THE HEFFERNAN GROUP

P.O. Box 1180
Surrey Hills North VIC 3127
Phone 03 9836 5021
Fax 03 9836 5025
info@theheffernangroup.com.au
www.theheffernangroup.com.au

Student Name……………………………………

MATHEMATICAL METHODS UNITS 3 & 4

TRIAL EXAMINATION 1

2009

Reading Time: 15 minutes
Writing time: 1 hour

Instructions to students

This exam consists of 11 questions.
All questions should be answered in the spaces provided.
There is a total of 40 marks available.
The marks allocated to each of the questions are indicated throughout.
Students may **not** bring any calculators or notes into the exam.
Where an exact answer is required a decimal approximation will not be accepted.
Where more than one mark is allocated to a question, appropriate working must be shown.
Diagrams in this trial exam are not drawn to scale.
A formula sheet can be found on page 12 of this exam.

Question 1

Let $f: R \rightarrow R, f(x) = x + 1$ and $g: (0,\infty) \rightarrow R, g(x) = \log_e(2x)$.

a. Write down the rule of $f(g(x))$.

b. Explain why the function $g(f(x))$ does not exist.

$1 + 1 = 2$ marks

Question 2

a. Let $f(x) = x\log_e(x^2 + 5)$. Find $f'(x)$.

b. Let $y = \frac{\tan(x)}{e^{2x}}$. Evaluate $\frac{dy}{dx}$ when $x = 0$.

$2 + 3 = 5$ marks

Question 3

Solve the equation $\sqrt{3}\tan(2x) = 1$ for $x \in [0, 2\pi]$.

2 marks

Question 4

From a particular event space, two events A and B are such that $\Pr(A) = 0 \cdot 3$ and $\Pr(B) = 0 \cdot 4$
.

a. If $\Pr(A \cap B) = 0 \cdot 15$, calculate

i. $\Pr(A' \cap B')$

ii. $\Pr(A|B)$

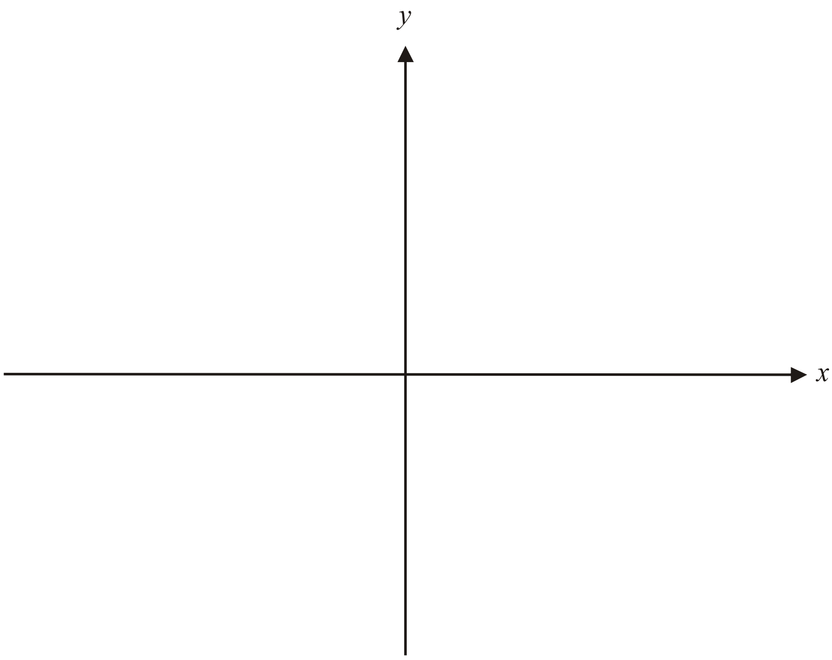
b. If A and B are mutually exclusive events find $\Pr(A|B)$.

$1 + 1 + 1 = 3$ marks

Question 5

Let $f: R \rightarrow R,\ f(x) = \left| x^2 - 6x + 5 \right|$.

a. Sketch the graph of $y = f(x)$ on the set of axes below. Indicate clearly any axes intercepts or turning points.



b. Write down the domain of the derivative function f'.

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c. Write down the values of x for which $f'(x) > 0$.

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2 + 1 + 1 = 4 marks

Question 6

The continuous random variable X has a probability density function given by

$$f(x)=\begin{cases}\dfrac{1}{2\sqrt{x}} & \text{if } x\in[1,4]\\ 0 & \text{otherwise}\end{cases}$$

a. Find $\Pr(X<2)$.

b. Find the mean value of X.

$2+2=4$ marks

Question 7

Let X be a random variable with a normal distribution.
The mean of X is 20 and the standard deviation is 5.
Let Z be a continuous random variable with a standard normal distribution.

a. Find m such that $\Pr(X > 20) = \Pr(Z < m)$.

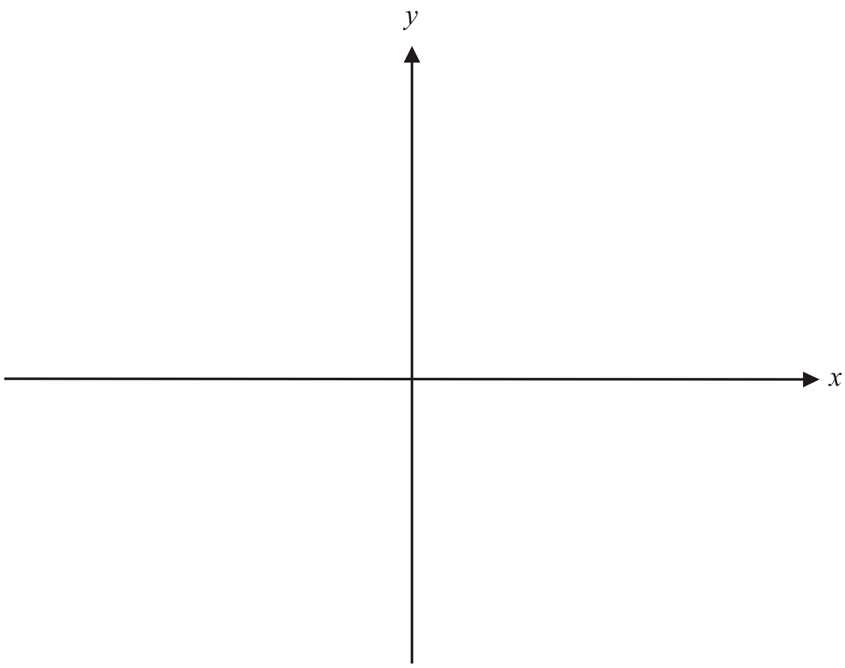
b. Find n such that $\Pr(X < 18) = \Pr(Z > n)$.

$1 + 2 = 3$ marks

Question 8

Let $h:(2,\infty)\to R, h(x)=\dfrac{1}{x-2}+1$.

a. On the axes below, sketch the graph of $y=h(x)$. Label any asymptotes with their equation.

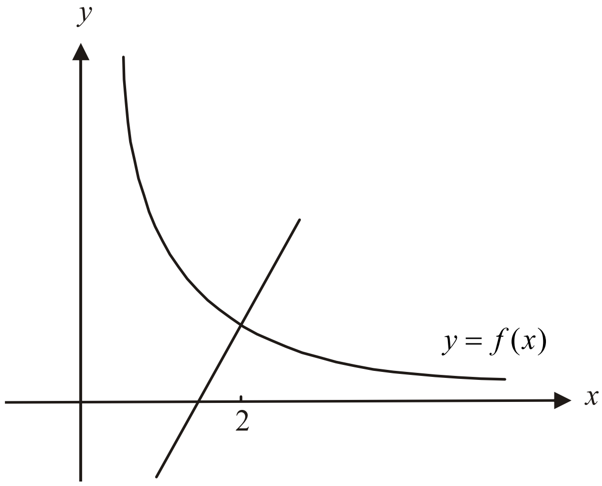


b. Find the rule and the domain of the inverse function h^{-1}.

2 + 2 = 4 marks

Question 9

The graph of $f:(0,\infty)\to R, f(x)=\dfrac{2}{x}$ is shown below. The normal to the graph of *f* at the point where $x=2$ is also shown.



a. Find the equation of the normal to the graph of *f* at the point where $x=2$.

b. Find the area of the region enclosed by the graph of the normal described in part **a.**, the *x*-axis and the line $x=2$.

$2+2=4$ marks

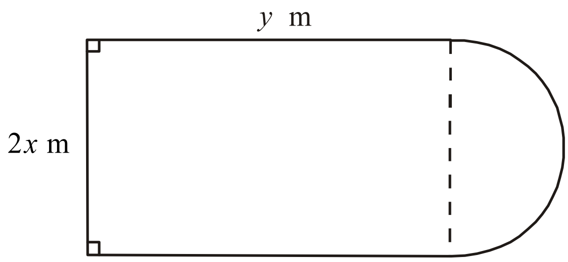
Question 10

Given that $f(x) = x\sqrt{1-x}$ and $f'(x) = \frac{-x}{2\sqrt{1-x}} + \sqrt{1-x}$, find an antiderivative of $\frac{x}{\sqrt{1-x}}$.

3 marks

Question 11

A pool complex is made up of a rectangular swimming pool with side lengths $2x$ m and y m attached to a semi-circular spa of radius x m.



The perimeter of the pool complex is 100m.

a. Express y in terms of x.

b. Show that the surface area of the pool complex is given by

$$A = 100x - \frac{x^2}{2}(\pi + 4).$$

c. Find the value of x for which the surface area of the pool complex is a maximum. It is not necessary to find this maximum surface area.

d. Using the result from part **b.** or otherwise, explain why the value of x found in part **c.** gives a maximum rather than a minimum surface area.

$2+1+2+1=6$ marks

END OF EXAM 1

Mathematical Methods and Mathematical Methods CAS Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$	volume of a pyramid:	$\frac{1}{3}Ah$
curved surface area of a cylinder:	$2\pi rh$	volume of a sphere:	$\frac{4}{3}\pi r^3$
volume of a cylinder:	$\pi r^2 h$	area of a triangle:	$\frac{1}{2}bc\sin A$
volume of a cone:	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$

$\int x^n dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$

$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$

$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$

$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$

$\int \frac{1}{x} dx = \log_e|x| + c$

$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$

$\int \sin(ax)dx = -\frac{1}{a}\cos(ax) + c$

$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$

$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + c$

$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\sec^2(ax)$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

approximation: $f(x+h) \approx f(x) + hf'(x)$

Probability

$\Pr(A) = 1 - \Pr(A')$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A/B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

mean: $\mu = E(X)$

variance: $\text{var}(X) = \sigma^2 = E((X-\mu)^2) = E(X^2) - \mu^2$

probability distribution		mean	variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \Sigma x\, p(x)$	$\sigma^2 = \Sigma (x-\mu)^2 p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)dx$	$\mu = \int_{-\infty}^{\infty} x f(x)dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx$