THE SCHOOL FOR EXCELLENCE (TSFX)
UNIT 4 MATHEMATICAL METHODS 2008

WRITTEN EXAMINATION 2

Reading Time: 15 minutes
Writing time: 2 hours

QUESTION AND ANSWER BOOKLET

Structure of Booklet

Section	Number of questions	Number of questions to be answered	Number of marks
1	22	22	22
2	4	4	58
			Total 80

This examination has two sections: Section 1 (multiple-choice questions) and Section 2 (extended-answer questions).

You must complete both parts in the time allocated. When you have completed one part continue immediately to the other part.

Students are permitted to bring into the examination rooms: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved **graphics** calculator (memory **DOES NOT** need to be cleared) and, if desired, one scientific calculator. For approved computer based CAS, their full functionality may be used.

Students are **NOT** permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Students are **NOT** permitted to bring mobile phones and/or any electronic communication devices into the examination room.

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SECTION 1 – MULTIPLE CHOICE QUESTIONS

Instructions for Section 1

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers. You should attempt every question.

No marks will be given if more than one answer is completed for any question.

QUESTION 1

The average rate of change of the function $f(x) = x^3 - 2x$ with respect to x over the interval $[-2, 0]$ is:

A. –2

B. 2

C. 5

D. –3

E. 6

QUESTION 2

$f(x) = x^3 - 12x$ is a decreasing function for

A. $x < -2$ or $x > 2$

B. $x < -2\sqrt{3}$ or $x > 2\sqrt{3}$

C. $-1 < x < 2$

D. $-2\sqrt{3} < x < 2\sqrt{3}$

E. $-2 \leq x \leq 2$

QUESTION 3

The function $f : D \to R,\ f(x) = 3x - x^2$ has the range $(-18, 2)$.
The domain D could be

A. $[1, 6)$

B. $\left(-\infty, \frac{3}{2}\right]$

C. $\left[\frac{3}{2}, +\infty\right)$

D. $\left[\frac{3}{2}, 6\right)$

E. $(2, 6)$

QUESTION 4

If the two lines $3x - y + 4 = 0$ and $ax + 2y - 3 = 0$ are perpendicular then

A. $a = \frac{2}{3}$

B. $a = -\frac{1}{3}$

C. $a = -\frac{2}{3}$

D. $a = \frac{1}{3}$

E. $a = \frac{1}{6}$

QUESTION 5

The implied domain of the function $y = \sqrt{\frac{x^2 - x - 6}{4 + 3x - x^2}}$ is

A. $\{x : -1 < x < 4\}$

B. $x \in R / \{-1, 4\}$

C. $\{x : 3 \le x \le 4\} \cup \{x : -2 \le x \le -1\}$

D. $\{x : 3 \le x < 4\} \cup \{x : -2 \le x < -1\}$

E. $\{x : 3 < x < 4\} \cup \{x : -2 < x \le -1\}$

QUESTION 6

If $\int_b^a g(x)\,dx = 2$ and $\int_b^a h(x)\,dx = -3$ then $\int_b^a 3g(x) - x - h(x)\,dx$ is equal to

A. $9 + \dfrac{b^2 - a^2}{2}$

B. $9 + \dfrac{a^2 - b^2}{2}$

C. $3 + \dfrac{a^2 - b^2}{2}$

D. $3 + \dfrac{b^2 - a^2}{2}$

E. $3 + b^2 - a^2$

QUESTION 7

The curve with equation $y = ax^2 + bx$ has a gradient of 4 at the point $(1, 2)$.
The values of a and b are

A. $a = -4$ and $b = 2$

B. $a = 2$ and $b = 0$

C. $a = 1$ and $b = 1$

D. $a = -4$ and $b = 6$

E. $a = 0$ and $b = 2$

QUESTION 8

If the point $\left(\log_e 3,\ \dfrac{82\sqrt{3}}{9}\right)$ lies on the curve $y = 3e^{x/a} + e^{-x/a}$ then

A. $a = -2$

B. $a = 2$

C. $a = -\dfrac{2}{3}$

D. $a = \dfrac{2}{3}$

E. $a = \dfrac{1}{2}$

QUESTION 9

The function with rule $f(x) = \begin{cases} (x-a)^3 + 2, & x \le 0 \\ bx + \cos x, & x > 0 \end{cases}$ is differentiable for all values of x if

A. $a = 0$ and $b = 1$

B. $a = 1$ and $b = -3$

C. $a = 1$ and $b = 3$

D. $a = -1$ and $b = -3$

E. $a = -1$ and $b = 3$

QUESTION 10

Let $f:(-2,12) \to R,\ f(x) = \left|(x-5)^2 - 4\right|$. The range of $y = f'(x)$ is

A. $(-2,3) \cup (3,7) \cup (7,12)$

B. $(-14,14)$

C. $(-45,45)$

D. $(-14,4) \cup (4,14)$

E. $(-14,-4) \cup (-4,4) \cup (4,14)$

QUESTION 11

The range of the function $f:(-1,5] \to R,\ f(x) = |2(x+2)(4-x)| - 15$ is

A. $[-5,\ -1]$

B. $[-15,\ 3]$

C. $[-1,\ 3]$

D. $[-15,\ +\infty)$

E. $[-15,\ -1]$

QUESTION 12

Let $y = \sin(\log_e[f(cx)])$ where c is a positive integer. $\frac{dy}{dx}$ is given by

A. $\cos(\log_e[f(cx)])$

B. $\frac{f'(cx)\cos(\log_e[f(cx)])}{f(cx)}$

C. $\frac{f'(cx)}{\cos(f(cx))}$

D. $\frac{cf'(cx)\cos(\log_e[f(cx)])}{f(cx)}$

E. $\frac{\cos(\log_e[f(cx)])}{f'(cx)}$

QUESTION 13

An approximation to $\int_1^2 ax^2 + b\,dx$ using three left rectangles of equal width is

A. $\frac{77a + 27b}{27}$

B. $\frac{4a + 3b}{3}$

C. $\frac{44a + 27b}{27}$

D. $\frac{50a + 27b}{27}$

E. $\frac{43a + 27b}{27}$

QUESTION 14

The function $f(x) = a\sin(x) + b\cos(x)$ will have a maximum turning point at $x = -\frac{\pi}{4}$ if

A. $a = -b$ and $a < 0$

B. $a = b$ and $a < 0$

C. $a = -b$ and $a > 0$

D. $a = b$ and $b > 0$

E. $a = -b$ and $b < 0$

QUESTION 15

Four solutions to $2\cos\left(\frac{x}{a}\right)+\sqrt{3}=0$, where $1<a<3$, are

A. $x=-\frac{11\pi a}{6}$ $x=-\frac{\pi a}{6}$, $x=\frac{\pi a}{6}$ and $x=\frac{23\pi a}{6}$

B. $x=-\frac{7\pi a}{6}$, $x=-\frac{5\pi a}{6}$ $x=\frac{17\pi a}{6}$ and $x=\frac{19\pi a}{6}$

C. $x=-\frac{13\pi a}{6}$ $x=-\frac{11\pi a}{6}$, $x=\frac{\pi a}{6}$ and $x=\frac{23\pi a}{6}$

D. $x=-\frac{19\pi a}{6}$ $x=-\frac{13\pi a}{6}$, $x=\frac{\pi a}{6}$ and $x=\frac{11\pi a}{6}$

E. $x=-\frac{17\pi a}{6}$ $x=-\frac{\pi a}{6}$, $x=\frac{\pi a}{6}$ and $x=\frac{17\pi a}{6}$

QUESTION 16

Let $f(x)=\frac{1}{x}-3$ and $g(x)=-ax$.

The values of a for which the graphs of $y=f(x)$ and $y=g(x)$ have two unique intersection points are

A. $a<\frac{9}{4}$

B. $a>\frac{9}{4}$

C. $a>\frac{4}{9}$

D. $a<\frac{4}{9}$

E. $-\infty<a<0$ and $0<a<\frac{9}{4}$

QUESTION 17

Let X be a normally distributed random variable with mean μ and variance σ^2.
If $\Pr(X < a) = 0\cdot9332$ and $\Pr(X > b) = 0\cdot841345$ then

A. $\mu = \dfrac{2a - 3b}{5}$ and $\sigma = \dfrac{2a + 2b}{5}$

B. $\mu = \dfrac{3a + 2b}{5}$ and $\sigma = \dfrac{2a - 3b}{5}$

C. $\mu = \dfrac{3a + 2b}{5}$ and $\sigma = \dfrac{3a - 2b}{5}$

D. $\mu = \dfrac{3a - 2b}{5}$ and $\sigma = \dfrac{2a + 2b}{5}$

E. $\mu = \dfrac{2a + 3b}{5}$ and $\sigma = \dfrac{2a - 2b}{5}$

QUESTION 18

If a random variable *X* has probability density function $f(x) = \begin{cases} ax^3 & \text{if } x \in [0, b] \\ 0 & \text{otherwise} \end{cases}$

and $E(X^2) = \dfrac{3}{8}$ then

A. $a = \dfrac{1024}{625}$ and $b = \dfrac{5}{4}$

B. $a = \dfrac{81}{4}$ and $b = \dfrac{2}{3}$

C. $a = \dfrac{64}{81}$ and $b = \dfrac{3}{2}$

D. $a = \dfrac{625}{64}$ and $b = \dfrac{4}{5}$

E. $a = \dfrac{1024}{81}$ and $b = \dfrac{3}{4}$

QUESTION 19

The amount of kitty litter in a bag is a normally distributed random variable with mean 1 kg and standard deviation 0.1 kg.

In a box containing 40 bags of kitty litter the expected number of bags that have more than 0.95 kg of kitty litter is, correct to the nearest whole number,

A. 29

B. 28

C. 27

D. 26

E. 25

QUESTION 20

A discrete random variable X has a mean of 4 and a standard deviation of 3. The value of $E(X^2)$ is closest to

A. 25

B. 19

C. 16

D. 9

E. 7

QUESTION 21

Consider the following probability distribution table:

x	0	1	2	3	4
$\Pr(X = x)$	k/2	k/4	k	k/4	k/2

The standard deviation of X is closest in value to

A. 1.23

B. 1.34

C. 1.4

D. 1.62

E. 1.8

QUESTION 22

Suppose that the probability of rain on any given day is conditional on whether or not it rained on the preceding day. The probability that it will rain on a particular day given that it rained on the preceding day is 0.55, and the probability that it will rain on a particular day given that it did not rain on the preceding day is 0.31. If it rains on Thursday, the probability that it will not rain on Saturday is closest to

A. 0.5580

B. 0.4420

C. 0.3025

D. 0.1705

E. 0.1395

SECTION 2 – EXTENDED ANSWER QUESTIONS

Instructions For Section 2

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

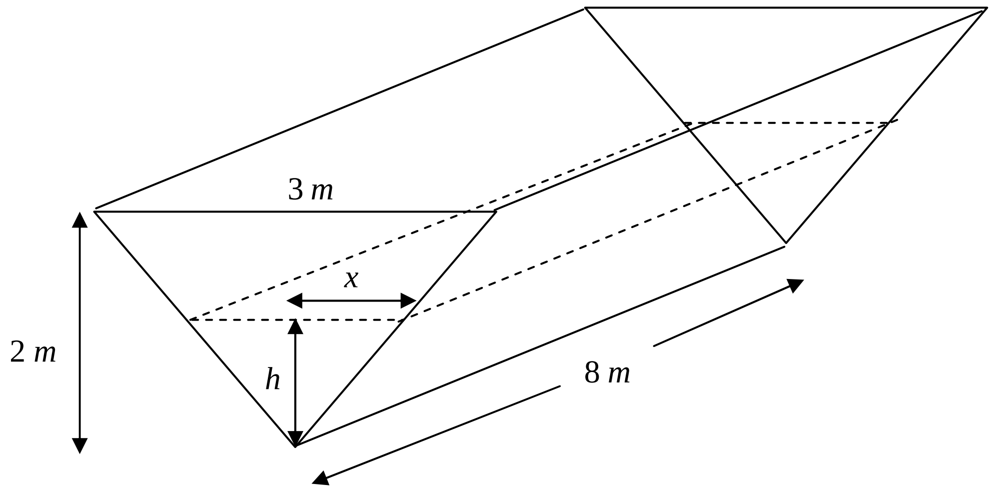
In questions where more than one mark is available, appropriate working **must** be shown.

Where an instruction to **use calculus** is stated for a question, you must show an appropriate derivative or anti-derivative.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

QUESTION 1

The diagram below shows a trough, open at the top, and that is supported by vertical beams that allow the trough to stand on its vertex on the ground.



The trough is used as a water reservoir for farm animals and consists of an isosceles triangular cross section. The trough is initially filled with water from the dam at a rate of $0.05\ m^3$ per minute and topped up periodically with dam water if rainfalls are insufficient.

a. Find an expression for the volume of water in the trough at any time t minutes in terms of h.

2 marks

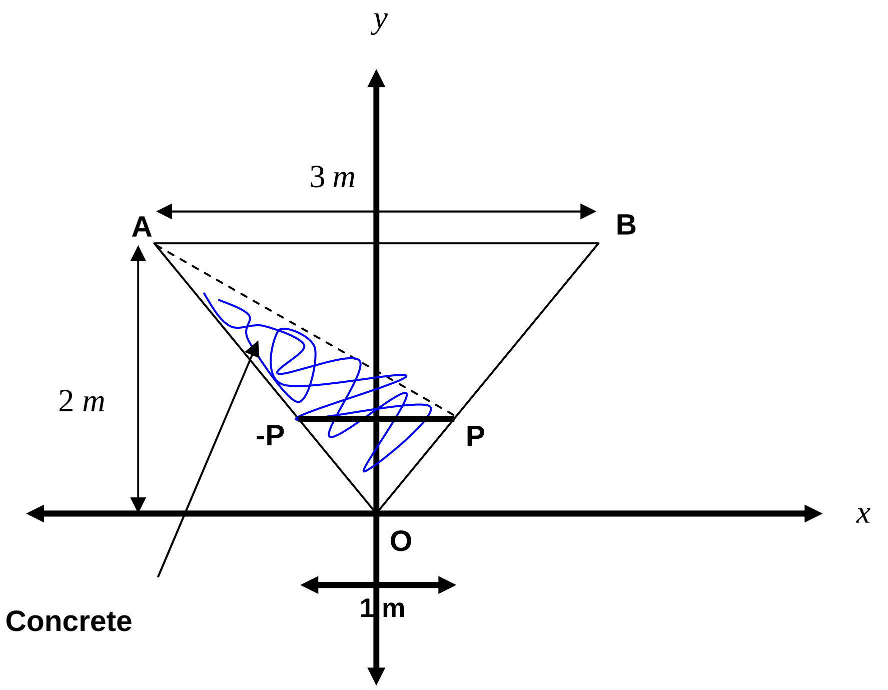
b. (i) Find an expression in terms of h for the rate at which the height of water in the trough is increasing with respect to time.

2 marks

(ii) Hence find the rate in $m/\min$ at which the water level is rising when the trough is half full.

2 marks

Some hooligans decide to pour concrete into the trough, markedly reducing the capacity of the reservoir.



c. (i) Using the x- and y-axes as shown, find the equation of the curve AOB, stating the implied domain.

2 marks

(ii) Find the exact coordinates of point P on the graph.

1 mark

d. (i) Use calculus to find the exact cross-sectional area of concrete in the xy-plane.

3 marks

(ii) Hence find the volume of concrete that was poured into the trough.

1 mark

Total 13 marks

QUESTION 2
'Targette,' a new chain of department stores has fitted its latest retail shop with 8 cash registers. In order to save some money, the executives purchased the registers from **Shone-Key Electricals**; a bargain basement distributor.

If three of every five registers distributed by **Shone-Key Electricals** is not working effectively:

a. Find the expected number of faulty registers in the new store.

1 mark

b. (i) Find the probability that five registers in the new store are faulty. State your answer to 3 decimal places.

1 mark

`(ii) Find the probability that at least two registers in the store are faulty. State your answer to 3 decimal places.

1 mark

(iii) It is known that on any given day at least 2 registers at a Targette store have faults. What is the probability that 5 registers are faulty on a particular day? Give your answer correct to 3 decimal places.

1 mark

c. Find the smallest number of registers that need to be installed in each store so that the probability that at least one register is faulty is at least 0.95

3 marks

d. The Greenville Targette store demands that faulty registers are exchanged immediately. If a faulty register results in a loss of $800 and a working register brings in a profit of $1300, find the expected profit per register for the **Shone-Key** Electrical store.

1 mark

A continuous random variable X has a probability distribution function given by

$$f(x) = \begin{cases} \frac{1}{2} - \frac{1}{4}|x-1| & \text{if } -1 \le x \le 3 \\ 0 & \text{otherwise} \end{cases}$$

e. (i) Find the exact value of a if $\Pr(X \ge a) = \frac{3}{4}$.

2 marks

(ii) Find the variance of X .

3 marks

Another continuous random variable Y has a probability distribution function given by

$$g(y) = \begin{cases} kf(2y-1) & \text{if } \alpha \le y \le \beta \\ 0 & \text{otherwise} \end{cases}$$

f. (i) Find the smallest possible exact value of α and the largest possible exact value of β.

2 marks

(ii) Hence find the exact value of k.

2 marks

Total 17 marks

QUESTION 3

The temperature, T degrees Celsius, of an indoor swimming pool in a gymnasium is modelled by the formula $T_1 = 20 - 4\cos\left(\frac{\pi(t-1)}{12}\right)$, where t is the number if hours after midnight on 31 December 2007.

a. State the maximum and minimum temperatures of the swimming pool.

1 mark

b. (i) Using algebra, show that the first time that the temperature reaches its maximum value is at 1pm.

2 marks

(ii) Hence state a generalised equation to describe the times at which the temperature is at a maximum for $[0, \infty)$.

1 mark

The temperature, T degrees Celsius, of an indoor swimming pool may also be modelled by the formula $T_2 = A + B\sin(Ct + D)$.

Given that $T_2 - T_1 = 0$:

c. (i) Determine the values of A, B and C.

1 mark

(ii) State the smallest positive value of D.

1 mark

The equation $T_3 = \frac{8}{\sqrt{3}}\cos\left(\frac{\pi(t-1)}{12}\right)\sin\left(\frac{\pi(t-1)}{12}\right) + 20$ models the temperature of a second pool in the same gymnasium.

d. Use algebra to find the first two times that the temperatures of the two swimming pools are equal. State your answers as exact values.

3 marks

e. (i) Given $f:(0,1] \to R$ where $f(t) = \log_e t$ and

$g: S \to R$ where $g(t) = \cos\left(\frac{\pi(t-1)}{12}\right)$, find the largest possible subset S of R such that $f\left[g(t)\right]$ is defined.

2 marks

(ii) Hence state the equation for $f\left[g(t)\right]$ stating the corresponding domain.

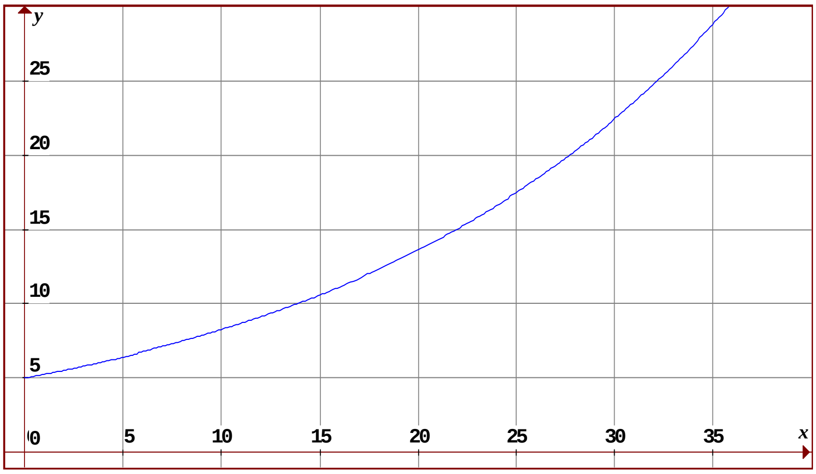
2 marks

Total = 13 Marks

QUESTION 4
The amount of fuel consumed by a steam ship increases exponentially but is modified by an idling factor that operates when the speed of the ship lies between 5 and 60 km/hr.

A model for the amount of fuel consumed (y tonnes) travelling at a speed of $x \ \ km/hr$ is given by the equation $y = y_1 + y_2$ where $y_1 = ae^{kx}$ and $y_2 = \dfrac{b}{x+c}$ and $x \geq 0$.

a. The graph of $y_1 = ae^{kx}$ is shown below.



(i) Find the exact value of a

____

____

1 mark

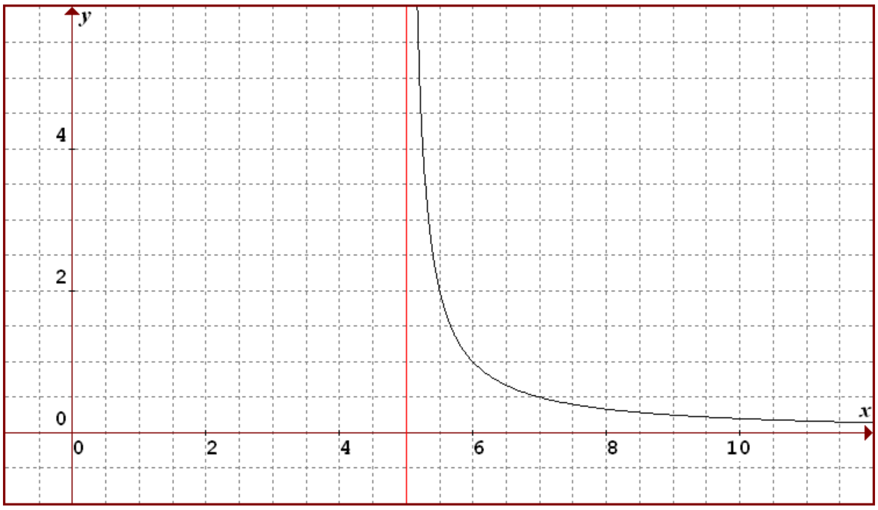
(ii) Find the value of k , correct to 3 decimal places.

____

____

1 mark

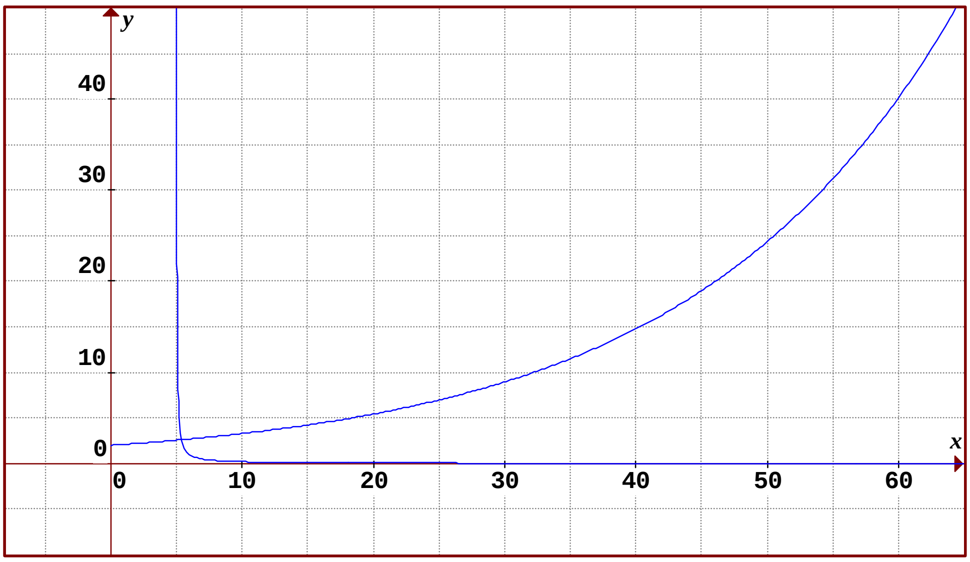
b. The idling factor (y_2) is modelled by the equation $y_2 = \frac{b}{x+c}$ where b and c are constants and $b > 0$. The constant c represents a factor that is common to all steam ships. The value of b varies from ship to ship.



(i) Use the graph above to determine the value of c.

1 mark

(ii) The figure below shows the graphs of $y_1 = ae^{kx}$ and $y_2 = \frac{b}{x+c}$ for a particular steamship. On the axes provided, sketch the graph describing the amount of fuel consumed by the steamship and state the domain.



2 marks

c. (i) Find in terms of b, an equation that describes the rate of fuel consumption with respect to speed.

2 marks

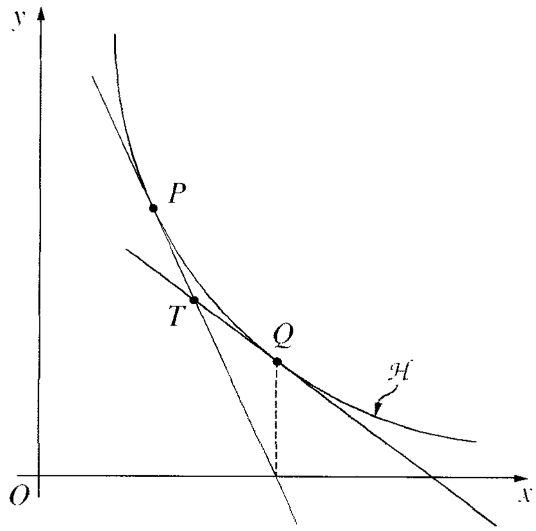
(ii) Find, correct to two decimal places, the smallest and largest values of b so that the minimum fuel consumption of a steamship occurs for speeds lying between 6 km/hr and 14 km/hr.

3 marks

Developers at **Great Steam Ships Incorporated** are investigating different engines and fuels that could improve the efficiency and performance of their steam ships.

The basic model for these engines is given by $xy = c^2$, where c represents an arbitrary real number constant that varies from engine to engine and y represents the amount of fuel consumed (in tonnes) when a ship is travelling at a speed of x km/hr.

The graph of $xy = c^2$ is given below.



The points $P\left(cp, \frac{c}{p}\right)$ and $Q\left(cq, \frac{c}{q}\right)$ lie on the curve $xy = c^2$ and the tangents to the curve at P and Q meet at the point T.

The tangents to the curve $xy = c^2$ provide researchers with valuable information regarding the efficiency of the engine being investigated, as well as the likely modifications that may be required.

d. (i) Use calculus to show that the equation of the tangent at P is $x + p^2 y = 2cp$.

3 marks

(ii) Hence or otherwise, find the equation of the tangent at Q.

1 mark

(iii) Hence show that the point T has coordinates $\left(\frac{2cpq}{p+q}, \frac{2c}{p+q}\right)$.

3 marks

Total 17 marks

End of Paper