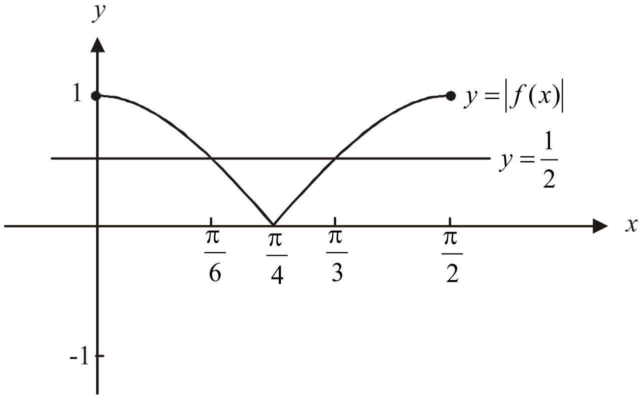
(1 mark)

ii. For at least one solution $a \in [0,1]$.

(1 mark)

f. Method 1

$$|f(x)| = \frac{1}{2}$$
$$\cos(2x) = \frac{1}{2}$$
$$2x = \frac{\pi}{3}$$
$$x = \frac{\pi}{6}$$ **(1 mark)**



From the graphs of $y = |f(x)|$ and $y = \frac{1}{2}$ we see that there is a point of intersection at $x = \frac{\pi}{6}$. The graph of $y = |f(x)|$ is symmetrical about the line $x = \frac{\pi}{4}$.

Now $\frac{\pi}{4} - \frac{\pi}{6} = \frac{3\pi}{12} - \frac{2\pi}{12} = \frac{\pi}{12}$ so there is another point of intersection at the point where $x = \frac{\pi}{4} + \frac{\pi}{12} = \frac{3\pi}{12} + \frac{\pi}{12} = \frac{4\pi}{12} = \frac{\pi}{3}$.

So $f(x) = \frac{1}{2}$ for $x = \frac{\pi}{6}, \frac{\pi}{3}$.

(1 mark)

Method 2

$$|f(x)| = \frac{1}{2}$$
$$f(x) = \pm\frac{1}{2}$$
$$\cos(2x) = \pm\frac{1}{2} \qquad x \in \left[0, \frac{\pi}{2}\right]$$ **(1 mark)**
$$2x = \frac{\pi}{3}, \frac{2\pi}{3} \qquad 2x \in [0, \pi]$$
$$x = \frac{\pi}{6}, \frac{\pi}{3}$$

(1 mark)

Total 14 marks

Question 4

a. Solve $f(x)=0$

$5\log_e(x-10)=0$

$e^0=x-10$

$1=x-10$

$x=11$

So $a=11$ as required.

(1 mark)

b. $f(x)=5\log_e(x-10)$

Let $y=5\log_e(x-10)$

Swap *x* and *y* for inverse.

$x=5\log_e(y-10)$

$\frac{x}{5}=\log_e(y-10)$

$e^{\frac{x}{5}}=y-10$

$y=e^{\frac{x}{5}}+10$

$f^{-1}(x)=e^{\frac{x}{5}}+10$ **(1 mark)** – correct rule

$d_f=[11,50]$

$r_f=[0,f(50)]$

$=[0,\ 18\cdot 44...]$

So $d_{f^{-1}}=r_f$

$=[0,\ 18\cdot 4]$ (correct to 1 decimal place)

(1 mark) – correct domain

c. Since the graph of $y=f^{-1}(x)$ is a reflection of the graph of $y=f(x)$ in the line $y=x$, the floodlit areas north and east are the same.

Total area $=2\int_{11}^{50} f(x)\,dx$ **(1 mark)**

$=10\int_{11}^{50}\log_e(x-10)dx$

$=1086\text{m}^2$ (to nearest square metre)

(1 mark)

d. Find the *x*-coordinates of the points of intersection between $y = 15$ and $y = f(x)$ and between $y = 15$ and $y = f^{-1}(x)$.

