The Mathematical Association of Victoria

MATHEMATICAL METHODS (CAS)

Trial written examination 2

2008

Reading time: **15 minutes**
Writing time: **2 hours**

Student's Name: ....

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	4	4	58
			Total 80

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

These questions have been written and published to assist students in their preparations for the 2008 Mathematical Methods (CAS) Examination 2. The questions and associated answers and solutions do not necessarily reflect the views of the Victorian Curriculum and Assessment Authority. The Association gratefully acknowledges the permission of the Authority to reproduce the formula sheet.

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SECTION 1

Instructions for Section 1

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.

Question 1

The equation of the cubic function for which the graph passes through the points (0, 1), $(1, \frac{4}{3})$, $(-1, -\frac{4}{3})$ and $(2, \frac{5}{3})$ is

A $y = 0.33x^3 - x^2 + x + 1$

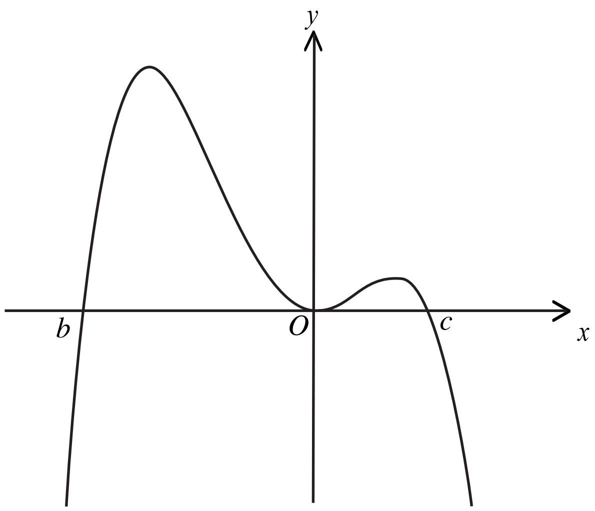
B. $y = 0.3x^3 - x^2 + x + 1$

C. $y = x^3 - 3x^2 + 3x + 3$

D. $3y = x^3 - 3x^2 + 3x + 3$

E. $y = 0.33x^3 + x^2 + x + 1$

Question 2



Part of the graph of $y = h(x)$ is shown above. If a, b and c are real constants and $a > 0$ which one of the following could be the rule for h?

A. $h(x) = ax(x-b)(x-c)$

B. $h(x) = -ax(x+b)(x-c)$

C. $h(x) = ax^2(x+b)(x-c)$

D. $h(x) = -ax^2(x+b)(x-c)$

E. $h(x) = -ax^2(x-b)(x-c)$

Question 3

If $f(x) = e^{3x}$ and $g(x) = \log_e|x|$ then the rule, domain and range of $f(g(x))$ are respectively

A. $f(g(x)) = |x^3|$, $R\backslash\{0\}$, R^+

B. $f(g(x)) = |x^3|$, $R\backslash\{0\}$, $R^+ \cup \{0\}$

C. $f(g(x)) = 3x$, R, R

D. $f(g(x)) = |x^3|$, R^+, R^+

E. $f(g(x)) = |x^3|$, R^+, $R\backslash\{0\}$

Question 4

If $f : (-\infty, 3] \to R$ where $f(x) = (x-3)^{\frac{2}{3}} + 1$ then f^{-1} is

A. $f^{-1} : R \to R$, where $f^{-1}(x) = (x-1)^{\frac{3}{2}} + 3$

B. $f^{-1} : [1, \infty) \to R$, where $f^{-1}(x) = (x-1)^{\frac{3}{2}} + 3$

C. $f^{-1} : [1, \infty) \to R$, where $f^{-1}(x) = 3 - (x-1)^{\frac{3}{2}}$

D. $f^{-1} : (-\infty, 3] \to R$, where $f^{-1}(x) = 3 - (x-1)^{\frac{3}{2}}$

E. $f^{-1} : R\backslash\{3\} \to R$, where $f^{-1}(x) = \dfrac{2}{3(x-3)^{\frac{1}{3}}}$

Question 5

The values of a and b for which the matrix equation $\begin{bmatrix} a & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ b \end{bmatrix}$ has a unique solution is

A. $a \in R \backslash \left\{\frac{1}{2}\right\}$ and $b \in R$

B. $a \in R \backslash \left\{\frac{1}{2}\right\}$ and $b = 8$

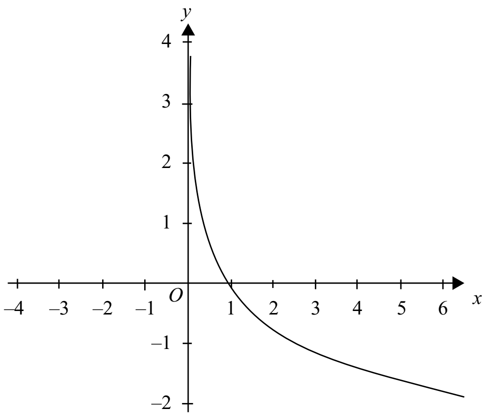
C. $a \in R \backslash \left\{\frac{1}{2}\right\}$ and $b \in R \backslash \{8\}$

D. $a = \frac{1}{2}$ and $b \in R \backslash \{8\}$

E. $a = \frac{1}{2}$ and $b = \{8\}$

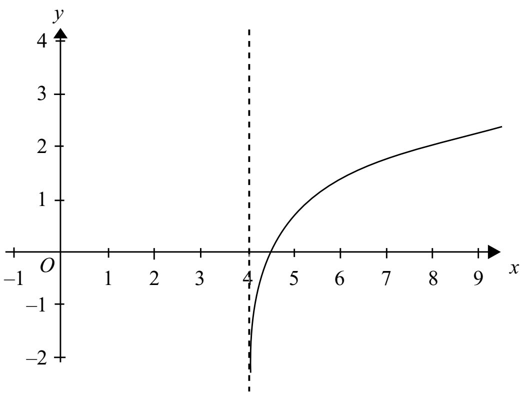
Question 6

The graph of $y = h(x)$ is shown below.

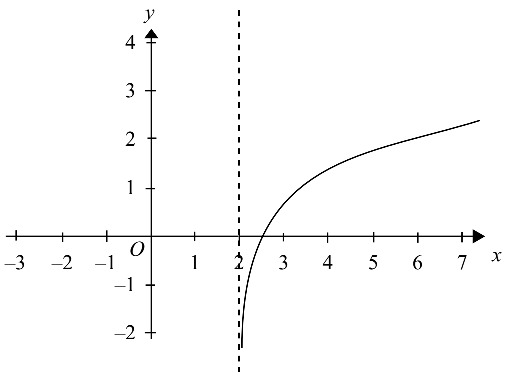


The graph of $y = -h(2x - 4)$ could be

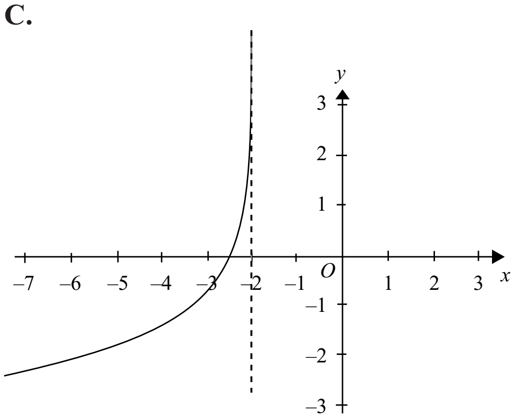
A.



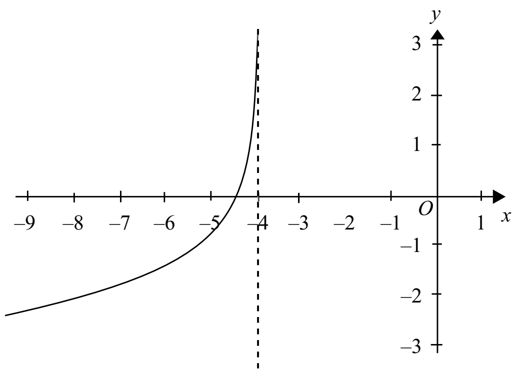
B.



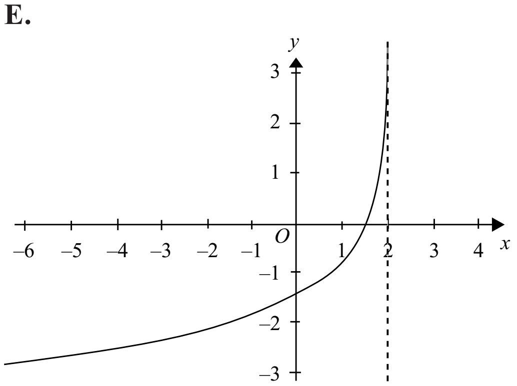
C.



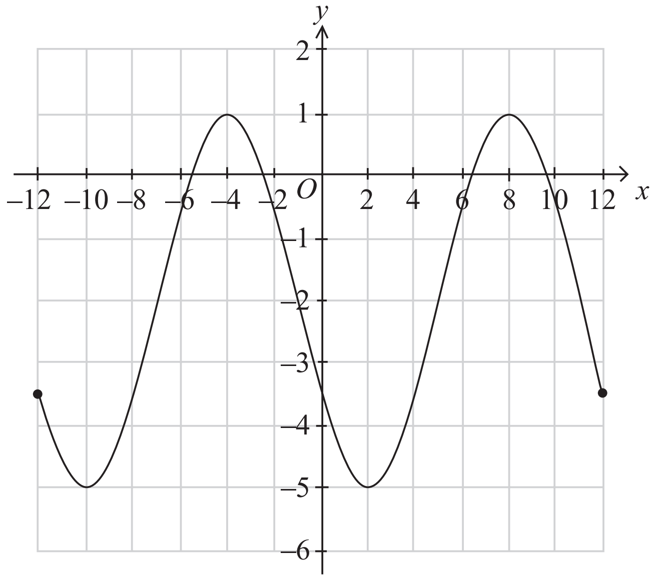
D.



E.



Question 7



The diagram above shows the graph of $y = f(x)$. Which one of the following could be the rule for f?

A. $f(x) = 6\sin\left(\frac{\pi}{6}(x-5)\right) - 2$

B. $f(x) = 3\sin\left(\frac{\pi}{12}(x-5)\right) - 2$

C. $f(x) = 3\sin\left(\frac{\pi}{6}(x+5)\right) - 2$

D. $f(x) = 3\sin\left(\frac{\pi}{12}(x+2)\right) - 5$

E. $f(x) = 3\sin\left(\frac{\pi}{6}(x-5)\right) - 2$

Question 8

Consider the function $f: R \to R, f(x) = \cos(x)$. Which one of the following functional equations is true?

A. $f(-x) = -f(x)$

B. $-f(-x) = f(-x)$

C. $f(x + \pi) = -f(x)$

D. $f(x + \pi) = f(x)$

E. $f(x^2) = (f(x))^2$

Question 9

The transformation $T : R^2 \rightarrow R^2$ that could be used to obtain the graph of $y = e^{-2(x+1)}$ from the graph of $y = e^x$ is

A. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

B. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -\frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

C. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

D. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

E. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -\frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \end{bmatrix}$

Question 10

$e^{2x} + be^x + 1 = 0$, where b is real constant, will have no real solutions if

A. $\{b : -2 \le b < 2\}$

B. $\{b : -2 < b \le 2\}$

C. $\{b : b < -2\} \cup \{b : b > 2\}$

D. $\{b : b \le -2\} \cup \{b : b \ge 2\}$

E. $\{b : -2 < b < 2\}$ only

Question 11

If $f(x) = \begin{cases} x^2 + 4x + a & \text{for } x \ge 0 \\ bx + c & \text{for } x < 0 \end{cases}$, where a, b and c are real constants, then $f(x)$ will be differentiable at $x = 0$ if

A. $a = c$ and $b \in R$

B. $a = c$ and $b = 4$ only

C. $b = 4$ and $a, c \in R$

D. $b = 2$ and $c = 4$

E. $a = b = c$

Question 12

Using the approximation formula, $f(x + h) = hf'(x) + f(x)$ then the change in f, where $f(x) = \log_e(2x + 1)$, as x decreases from 1 to 0.9 is

A. $-\dfrac{1}{15}$

B. $\dfrac{1}{15}$

C. $\log_e\left(\dfrac{14}{15}\right)$

D. $-\dfrac{1}{15} + \log_e(3)$

E. $\dfrac{1}{15} + \log_e(3)$

Question 13

The equation of the normal to the curve with equation $y = x^5 + 2x$ at $x = 0$ is

A. $x = 0$

B. $y = 0$

C. $y = 2x$

D. $y = -\frac{1}{2}x$

E. $y = 2$

Question 14

If $\frac{x^2}{y} = 1$ and $p = 5x + 3y$ then p will have

A. a maximum value when $x = -\frac{5}{3}$

B. a minimum value when $x = -\frac{5}{3}$

C. a maximum value when $x = \frac{25}{36}$

D. a maximum value when $x = -\frac{5}{6}$

E. a minimum value when $x = -\frac{5}{6}$

Question 15

If a particle is moving in a straight line such that its position x cm from a point O at time t seconds is given by $x = \sqrt{(1-t)}$ for $0 < t < 1$ then its acceleration, a cms^{-2}, where acceleration is the rate of change of velocity, is given by

A. $a = -\frac{1}{2x}$

B. $a = -\frac{1}{4x^3}$

C. $a = -\frac{2x^3}{3}$

D. $a = -\frac{1}{4(1-x)^{\frac{3}{2}}}$

E. $a = \frac{1}{4x^3}$

Question 16

The average value of the function $g : [0, a] \to R, g(x) = x^2 - 4$ is 8. The value of a is

A. 4

B. 6

C. 8

D. 10

E. 12

Question 17

The area bounded by the curves $y = \cos(2x)$ and $y = \sin(x)$ for $0 \le x \le \pi$ can be found by evaluating

A. $\displaystyle\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \left(\sin(x) - \cos(2x)\right) dx$

B. $\displaystyle\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \left(\cos(2x) - \sin(x)\right) dx$

C. $\displaystyle\int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \left(\sin(x) - \cos(2x)\right) dx$

D. $\displaystyle\int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \left(\cos(2x) - \sin(x)\right) dx$

E. $\displaystyle 2\int_{0}^{\frac{5\pi}{6}} \left(\cos(2x) - \sin(x)\right) dx + \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \left(\sin(x) - \cos(2x)\right) dx$

Question 18

The probability distribution for a continuous random variable, T, is defined by the probability density function $f(t) = \begin{cases} k & -\pi \le t \le \dfrac{\pi}{2} \\ 0 & \text{otherwise} \end{cases}$

If k is a real constant, the value of k is

A. $\dfrac{2}{3}$

B. $\dfrac{3\pi}{2}$

C. $\dfrac{2}{3\pi}$

D. $\dfrac{2\pi}{3}$

E. $\sin(t)$

Question 19

Suppose that in a particular year the percentage increase in the value of stocks listed on the Australian Stock Exchange was a normally distributed random variable with a mean of 5% and standard deviation of 10%. The proportion of stocks that would have **decreased** in value, in that particular year, is closest to

A. 0.31

B. 0.5

C. 0.69

D. 0.05

E. 0.1

Question 20

In the telephone directory there are eight people with the same initial and surname as that of my friend, Stewart. I randomly select and dial telephone numbers from the eight listed. If I dial a number once only, the probability that the third number dialled is Stewart's number is given by

A. $\frac{7}{8}\times\frac{6}{8}\times\frac{1}{8}$

B. $\frac{7}{8}\times\frac{6}{7}\times\frac{1}{6}$

C. $\frac{1}{8}\times\frac{1}{7}\times\frac{5}{6}$

D. $\left(\frac{7}{8}\right)^2\times\frac{1}{8}$

E. $\left(\frac{7}{8}\right)^3$

Question 21

The probability distribution of a discrete random variable, X, is shown below.

x	0	1	$\sqrt{2}$
$\Pr(X = x)$	h	h^2	$\frac{1}{2} - h$

If h is a real constant, the mean of X is

A. $\frac{\sqrt{2}}{2}$

B. $\frac{\sqrt{2}-1}{2}$

C. $\frac{1-\sqrt{2}}{2}$

D. $\frac{\sqrt{2}}{2}-1$

E. $\frac{2}{\sqrt{2}}$

Question 22

A Markov process has transition matrix $T = \begin{bmatrix} 0.7 & 0.2 \\ 0.3 & 0.8 \end{bmatrix}$ and initial state matrix $S_0 = \begin{bmatrix} 25 \\ 75 \end{bmatrix}$, such that the n^{th} state is given by $S_n = T^n S_0$. In the long term, the steady-state matrix will be

A. $\begin{bmatrix} 60 \\ 40 \end{bmatrix}$

B. $\begin{bmatrix} 0.6 \\ 0.4 \end{bmatrix}$

C. $\begin{bmatrix} 40 \\ 60 \end{bmatrix}$

D. $\begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$

E. $\begin{bmatrix} 50 \\ 50 \end{bmatrix}$

END OF SECTION 1

Working space

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.

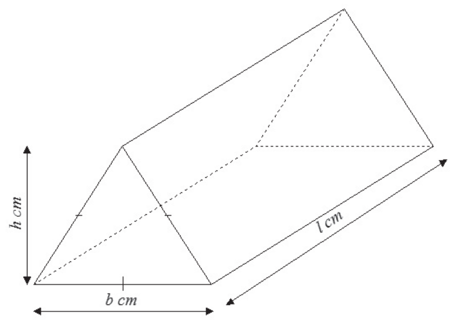
A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

A chocolate manufacturer wants to promote a particular brand of chocolate called Toby by maximising the amount of chocolate in their new Toby bar. The Toby bar is in the shape of a triangular prism as shown below, where the triangle has base, b cm and height, h cm and the length of the bar is l cm.



The surface area of the Toby bar is 120 cm^2 and the triangle is an **equilateral** triangle, where $3 \le b \le 9$.

a. Show that $l = \dfrac{240 - \sqrt{3}b^2}{6b}$.

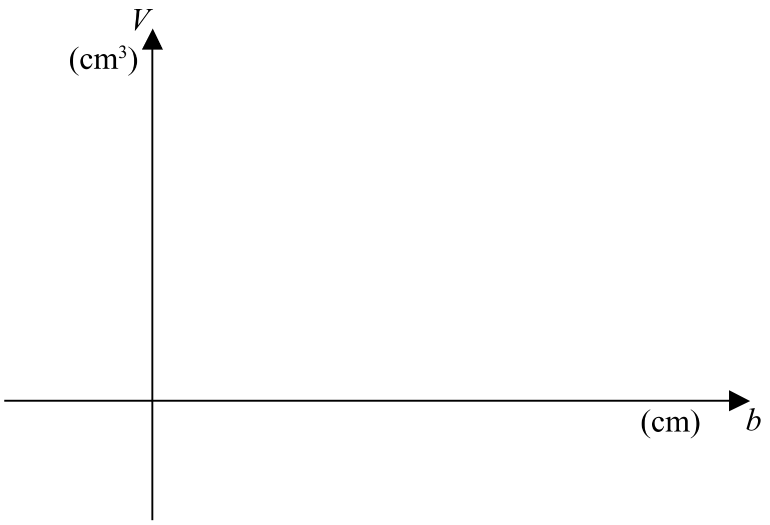
2 marks

b. Hence, show that the volume, V cm^3, of the chocolate bar in terms of b is given by

$$V = \frac{b(80\sqrt{3} - b^2)}{8}.$$

2 marks

c. Sketch the graph of V against b on the set of axes below. Give the coordinates of the end points and turning point correct to two decimal places.



3 marks

d. Find the dimensions of the chocolate which will give maximum volume.

4 marks

e. What is the maximum amount of chocolate, in cm^3, the manufacturer can put in the bar? Give your answer in the form $\dfrac{C\sqrt{D}}{E^F}$, where C, D, E and F are real constants.

1 mark

f. Find the average value of V.

2 marks

Total 14 marks

Question 2

Glottochronology is the study of the age of languages.
The number of ancient words retained in a language can be modelled by the function $N = N_0 e^{-mt}$, $t \geq 0$, where N is the number of ancient words retained after t **thousand** years, N_0 is the initial number of ancient words and m is a positive real constant.

a. In 2005, Graham, a glottochronology scholar, studied a list of 200 words from an ancient Chinese text dated 1005 AD. He found that 140 words were still used, 1000 years later, in modern Mandarin Chinese. According to these results, show that $m = \log_e\left(\frac{10}{7}\right)$.

2 marks

b. According to this model, what **percentage** of ancient words, from the 1005 AD text, were still in use in 1600 AD? Give your answer correct to the nearest integer.

2 marks

Graham also studied the divergence of the Italian language from Classical Latin. Again, he used the function $N = N_0 e^{-mt}$, $t \geq 0$, to model the new situation.

c. Graham found that 80% of words in a Classical Latin manuscript dated 200 AD were retained in the language of the Tuscany region of Italy in 700 AD. At this rate of word replacement, how many years will it take for 75% of words from Classical Latin to be replaced in Tuscan Italian? Answer correct to the nearest year.

3 marks

As a diversion during tedious monthly staff meetings, Graham kept count of the number of times that a "jargon" word was used by his new boss, John. He found that the number of times a "jargon" word was used in a meeting, *B*, *t* months after John's appointment as boss, could be modelled by the function $B(t)=\dfrac{a}{1+3e^{-kt}}$, $t \geq 0$, where *a* and *k* are positive real constants.

d. John used a "jargon" word 15 times in the initial meeting, on the day of his appointment, and 22 times in the 11th meeting. According to this model, how many months after his appointment will John use a "jargon" word 40 times in a meeting? (Answer correct to the nearest integer)

3 marks

Graham then studied the divergence of the English language from German. He found that the life of an old Germanic word in English (that is, the time taken for an old Germanic word to be replaced) is a random variable, *T* years, with the probability density function shown below, where *k* is a real constant.

$$f(t)=\begin{cases} k\,e^{-\frac{t}{200}} & , t \geq 0 \\ 0 & , t < 0 \end{cases}$$

e. Write down an equation which when solved will give the value of *k* as 0.005.

1 mark

f. According to this model, what is the expected or mean life of an old Germanic word in the English language?

2 marks

g. According to this model, what proportion of old Germanic words, have a life greater than 300 years? Answer correct to three decimal places.

2 marks

h. According to this model, what is the median life of an old Germanic word in the English language?

2 marks

Total 17 Marks

Working space

Question 3

The *Yolks On You* Company markets its large-sized eggs in cartons labelled 700 g. However, quality controllers have found that the total mass of eggs in each of the cartons, X grams, is a normally distributed random variable with a mean of 712 grams and a standard deviation of 20 grams.

a. In what proportion of cartons, correct to three decimal places, do the eggs have a total mass of less than 700grams?

2 marks

b. 60% of cartons contain eggs with a total mass greater than m. What is the value of m, in grams, correct to one decimal place?

2 marks

Cartons that are found to contain less than 700 grams of eggs are classified as underweight and are relabelled as medium-sized.

c. What is the probability that a carton of eggs that is relabelled as medium-sized contain less than 695 grams of eggs? Answer correct to three decimal places.

2 marks

From the thousands of cartons that are labelled as large-sized on a particular day, a quality controller weighs the contents of 8 cartons in turn.

d. What is the probability, correct to three decimal places, that at least three cartons will be classified as underweight?

2 marks

Celebrity chef, Jamie Oliveloaf, buys his eggs daily from *Yolks On You* for his restaurant in Thyme Square. Each day Jamie orders medium-sized or large-sized eggs. The transition matrix for the probabilities of Jamie ordering medium-sized or large-sized eggs is $\begin{bmatrix} 0.45 & 0.75 \\ 0.55 & 0.25 \end{bmatrix}$.

e. The probability that Jamie orders large-sized eggs today and medium-sized eggs tomorrow is 0.55. What is the probability that Jamie orders medium-sized today and large-sized tomorrow?

1 mark

f. Jamie orders medium-sized eggs on Tuesday.

i. What is the probability, correct to four decimal places, that he will order large-sized eggs on Wednesday and Thursday and medium-sized eggs on Friday of that week?

ii. What is the probability, correct to four decimal places, that he will order large-sized eggs on Tuesday of the following week?

1 + 2 = 3 marks

Total 12 marks

Question 4

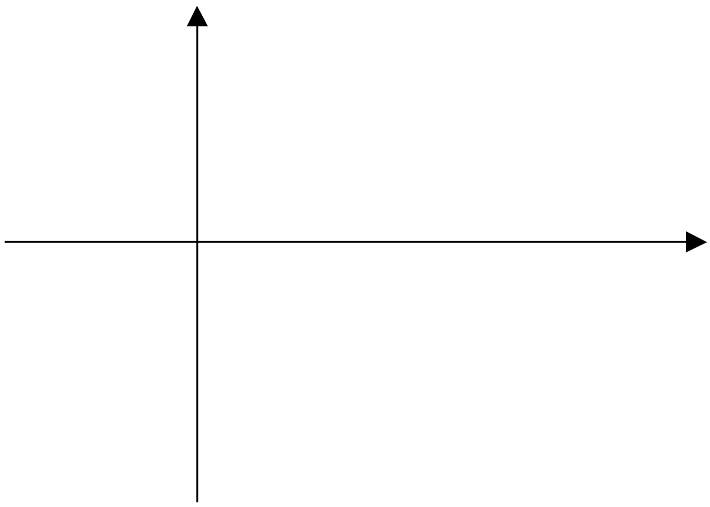
a. Find the general solution to $2\sin(x)\cos(x) = \frac{1}{2}$.

2 marks

b. **i.** State the transformations of the graph of $g(x) = \sin(2x)$ to get the graph of $h(x) = 3\sin\left(2(x+\frac{\pi}{12})\right)$.

2 marks

ii. Sketch the graph of $f: [0, \pi] \to R$, where $f(x) = 3\sin\left(2(x+\frac{\pi}{12})\right)$ on the set of axes below, clearly labelling intercepts and turning points with their coordinates.

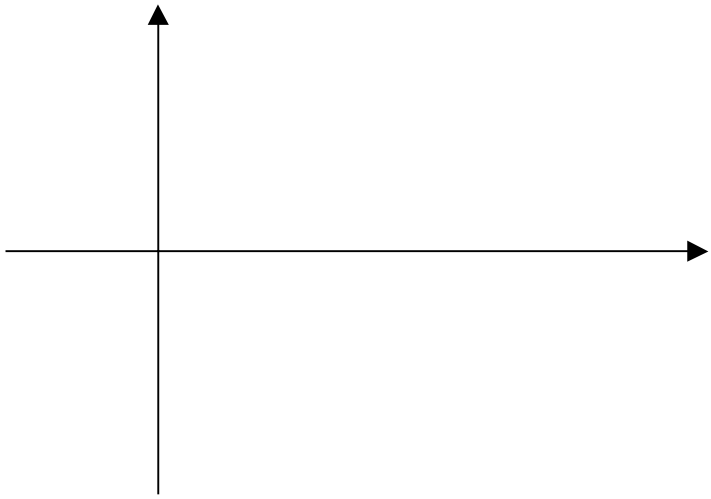


4 marks

c. **i.** Write down the derivative of $p(x) = |\sin(2x)|$ as a hybrid function.

2 marks

ii. Sketch the graphs of p and p' for $x \in (0, \pi)$ on the set of axes below.



3 marks

iii. Find the area bounded by p, p' and the lines $x = \frac{\pi}{4}$ and $x = \frac{\pi}{2}$.

2 marks

Total 15 marks

END OF QUESTION AND ANSWER BOOK

MULTIPLE CHOICE ANSWER SHEET

Student Name: ....

Circle the letter that corresponds to each correct answer

1	**A**	**B**	**C**	**D**	**E**
2	**A**	**B**	**C**	**D**	**E**
3	**A**	**B**	**C**	**D**	**E**
4	**A**	**B**	**C**	**D**	**E**
5	**A**	**B**	**C**	**D**	**E**
6	**A**	**B**	**C**	**D**	**E**
7	**A**	**B**	**C**	**D**	**E**
8	**A**	**B**	**C**	**D**	**E**
9	**A**	**B**	**C**	**D**	**E**
10	**A**	**B**	**C**	**D**	**E**
11	**A**	**B**	**C**	**D**	**E**
12	**A**	**B**	**C**	**D**	**E**
13	**A**	**B**	**C**	**D**	**E**
14	**A**	**B**	**C**	**D**	**E**
15	**A**	**B**	**C**	**D**	**E**
16	**A**	**B**	**C**	**D**	**E**
17	**A**	**B**	**C**	**D**	**E**
18	**A**	**B**	**C**	**D**	**E**
19	**A**	**B**	**C**	**D**	**E**
20	**A**	**B**	**C**	**D**	**E**
21	**A**	**B**	**C**	**D**	**E**
22	**A**	**B**	**C**	**D**	**E**

MATHEMATICAL METHODS (CAS)

Trial written examination 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time This formula sheet is provided for your reference

Mathematical Methods and Mathematical Methods CAS Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$	volume of a pyramid:	$\frac{1}{3}Ah$
curved surface area of a cylinder:	$2\pi rh$	volume of a sphere:	$\frac{4}{3}\pi r^3$
volume of a cylinder:	$\pi r^2 h$	area of a triangle:	$\frac{1}{2}bc\sin A$
volume of a cone:	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$

$\int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$

$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$

$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$

$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$

$\int \frac{1}{x} dx = \log_e |x| + c$

$\frac{d}{dx}\left(\sin(ax)\right) = a \ \cos(ax)$

$\int \sin(ax) dx = -\frac{1}{a}\cos(ax) + c$

$\frac{d}{dx}\left(\cos(ax)\right) = -a \ \sin(ax)$

$\int \cos(ax) dx = \frac{1}{a}\sin(ax) + c$

$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a \ \sec^2(ax)$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

approximation: $f(x+h) \approx f(x) + hf'(x)$

Probability

$\Pr(A) = 1 - \Pr(A')$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

mean: $\mu = \mathrm{E}(X)$

variance: $\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X-\mu)^2) = \mathrm{E}(X^2) - \mu^2$

probability distribution		mean	variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x\,p(x)$	$\sigma^2 = \sum (x-\mu)^2\,p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)dx$	$\mu = \int_{-\infty}^{\infty} x\,f(x)dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx$