Neap:

Trial Examination 2008

VCE Mathematical Methods Units 3 & 4

Written Examination 1

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 1 hour

Student's Name: _____

Teacher's Name: _____

Structure of Booklet

Number of questions	Number of questions to be answered	Number of marks
12	12	40

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

Materials supplied

Question and answer booklet of 10 pages, with a detachable sheet of miscellaneous formulas in the centrefold.
Working space is provided throughout the booklet.

Instructions

Detach the formula sheet from the centre of this booklet during reading time.
Write your **name** and **teacher's name** in the space provided above on this page.
All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2008 VCE Mathematical Methods Units 3 & 4 Written Examination 1.

Instructions

Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working must be shown.
Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

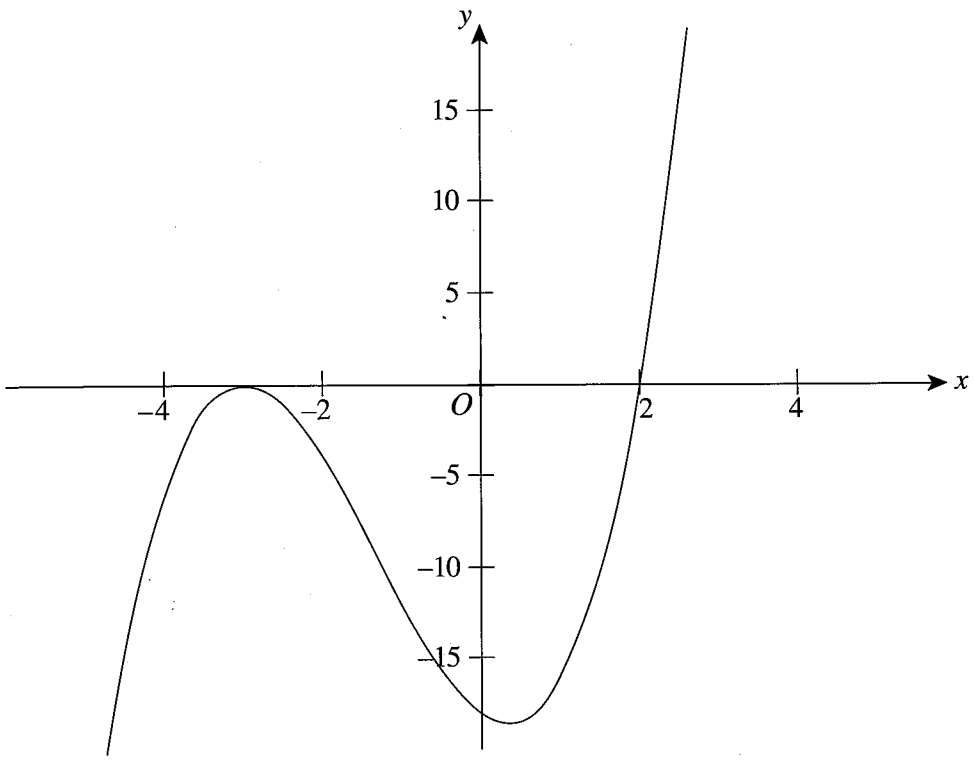
Question 1

Let $f(x) = \sin^2(x^2)$. Find $f'(x)$.

2 marks

Question 2

The graph of $f: R \to R,\ f(x) = x^3 - ax^2 - 3x - 18$ where $a \in R,$ is shown below.



The x-axis is tangential to the graph at $(-3, 0)$ and there is an x intercept at $(2, 0)$.

a. Find the value of a.

1 mark

b. Find $\{x: f(1-x) \geq 0\}$.

2 marks

Question 3

The equation $e^{-2x} - 6e^{-x} + 8 = 0$ has 2 real roots. Express the sum of these roots in the form $\log_e\left(\frac{a}{b}\right)$, where a and b are positive integers with no common factor.

3 marks

Question 4

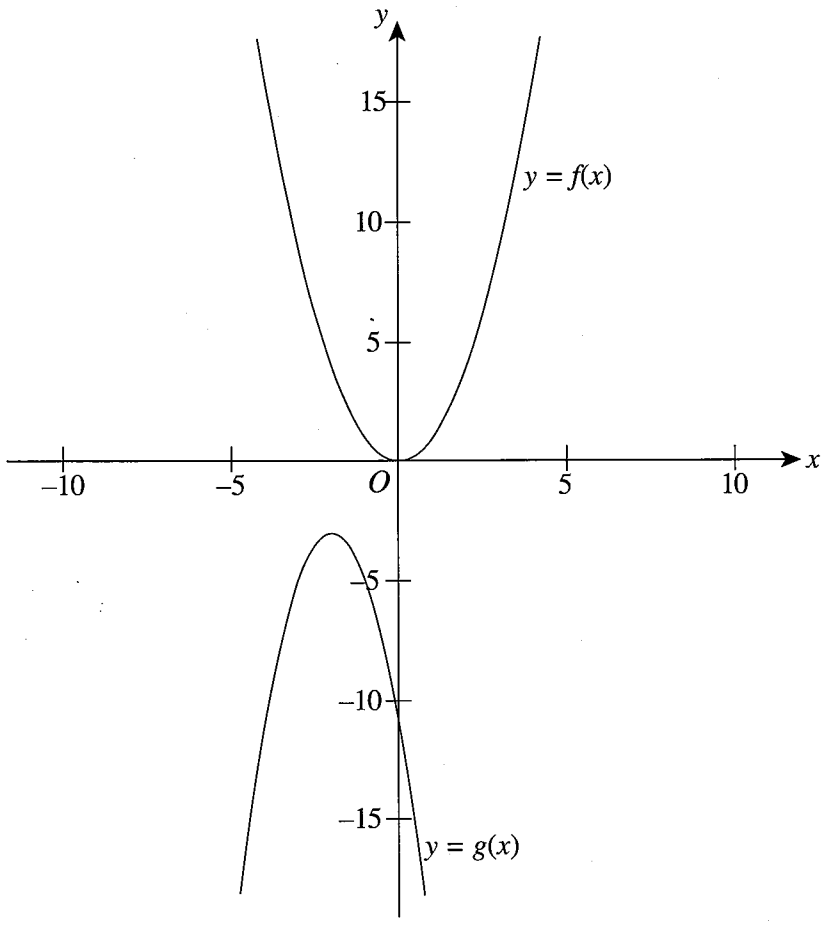
Let $f: [0, \infty) \rightarrow R, f(x) = \sin\left(\frac{3\pi x}{5}\right)$.

Find the minimum gradient of the graph of f and the least value of x for which this occurs.

2 marks

Question 5

The graphs of $y = f(x)$ and $y = g(x)$, where $f(x) = x^2$ and $g(x) = -2x^2 - 8x - 11$, are shown below.



a. Show that the coordinates of the maximum turning point on the graph of $y = g(x)$ are $(-2, -3)$.

2 marks

b. The graph of $y = g(x)$ can be obtained from the graph of $y = f(x)$ by a sequence of transformations. Fully describe such a sequence of transformations in the correct order.

2 marks

Question 6

Let $f(x) = \tan(x)$ and $g(x) = 2\left(x + \frac{\pi}{2}\right)$.

a. Calculate the exact value of $f\left(g\left(\frac{\pi}{3}\right)\right)$.

1 mark

b. Solve $f(g(x)) = -1$ for $0 \leq x \leq 2\pi$.

2 marks

Question 7

The volume, V cm^3, of a particular tyre tube is given by the formula $V = 2\pi^2(30r^2 + r^3)$ where r cm is the cross-sectional radius of the tyre tube. If the tyre tube is being filled with air at a rate of 20 cm^3/s, find the **exact** rate at which the radius is increasing when the radius is 2 cm.

3 marks

Question 8

If $f(x) = x^2 \log_e(2x)$ then $f'(x) = 2x\log_e(2x) + x$.

Use this fact to find an antiderivative of $x\log_e(2x)$.

2 marks

Question 9

A discrete random variable X may take values 0, 1, 2 and 3. The probability distribution of X is given by

$$\Pr(X = x) = a(x - 4)(1 + 2x), \text{ where } a \in R.$$

a. Find the value of a.

2 marks

b. Find the probability of X exceeding its mean.

3 marks

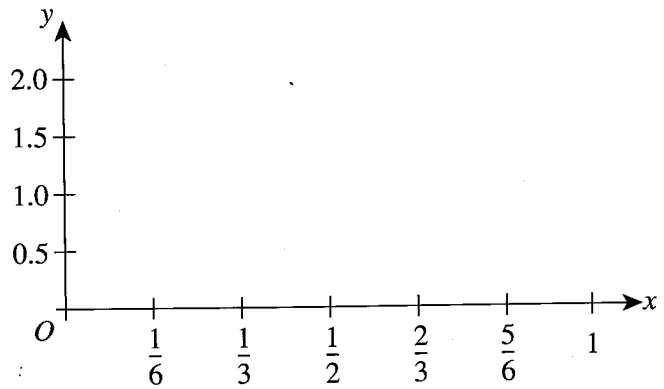
Question 10

The continuous random variable X has the probability function given by

$$f(x) = \begin{cases} 12x^2(1-x) & \text{for } 0 \le x \le 1 \\ 0 & \text{elsewhere} \end{cases}$$

The mode of X is $\frac{2}{3}$.

a. On the axes below sketch the graph of $y = f(x)$, for $0 \le x \le 1$.



1 mark

b. Find $\Pr\left(\frac{1}{2} \le X \le 1\right)$.

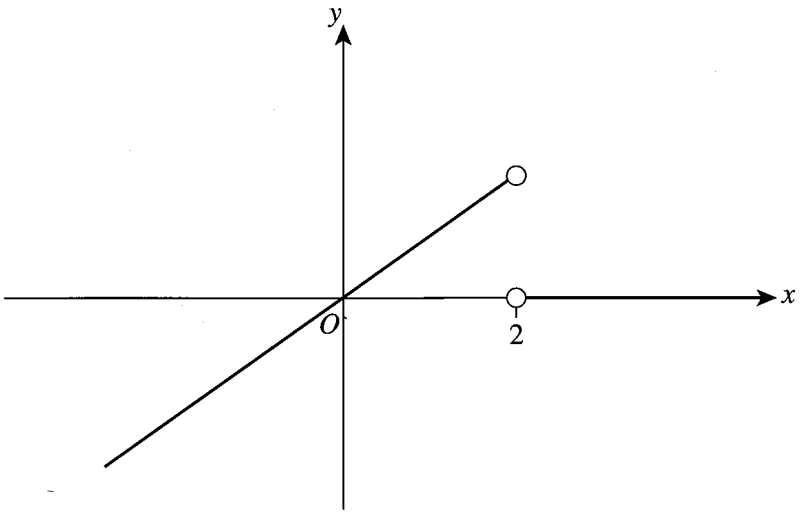
2 marks

c. Let X have median m. Write, **but do not attempt to solve**, an equation involving an integral from which the value of m can be obtained.

1 mark

Question 11

The function f, which is continuous on R, has derivative function f'. The graph of $y = f'(x)$ is shown below.



a. Given that $f(0) = -2$, sketch on the same set of axes the graph of $y = f(x)$.

3 marks

b. Write down the range of f.

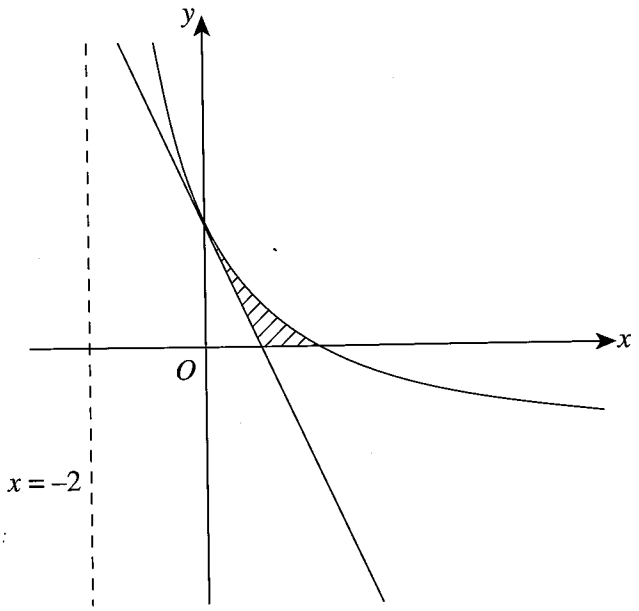
_____

_____

1 mark

Question 12

The graph of f: $(-2, \infty) \rightarrow R, f(x) = \frac{12}{x+2} - 3$ is shown below. The tangent to the graph of f at the point where the graph crosses the y-axis is also shown.



a. Find the equation of this tangent.

2 marks

b. Find the exact area of the shaded region shown in the diagram above.

3 marks

END OF QUESTION AND ANSWER BOOKLET