Method 1 – by calculator

$y = 15$ and $y = 5\log_e(x - 10)$ intersect when $x = 30 \cdot 09$ (to 2 decimal places)

$y = 15$ and $y = e^{\frac{x}{5}} + 10$ intersect when $x = 8 \cdot 05$ (to 2 decimal places)

So $b \in (8 \cdot 05, 30 \cdot 09)$ or $8 \cdot 05 < b < 30 \cdot 09$.

(1 mark) correct values

(1 mark) correct brackets or inequality signs

Method 2 – by hand

$$15 = 5\log_e(x - 10) \qquad 15 = e^{\frac{x}{5}} + 10$$

$$3 = \log_e(x - 10) \qquad 5 = e^{\frac{x}{5}}$$

$$e^3 = x - 10 \qquad \log_e(5) = \frac{x}{5}$$

$$x = 10 + e^3 \qquad x = 5\log_e(5)$$

$$= 30 \cdot 09 \text{ (to 2 dec. places)} \qquad = 8 \cdot 05 \text{ (to 2 dec. places)}$$

So $b \in (8 \cdot 05, 30 \cdot 09)$ or $8 \cdot 05 < b < 30 \cdot 09$.

(1 mark) correct values

(1 mark) correct bracket or inequality signs

e. **i.**

$$T = \sqrt{x^2 + 2500} + \frac{50 - x}{2}$$

$$\frac{dT}{dx} = \frac{1}{2}(x^2 + 2500)^{-\frac{1}{2}} \times 2x - \frac{1}{2}$$

$$= \frac{x}{\sqrt{x^2 + 2500}} - \frac{1}{2}$$

$$= \frac{2x - \sqrt{x^2 + 2500}}{2\sqrt{x^2 + 2500}}$$ **(1 mark)**

$\frac{dT}{dx} = 0$ for minimum.

So $2x - \sqrt{x^2 + 2500} = 0$ **(1 mark)**

$$2x = \sqrt{x^2 + 2500}$$

$$4x^2 = x^2 + 2500$$

$$3x^2 = 2500$$

$$x = \pm 28 \cdot 8675$$

but $x \geq 0$

So $x = 28 \cdot 87$ (correct to 2 decimal places)

(1 mark)

ii. $T = 68 \cdot 3$ seconds correct to 1 decimal place.

(1 mark)

f. To take the quickest path, Victoria runs to $P(28 \cdot 87, 50)$. The straight line from $O(0,0)$ to $P(28 \cdot 87, 50)$ is given by $y = \frac{50}{28 \cdot 87} x$. **(1 mark)**

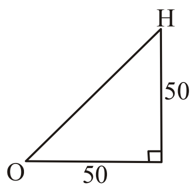
Check whether this intersects with the perimeter of the floodlit area to the north given by $f^{-1}(x) = e^{\frac{x}{5}} + 10$.

Graph the two functions.
They intersect at the point $(10 \cdot 3475..., 17 \cdot 9209...)$ and at the point $(11 \cdot 226152, 19 \cdot 44259...)$ so Victoria does enter the floodlit area.

(1 mark)

g. The dogs have to run $\sqrt{50^2 + 50^2} = 50\sqrt{2}\text{m}$.
They run at 7m/sec.
It takes the dogs

$$50\sqrt{2}\text{m} \div \frac{7\text{m}}{\text{sec}}$$
$$= \frac{50\sqrt{2}}{7}\text{secs}$$
$$= 10 \cdot 1015...\text{secs}$$



to get to *H* from *O*. **(1 mark)**
The dogs leave 60 secs after Victoria so they arrive 70.1015…secs after she leaves $O(0,0)$.
From part **e. ii.,** the shortest time it takes Victoria to get from *O* to *H* is 68.3 seconds (to 1 decimal place).
So Victoria escapes the dogs; but only just!

(1 mark)
Total 15 marks