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MATHEMATICAL METHODS UNITS 3 & 4

TRIAL EXAMINATION 2

2008

Reading Time: 15 minutes
Writing time: 2 hours

Instructions to students

This exam consists of Section 1 and Section 2.
Section 1 consists of 22 multiple-choice questions, which should be answered on the detachable answer sheet that can be found on page 26 of this exam.
Section 2 consists of 5 extended-answer questions that should be answered in the spaces provided.
Section 1 begins on page 2 of this exam and is worth 22 marks.
Section 2 begins on page 10 of this exam and is worth 58 marks.
There is a total of 80 marks available.
All questions in Section 1 and Section 2 should be answered.
Unless otherwise stated, diagrams in this exam are not drawn to scale.
Where more than one mark is allocated to a question, appropriate working must be shown.
Where an exact answer is required to a question, a decimal approximation will not be accepted.
Where you are asked to use calculus in a question you must show an appropriate derivative or antiderivative.
Students may bring one bound reference into the exam.
Students may bring an approved graphics calculator and if desired one scientific calculator into the exam.
A formula sheet can be found on page 25 of this exam.

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SECTION 1

Question 1

Consider the function *f* with rule $f(x)=1+\sqrt{x+5}$.
The maximal domain of *f* is

A. $R\setminus\{5\}$
B. R
C. $(1,\infty)$
D. $(-5,\infty)$
E. $[-5,\infty)$

Question 2

The function $f:[a,b]\rightarrow R, f(x)=4-2x$ has range $[-2,8]$.
The values of a and b are

A. $a=-12,\quad b=8$
B. $a=-3,\quad b=2$
C. $a=-3,\quad b=3$
D. $a=-2,\quad b=3$
E. $a=0,\quad b=20$

Question 3

$\log_3(2)$ is closest in value to

A. – 0.41
B. 0.41
C. 0.63
D. 0.67
E. 1.58

Question 4

If $\Pr(A)=\frac{1}{4}$, $\Pr(B)=\frac{1}{3}$ and $\Pr(A\cap B)=\frac{1}{6}$ then $\Pr(A'\cap B')$ is equal to

A. $\frac{1}{13}$
B. $\frac{5}{12}$
C. $\frac{1}{2}$
D. $\frac{7}{12}$
E. $\frac{2}{3}$

Question 5

A random variable X has a normal distribution with a mean of 6 and a standard deviation of 0.5.
The random variable Z has a standard normal distribution.
The probability that X is greater than 7 is equal to

A. $\Pr(Z < -2)$
B. $\Pr(Z < -1)$
C. $\Pr(Z > -1)$
D. $1 - \Pr(Z < -2)$
E. $1 - \Pr(Z > 2)$

Question 6

The range of the function $f : R \to R,\ f(x) = 2\cos\left(3x - \frac{\pi}{2}\right) - 1$ is

A. $[-3, 1]$
B. $[-2, 2]$
C. $[-1, 2]$
D. $[-1, 3]$
E. $[2, -1]$

Question 7

The average rate of change of the function $g(x) = \sqrt{\tan(x)}$ between $x = 0$ and $x = \frac{\pi}{4}$ is

A. $\frac{\pi}{4}$

B. $\frac{4}{\pi}$

C. $\frac{2\sqrt{2}}{\pi}$

D. $\frac{4\sqrt{3}}{\pi}$

E. $\frac{4\sqrt{3}}{3\pi}$

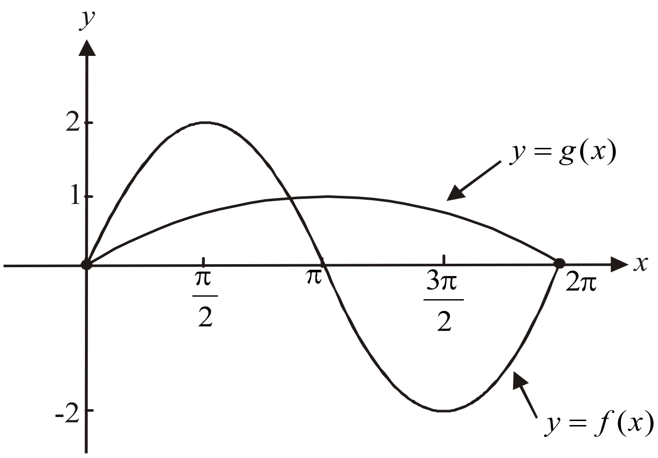
Question 8

For the function $f:(-1,\infty)\to R,\ f(x)=\left|\log_e(x+1)\right|$, it is true to say that

A. $f(0)=1$
B. $f'(x)>0$ for $x<0$
C. the graph of $y=f(x)$ is smoothly continuous at $x=0$
D. $f'(x)\neq 0$ for $x\in(-1,\infty)$
E. the function f has an inverse function

Question 9

The graph of $y=f(x)$ and part of the graph of $y=g(x)$ are shown below.



The transformations that the graph of $y=f(x)$ undergoes to become the graph of $y=g(x)$ are

A. a dilation by a scale factor of 2 from the y-axis followed by a translation by one unit down.
B. a dilation by a scale factor of $\frac{1}{2}$ from the y-axis followed by a translation by one unit down.
C. a dilation by a scale factor of $\frac{1}{2}$ from the x and y-axis.
D. a dilation by a scale factor of 2 from the x-axis followed by a dilation by a scale factor of $\frac{1}{2}$ from the y-axis..
E. a dilation by a scale factor of $\frac{1}{2}$ from the x-axis followed by a dilation by a scale factor of 2 from the y-axis

Question 10

The function $f:(-\infty, a] \to R, f(x) = \left|e^{-x} - 1\right|$ will have an inverse function if

A. $a \le 0$
B. $a \le 1$
C. $a \ge 0$
D. $a \ge 1$
E. $a \ge e$

Question 11

The derivative of $e^{2x}(\sin(2x) + x)$ with respect to x is

A. $2e^{2x}(2\cos(2x) + 1)$
B. $2e^{2x}(1 - 2\cos(2x))$
C. $2e^{2x}\sin(2x) + 2e^{2x}(\cos(2x)) + 1$
D. $2e^{2x}(\sin(2x) + x) + e^{2x}(1 - 2\cos(2x))$
E. $2e^{2x}(\sin(2x) + x) + e^{2x}(2\cos(2x) + 1)$

Question 12

The continuous function $f: R \to R$ has the following properties:

$$f'(x) = 0 \text{ at } x = 1 \text{ and at } x = 3$$
$$f'(x) < 0 \text{ for } x < 1$$
$$f'(x) > 0 \text{ for } 1 < x < 3 \text{ and for } x > 3$$

It is true to say that the graph of *f* has a

A. maximum turning point at $x = 1$
B. minimum turning point at $x = 3$
C. stationary point of inflection at $x = 1$
D. stationary point of inflection at $x = 3$
E. stationary point of inflection at $x = 1$ and a minimum turning point at $x = 3$

Question 13

In an animation, a pyramid is being shown. The pyramid has a square base of variable side length x cm and a fixed height of 90cm.
The rate at which the side lengths of the base are increasing is 2cm/sec.
The rate at which the volume of the pyramid is increasing when the side lengths of the base are 5cm is given by

A. $150\text{cm}^3/\text{sec}$
B. $600\text{cm}^3/\text{sec}$
C. $750\text{cm}^3/\text{sec}$
D. $1500\text{cm}^3/\text{sec}$
E. $2500\text{cm}^3/\text{sec}$

Question 14

$\int\left(\sqrt{x}-\cos(3x)\right)dx$ is equal to

A. $\dfrac{2x^{\frac{3}{2}}-\sin(3x)}{3}+c$

B. $\dfrac{2x^{\frac{3}{2}}+\sin(3x)}{3}+c$

C. $\dfrac{2x^{\frac{3}{2}}}{3}-3\sin(3x)+c$

D. $-\dfrac{2}{\sqrt{x}}-\dfrac{1}{3}\sin(3x)+c$

E. $-\dfrac{2}{\sqrt{x}}+3\sin(3x)+c$

Question 15

Consider the function $f: R \to R, f(x)=|x-1|$.
The area between the graph of $y=f(x)$, the *x*-axis and the lines $x=-1$ and $x=3$ is given by

A. $-1\int_{-1}^{1} f(x)dx$

B. $-1\int_{-1}^{3} f(x)dx$

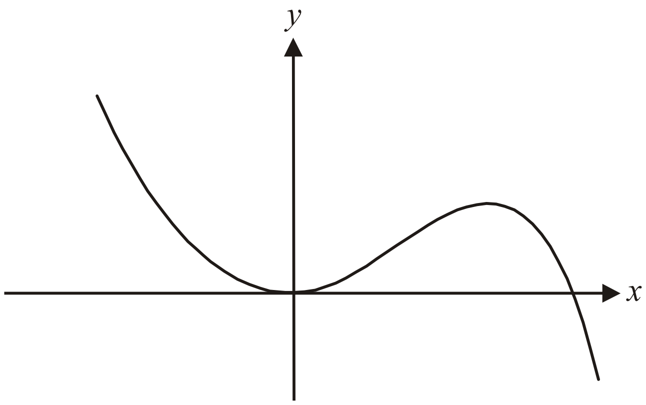
C. $\int_{-1}^{1} f(x)dx$

D. $2\int_{-1}^{1} f(x)dx$

E. $2\int_{-1}^{3} f(x)dx$

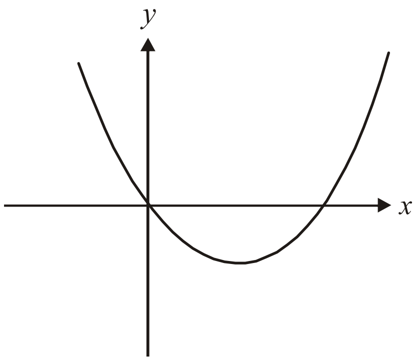
Question 16

The graph of $y = g(x)$ is shown below.

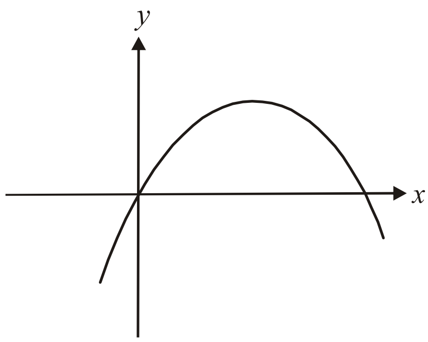


The graph of $y = g'(x)$ could be represented by

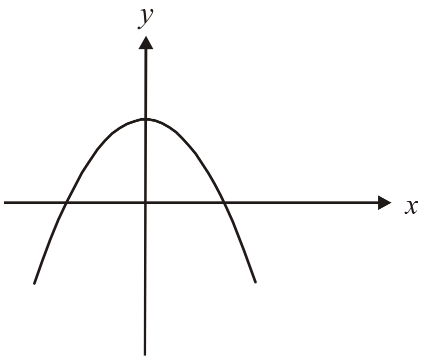
A.



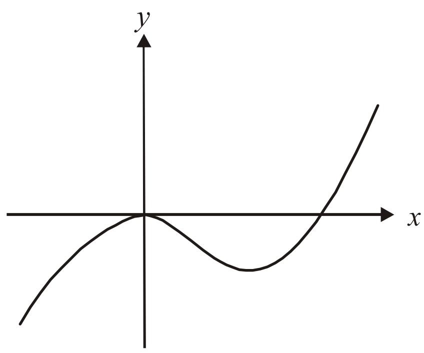
B.



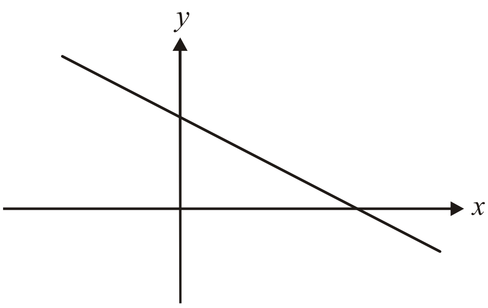
C.



D.



E.



Question 17

If $f(x)=h(x(x+2))$ then $f'(x)$ is equal to

A. $h'(2(x+1))$
B. $h'(x(x+2))$
C. $2(x+1)h'(x(x+2))$
D. $2(x+1)h'(2(x+1))$
E. $2(x+1)h(x(x+2))+2(x+1)h'(x(x+2))$

Question 18

If $f:[1,2]\to R,\ f(x)=(x-1)^2+2$, then the rule for the inverse function is

A. $f^{-1}(x)=1-\sqrt{x-2}$
B. $f^{-1}(x)=1+\sqrt{x-2}$
C. $f^{-1}(x)=\sqrt{x-1}-2$
D. $f^{-1}(x)=\sqrt{x+1}-2$
E. $f^{-1}(x)=(x+2)^2+1$

Question 19

The weights of a group of school children are normally distributed with a mean of 42kg and a standard deviation of 3.2kg. For this age group a child of more than 44kg is considered overweight. The proportion of overweight children in this group is

A. 5%
B. 10%
C. 27%
D. 73%
E. 90%

Question 20

The continuous random variable X has a probability density function given by

$$f(x)=\begin{cases}2\cos(2x) & \text{if} \quad 0<x<\dfrac{\pi}{4}\\ 0 & \text{elsewhere}\end{cases}$$

The value of a such that $\Pr(X<a)=0\cdot 4$ is closest to

A. -0.2
B. 0.1
C. 0.2
D. 0.4
E. 11.8

Question 21

If $\sin^2(x)-\frac{1}{2}\sin(x)=0$ and $x\in\left[0,\frac{\pi}{2}\right]$ then

A. $x=0$ only

B. $x=\frac{\pi}{6}$ only

C. $x=0$ or $x=\frac{\pi}{6}$

D. $x=0,\ x=\frac{\pi}{6}$ or $x=\frac{\pi}{2}$

E. $x=0,\ x=\frac{\pi}{6}$ or $x=\frac{5\pi}{6}$

Question 22

If $\int_a^b g(x)dx=4$ then $\int_b^a (3g(x)+5)dx$ is equal to

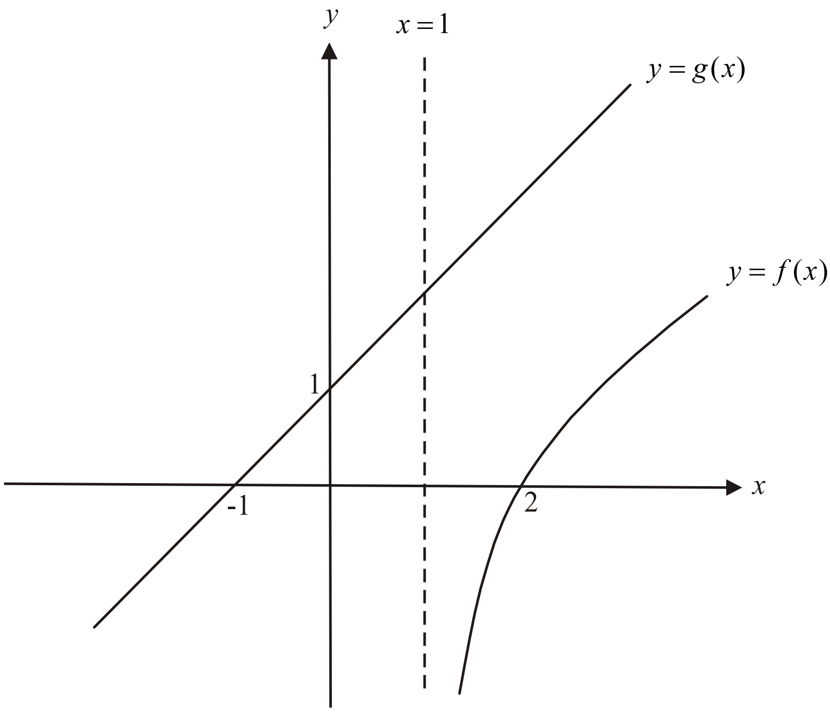
A. -7
B. 17
C. $5(a-b)-42$
D. $5(a-b)-12$
E. $3(a-b)-12$

SECTION 2

Answer all questions in this section.

Question 1

The graphs of the functions $f:(1,\infty)\rightarrow R,\ f(x)=2\log_e(x-1)$ and $g:R\rightarrow R,\ g(x)=x+1$ are shown below.



a. Find the rule and the domain of the inverse function f^{-1}.

3 marks

Consider the function h where $h(x)=g(x)-f(x)$.

b. Write down the domain of h.

1 mark

c. Use calculus to find the value of x for which $h(x)$ is a minimum. (You are not required to justify that the stationary point is a minimum.)

2 marks

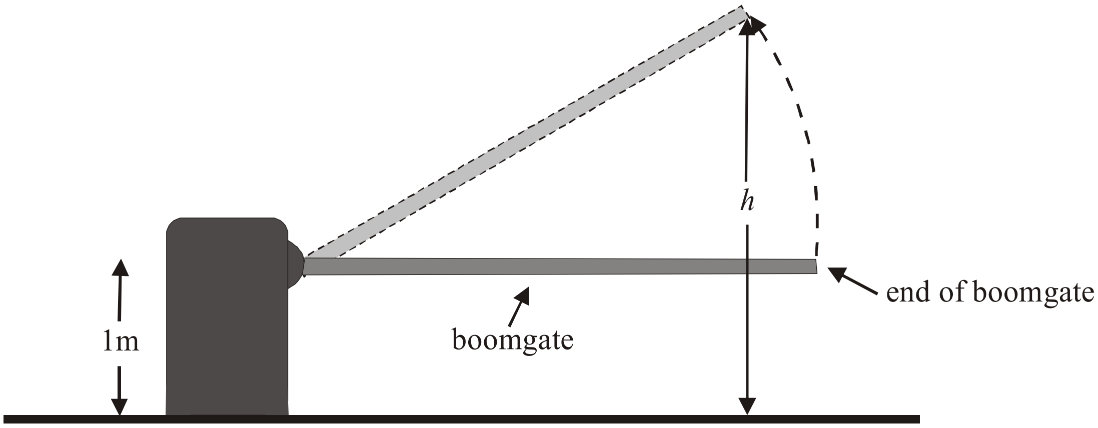
d. Hence find the minimum vertical distance between the graphs of $y = g(x)$ and $y = f(x)$. Express your answer as an exact value.

1 mark

Total 7 marks

Question 2

A boomgate at the entrance to a warehouse is malfunctioning. It is automatically opening and closing in a pattern that repeats each hour and will not respond to being manually opened or closed.



The height above the ground, *h*, in metres, of the end of the boomgate at time *t* minutes, is given by the continuous function

$$h(t)=\begin{cases}\left|3\sin\frac{\pi}{20}(t-10)\right|+1, & 10\le t\le 50\\ p, & 0\le t<10 \text{ or } 50<t\le 60\end{cases}$$

where $t=0$ corresponds to 8am on this particular day.

a. Find the value of *p*.

1 mark

b. Sketch the graph of the function h for $0 \le t \le 60$.

3 marks

c. For what value(s) of t does the boomgate reach its maximum height above the ground?

1 mark

A delivery truck can pass through the entrance to the warehouse when the end of the boomgate is 3.5 metres or higher above the ground.
Mal drives a delivery truck and pulls up at the entrance at 8:13am.

d. How long does Mal have to wait until he can first drive his delivery truck through the entrance? Express your answer in minutes correct to 2 decimal places.

1 mark

e. How many minutes in each hour would Mal be able to pass through the entrance to the warehouse? Express your answer correct to 1 decimal place.

2 marks

The time, T minutes, that Mal takes to pass through the entrance, unload and return to the boomgate is dependent on the mass, m tonnes, of his load where $T = m^2 + 12, \quad 0 \le m \le 5$.
Assume that Mal could drive through the entrance at the first possible opportunity after his 8:13am arrival.

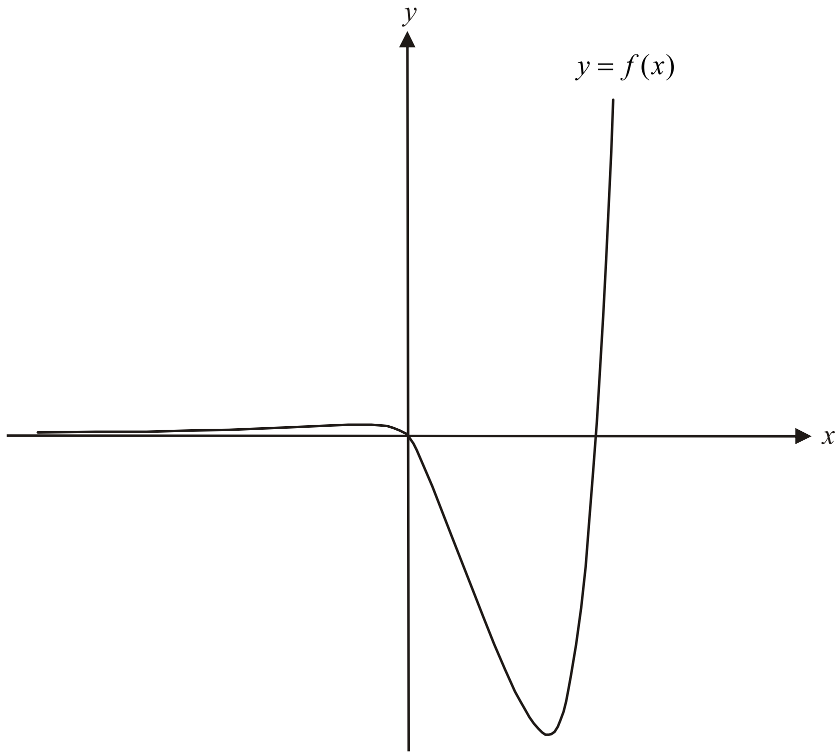
f. Find the total time in minutes that elapses between Mal entering and exiting at the boomgate entrance given that the mass of his load is 2 tonnes.

2 marks

Total 10 marks

Question 3

Consider the function $f : R \to R, f(x) = e^{2x}(x^2 - 2x)$.
The graph of $y = f(x)$ is shown below.



a. Find the values of x for which $f(x) > 0$.

2 marks

b. Given that $f'(x) = 2e^{2x}(x - a - b)(x - a + b)$ show that $a = \frac{1}{2}$ and $b = \frac{\sqrt{5}}{2}$.

2 marks

c. Hence find the coordinates of the point at which the graph of $y = f(x)$ is a minimum. Express your coordinates correct to 2 decimal places.

2 marks

d. Find the maximum value of the function *f* for $x \in [-3,3]$. Express your answer as an exact value

1 mark

e. **i.** Show that at the point on the graph of $y = f(x)$ where $x = -1$, the gradient is $2e^{-2}$

ii. Hence find the y-intercept of the tangent to the graph of $y = f(x)$ at $x = -1$.

$1 + 2 = 3$ marks
Total 10 marks

Question 4

Each morning Liv has either toast or cereal for breakfast.

If she has toast one morning then the probability that she has toast the next morning is 0.3.
If she has cereal one morning then the probability that she has toast the next morning is 0.6.

Liv has cereal for breakfast one Sunday morning.

a. **i.** What is the probability that she will have cereal for the next two mornings?

ii. What is the probability that she will have cereal just once over the next three mornings?

iii. What is the probability that she will have toast at least once over the next three mornings?

$1+2+2=5$ marks

When Liv has cereal for breakfast, the mass, X, (grams) of cereal that she pours into her bowl is a continuous random variable with probability density function given by

$$f(x)=\begin{cases}\dfrac{1}{1000}(x-20), & \text{if } 20 \le x \le 40\\ \dfrac{1}{4000}(120-x), & \text{if } 40 < x \le 120\\ 0 & \text{otherwise}\end{cases}$$

b. Sketch the graph of $y = f(x)$ on the set of axes below.

3 marks

c. One cup of Liv's breakfast cereal has a mass of 30g.
Show that the probability of Liv having less than a cup of cereal for breakfast is 0.05.

1 mark

d. Hence what is the probability that Liv has less than 1 cup of cereal for breakfast on 2 of the next 10 mornings that she has cereal? Express your answer correct to 3 decimal places.

1 mark

e. Find the probability that Liv has more than 2 cups of cereal given that she has more than one. Express your answer as an exact value.

2 marks

f. If $\Pr(X < n) = 0 \cdot 8$, find the value of n.

3 marks

Total 15 marks

Question 5

The cross-sectional view of an earth moving machine at a mine is shown in Diagram 1 below.

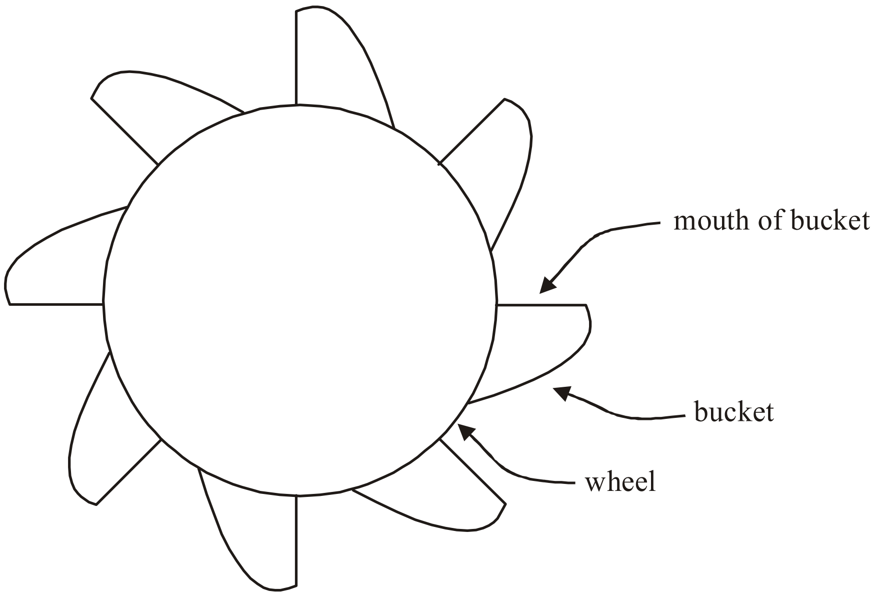


Diagram 1

Eight identically sized buckets; in which the earth is collected, are attached evenly around a wheel that turns anticlockwise. The mouth of the bucket is a straight line of length 1 metre.
Diagram 2 below shows the wheel, with centre located at $O(0,0)$ on the Cartesian plane, and two of the buckets. The mouth of both of these buckets is running vertically; that is, along the y-axis.

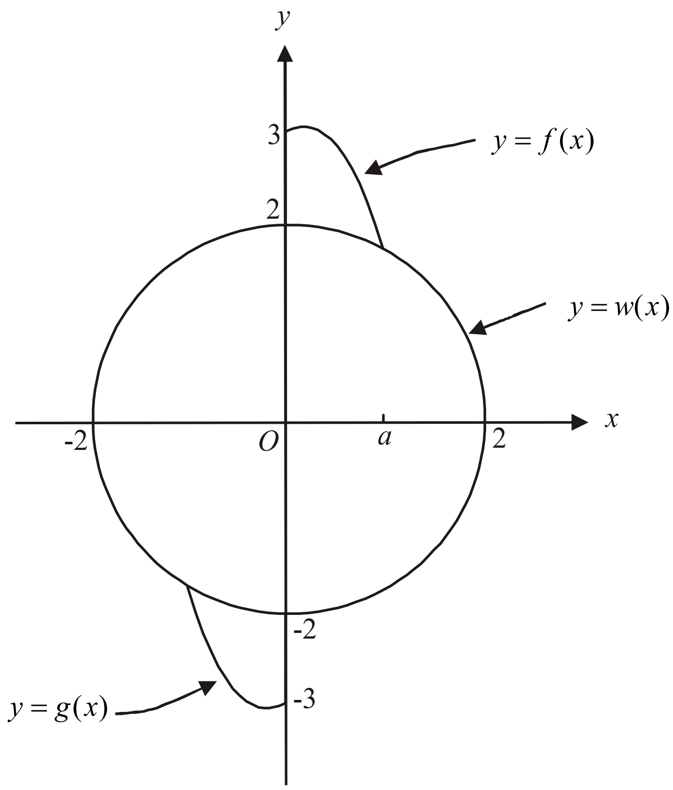


Diagram 2

The right hand edge of the top bucket can be defined by the function

$$f:[0,a] \to R,\, f(x) = -2x^2 + \left(\sqrt{3}-1\right)x + 3 \,.$$

The edge of the top half of the wheel can be defined by the function

$$w:[-2,2] \to R,\, w(x) = \sqrt{4-x^2} \,.$$

a. Show that $a = 1$.

1 mark

The left hand edge of the bottom bucket can be defined by the function *g*.

b. **i.** Write down a sequence of two transformations (excluding rotations) that map the graph of $y = f(x)$ on to the graph of $y = g(x)$.

ii. Hence find the rule for *g*.

iii. Write down the domain of *g*.

$2+2+1=5$ marks

The centre of the wheel is 5m above the ground and the machine has its buckets in the **position indicated in Diagram 2**. Also, the unit of measurement in Diagram 2 is the metre.

c. **i.** Use calculus to find the value of x for which the height of the machine above the ground is a maximum. Express your answer as an exact value.

ii. Hence find the maximum height of the machine above the ground. Express your answer in metres correct to 2 decimal places.

$2+1=3$ marks

d. Hence write down the range of the function f.

2 marks

The functions *f* and *w* are shown in Diagram 3 below.
The cross-sectional area of the top bucket is bounded by the *y*-axis and the graphs of $y = f(x)$ and $y = w(x)$.

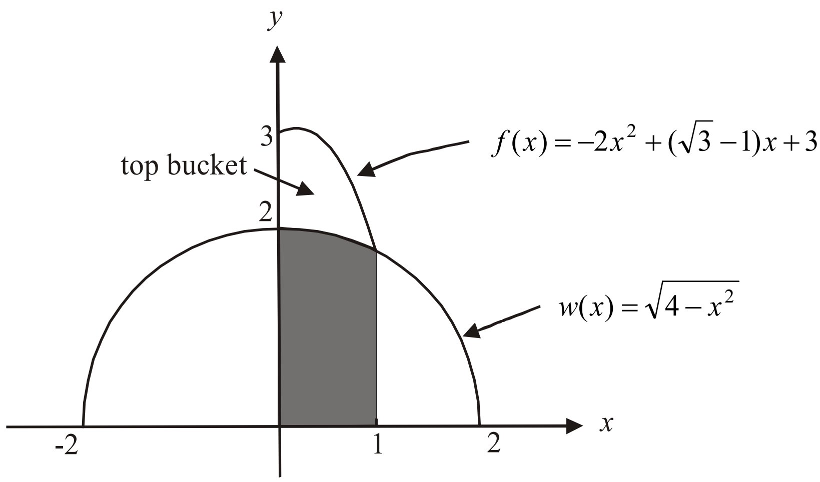


Diagram 3

e. Write down; but **do not evaluate**, a definite integral that gives the cross-sectional area of the top bucket shown in Diagram 3 above.

2 marks

The shaded area in Diagram 3 has an area of $\frac{\sqrt{3}}{2} + \frac{\pi}{3}$ square metres.

f. Find the cross-sectional area of the top bucket. Express your answer as an exact value.

3 marks
Total 16 marks

Mathematical Methods and Mathematical Methods CAS Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$	volume of a pyramid:	$\frac{1}{3}Ah$
curved surface area of a cylinder:	$2\pi rh$	volume of a sphere:	$\frac{4}{3}\pi r^3$
volume of a cylinder:	$\pi r^2 h$	area of a triangle:	$\frac{1}{2}bc\sin A$
volume of a cone:	$\frac{1}{3}\pi r^2 h$		

Calculus

$$\frac{d}{dx}\left(x^n\right)=nx^{n-1} \qquad \int x^n\,dx=\frac{1}{n+1}x^{n+1}+c,\ n\neq -1$$

$$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax} \qquad \int e^{ax}\,dx=\frac{1}{a}e^{ax}+c$$

$$\frac{d}{dx}\left(\log_e(x)\right)=\frac{1}{x} \qquad \int \frac{1}{x}\,dx=\log_e|x|+c$$

$$\frac{d}{dx}\left(\sin(ax)\right)=a\cos(ax) \qquad \int \sin(ax)\,dx=-\frac{1}{a}\cos(ax)+c$$

$$\frac{d}{dx}\left(\cos(ax)\right)=-a\sin(ax) \qquad \int \cos\ (ax)\,dx=\frac{1}{a}\sin(ax)+c$$

$$\frac{d}{dx}\left(\tan(ax)\right)=\frac{a}{\cos^2(ax)}=a\sec^2(ax)$$

product rule: $\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$

approximation: $f(x+h)\approx f(x)+hf'(x)$

Probability

$\Pr(A)=1-\Pr(A')$

$\Pr(A\cup B)=\Pr(A)+\Pr(B)-\Pr(A\cap B)$

$\Pr(A/B)=\frac{\Pr(A\cap B)}{\Pr(B)}$

mean: $\mu=E(X)$

variance: $\text{var}(X)=\sigma^2=E((X-\mu)^2)=E(X^2)-\mu^2$

probability distribution		mean	variance
discrete	$\Pr(X=x)=p(x)$	$\mu=\Sigma x\,p(x)$	$\sigma^2=\Sigma(x-\mu)^2\,p(x)$
continuous	$\Pr(a<X<b)=\int_a^b f(x)dx$	$\mu=\int_{-\infty}^{\infty} x\,f(x)dx$	$\sigma^2=\int_{-\infty}^{\infty}(x-\mu)^2 f(x)\,dx$

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The VCAA publish an exam issue supplement to the VCAA bulletin.

MATHEMATICAL METHODS

TRIAL EXAMINATION 2

MULTIPLE- CHOICE ANSWER SHEET

STUDENT NAME:..

INSTRUCTIONS

Fill in the letter that corresponds to your choice. Example: A ■ C D E

The answer selected is B. Only one answer should be selected.

1.	A	B	C	D	E	12.	A	B	C	D	E
2.	A	B	C	D	E	13.	A	B	C	D	E
3.	A	B	C	D	E	14.	A	B	C	D	E
4.	A	B	C	D	E	15.	A	B	C	D	E
5.	A	B	C	D	E	16.	A	B	C	D	E
6.	A	B	C	D	E	17.	A	B	C	D	E
7.	A	B	C	D	E	18.	A	B	C	D	E
8.	A	B	C	D	E	19.	A	B	C	D	E
9.	A	B	C	D	E	20.	A	B	C	D	E
10.	A	B	C	D	E	21.	A	B	C	D	E
11.	A	B	C	D	E	22.	A	B	C	D	E