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MATHS METHODS 3 & 4 TRIAL EXAMINATION 2 SOLUTIONS 2008

1. E	9. E	17. C
2. D	10. A	18. B
3. C	11. E	19. C
4. D	12. D	20. C
5. A	13. B	21. C
6. A	14. A	22. D
7. B	15. D	
8. D	16. B	

Section 1 – Multiple-choice solutions

Question 1

$f(x) = 1 + \sqrt{x+5}$

For *f* to be defined

$$x + 5 \geq 0$$
$$x \geq -5$$

So the maximal domain is given by $[-5, \infty)$.

The answer is E.

Question 2

<u>Method 1</u> – graphically

Sketch the graph of $y = 4 - 2x$. The range of *f* is restricted and hence the domain will be restricted.

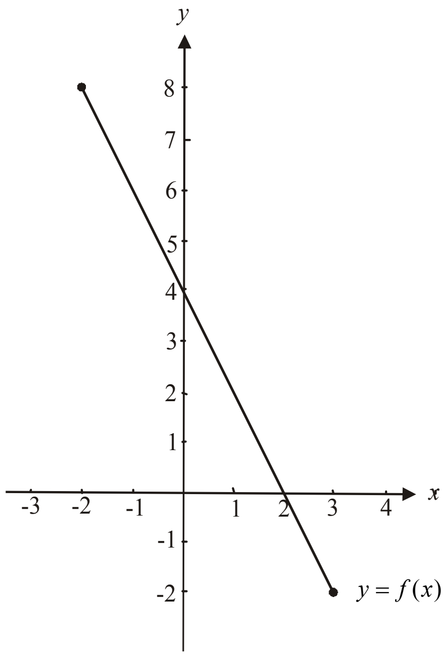
From the graph,

since $r_f = [-2, 8]$,

$d_f = [-2, 3]$

So $a = -2$ and $b = 3$.

The answer is D.



<u>Method 2</u> – algebraically

$f(x) = 4 - 2x$

Let $y = 4 - 2x$

When $y = -2$, $-2 = 4 - 2x$

$$2x = 6$$
$$x = 3$$

When $y = 8$, $8 = 4 - 2x$

$$2x = -4$$
$$x = -2$$

We are dealing with a linear function so $d_f = [-2, 3]$ and since $d_f = [a, b]$ then $a = -2$ and $b = 3$.

The answer is D.

Question 3

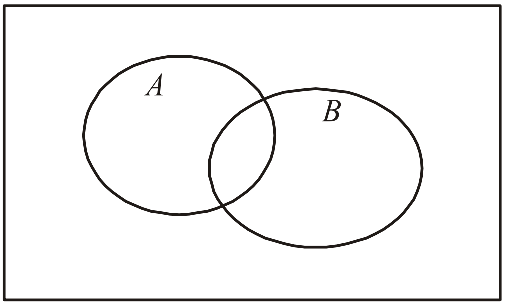
$$\log_3(2) = \frac{\log_e(2)}{\log_e(3)} \quad \text{(change of base rule)}$$
$$= 0 \cdot 63 \quad \text{(to 2 decimal places)}$$

Alternatively, $\log_3(2) = \dfrac{\log_{10}(2)}{\log_{10}(3)}$
$$= 0 \cdot 63 \quad \text{(to 2 decimal places)}$$

The answer is C.

Question 4

Method 1 - use a Venn diagram,



$$\Pr(A' \cap B') = \Pr(A \cup B)'$$
$$= 1 - \Pr(A \cup B)$$
$$= 1 - (\Pr(A) + \Pr(B) - \Pr(A \cap B))$$
$$= 1 - \left(\frac{1}{4} + \frac{1}{3} - \frac{1}{6}\right)$$
$$= 1 - \left(\frac{3 + 4 - 2}{12}\right)$$
$$= 1 - \frac{5}{12}$$
$$= \frac{7}{12}$$

The answer is D.

Method 2 – use a probability table.

	A	A'	
B	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{3}$
B'	$\frac{1}{12}$	$\frac{7}{12}$	$\frac{2}{3}$
	$\frac{1}{4}$	$\frac{3}{4}$	1

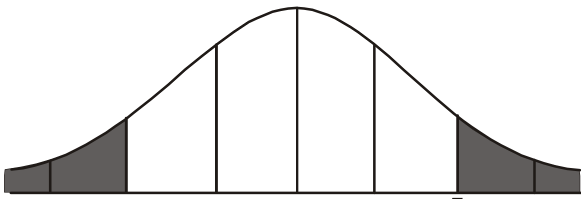
From the table, $\Pr(A' \cap B') = \dfrac{7}{12}$.

The answer is D.

Question 5

$$\Pr(X > 7) = \Pr(Z > 2)$$
$$= \Pr(Z < -2)$$

The answer is A.



Question 6

Method 1

Sketch the graph of $y = f(x)$.

The maximum value occurs when $y = 1$ and the minimum value occurs when $y = -3$.

The range is $[-3, 1]$.

The answer is A.

Method 2

The domain is not restricted (that is, the domain is *R*) so there is no restriction placed on the range. The amplitude is 2 and the graph of $y = 2\cos\left(3x - \frac{\pi}{2}\right)$ is translated 1 unit down so the range is $y \in [-3, 1]$. Note that the $\left(3x - \frac{\pi}{2}\right)$ has no influence on the range. It determines the period and the horizontal translation.

The answer is A.

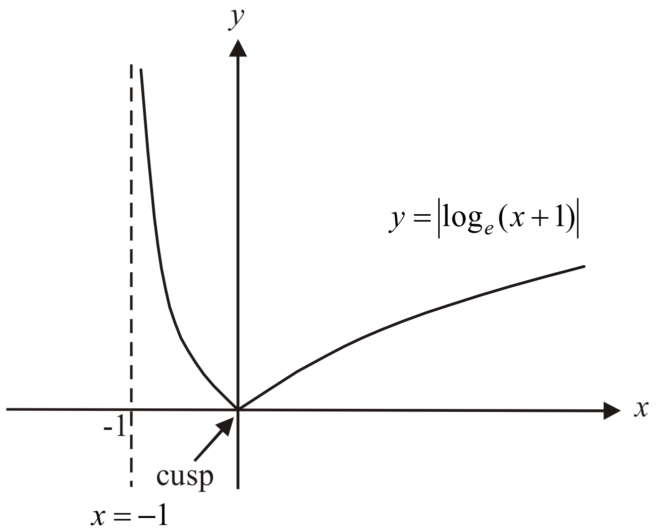
Question 7

$$\text{Average rate of change} = \frac{g\left(\frac{\pi}{4}\right) - g(0)}{\frac{\pi}{4} - 0}$$
$$= \frac{\sqrt{\tan\left(\frac{\pi}{4}\right)} - \sqrt{\tan(0)}}{\frac{\pi}{4}}$$
$$= \frac{\sqrt{1} - \sqrt{0}}{\frac{\pi}{4}}$$
$$= 1 \div \frac{\pi}{4}$$
$$= \frac{4}{\pi}$$

The answer is B.

Question 8

Sketch the graph.

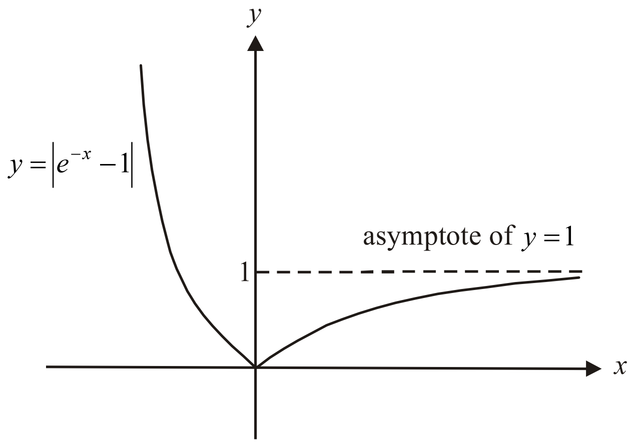


$f(0) = 0$ so option A is incorrect.
For $x < 0$, the gradient of the function is negative so option B is incorrect.
At $x = 0$, the graph of $y = f(x)$ has a cusp, i.e. a "pointy bit". So whilst the graph is continuous at $x = 0$, it is not smoothly continuous so option C is incorrect.
Option D is correct; that is, $f'(x) \neq 0$ for $x \in (-1, \infty)$, i.e. the gradient of *f* is not zero over its domain.
Also, the graph of $y = f(x)$ is not 1:1 and therefore an inverse function of *f* does not exist so option E is incorrect.
The answer is D.

Question 9

The graph of $y = f(x)$ is stretched horizontally; that is, dilated by a scale factor of 2 from the *y*-axis. For example, the local maximum that occurs at $x = \frac{\pi}{2}$, occurs on the graph of $y = g(x)$ at $x = \pi$.
Also, the graph of $y = f(x)$ is squashed vertically; that is, dilated by a scale factor of $\frac{1}{2}$ from the *x*-axis. For example the local maximum that has a *y* value of 2 occurs on the graph of $y = g(x)$ with a *y* value of 1.
The answer is E.

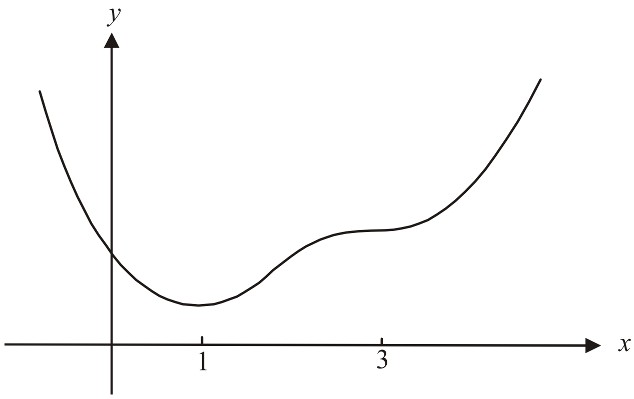
Question 10
Sketch the graph of $y=\left|e^{-x}-1\right|$.



For *f* to have an inverse function it must be a 1:1 function.
Since $d_f=(-\infty,a]$, we require that $a\le 0$ so that the graph of $y=f(x)$ is 1:1.
So $a\le 0$.
The answer is A.

Question 11
$\frac{d}{dx}\left(e^{2x}(\sin(2x)+x)\right)$
$=2e^{2x}(\sin(2x)+x)+e^{2x}(2\cos(2x)+1)$ (product rule)
The answer is E.

Question 12
Do a quick sketch of a possible graph of *f*.



The first clue tells us that we have stationary points at $x=1$ and at $x=3$. The second clue tells us that the gradient is negative for $x<1$. The third clue tells us that the gradient is positive for $1<x<3$ and for $x>3$.

Whilst we don't know the exact positioning of points, the gradients given can tell us the approximate shape of the graph. From this rough sketch we see that we have a minimum turning point at $x=1$ and a stationary point of inflection at $x=3$.
Only option D offers a correct alternative.
The answer is D.

Question 13

For a pyramid, $V=\frac{1}{3}Ah$ (formulae sheet)

Since the pyramid has a square base with sidelength *x*, and a height of 90 we have

$$V=\frac{1}{3}x^2\times 90$$

$$V=30x^2$$

$$\frac{dV}{dx}=60x$$

Also, $\frac{dx}{dt}=2$ which is given in the question.

Now, $\frac{dV}{dt}=\frac{dV}{dx}\cdot\frac{dx}{dt}$ (Chain rule)

$$=60x\cdot 2$$

$$=120x$$

When $x=5, \frac{dV}{dt}=600\text{cm}^3/\text{sec}$

The answer is B.

Question 14

$$\int\left(\sqrt{x}-\cos(3x)\right)dx$$

$$=\int\left(x^{\frac{1}{2}}-\cos(3x)\right)dx$$

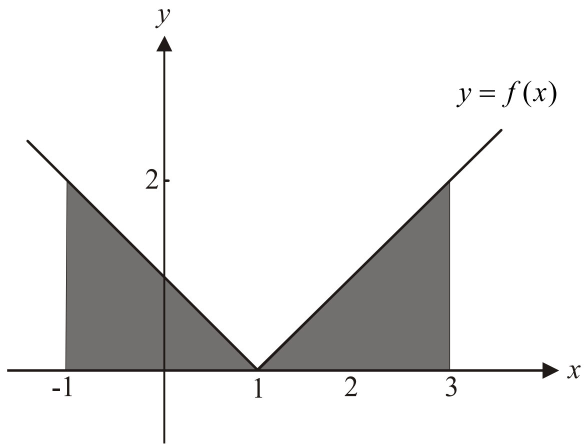
$$=\frac{2x^{\frac{3}{2}}}{3}-\frac{1}{3}\sin(3x)+c$$

$$=\frac{2x^{\frac{3}{2}}-\sin(3x)}{3}+c$$

The answer is A.

Question 15

Sketch the graph.



The shaded area gives the required area. The function is symmetrical about the line $x = 1$.
From the graph,

$$\int_{-1}^{1} f(x)dx = \int_{1}^{3} f(x)dx$$

So $\int_{-1}^{3} f(x)dx = 2\int_{-1}^{1} f(x)dx$

The answer is D.

Question 16

For $x < 0, g'(x) < 0$.
Between $x = 0$ and the local maximum to the right of the origin, $g'(x) > 0$.
For *x* greater than the value of *x* at the local maximum to the right of the origin, $g'(x) < 0$.
Only option B shows this.
The answer is B.

Question 17

f is a composite function and so we need to use the chain rule to differentiate it.

Method 1 – quick way

$$\begin{aligned} f(x) &= h(x(x+2)) \\ &= h(x^2 + 2x) \\ f'(x) &= h'(x(x+2)) \times (2x+2) \quad \text{(Chain rule)} \\ &= 2(x+1)h'(x(x+2)) \end{aligned}$$

The answer is C.

Method 2 – longer way

Let $y = h(x(x+2))$

Let $u = x(x+2)$

$= x^2 + 2x$

$\frac{du}{dx} = 2x + 2$

So $y = h(u)$

$\frac{dy}{du} = h'(u)$

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ (Chain rule)

$= h'(u) \cdot (2x+2)$

$= 2(x+1)h'(x(x+2))$

The answer is C.

Question 18

$f:[1,2] \to R, f(x) = (x-1)^2 + 2$

Let $y = (x-1)^2 + 2$

Swap x and y.

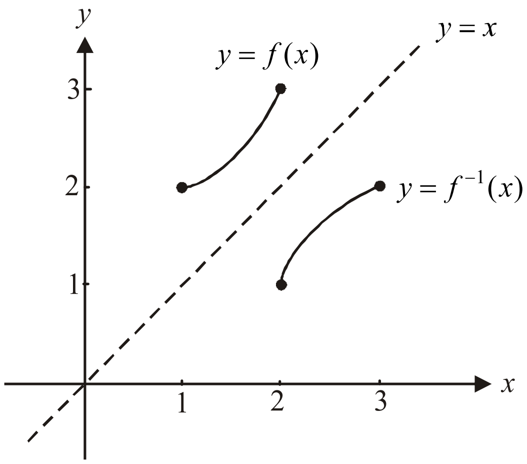
$x = (y-1)^2 + 2$

Rearrange

$x - 2 = (y-1)^2$

$\pm\sqrt{x-2} = y - 1$

$y = 1 \pm \sqrt{x-2}$



A quick sketch shows us that the rule is $y = 1 + \sqrt{x-2}$ because it gives us the image of $y = (x-1)^2 + 2$ in the line $y = x$. A function and it's inverse are mirror images in this line.

An alternative way to think of it is that we are given part of the right arm of the parabola and therefore the inverse will be part of the top arm of the side parabola (square root graph).
The answer is B.

Question 19

$\text{normal cdf}(44, 1000, 42, 3.2) = 0.265985...$
so $\Pr(X > 44) = 0 \cdot 265985...$
The closest proportion is 27%.
The answer is C.

Question 20

$$f(x) = \begin{cases} 2\cos(2x) & \text{if } 0 < x < \frac{\pi}{4} \\ 0 & \text{elsewhere} \end{cases}$$

$$\begin{aligned} \Pr(X < a) &= 0 \cdot 4 \\ \int_0^a 2\cos(2x)dx &= 0 \cdot 4 \\ [\sin(2x)]_0^a &= 0 \cdot 4 \\ \sin(2a) - \sin(0) &= 0 \cdot 4 \\ \sin(2a) &= 0 \cdot 4 \\ 2a &= 0 \cdot 4115... \\ a &= 0 \cdot 2057... \end{aligned}$$

The answer is C.

Question 21

$$\sin^2(x) - \frac{1}{2}\sin(x) = 0, \quad x \in \left[0, \frac{\pi}{2}\right]$$

$$\sin(x)\left(\sin(x) - \frac{1}{2}\right) = 0$$

$$\sin(x) = 0 \quad \text{or} \quad \sin(x) - \frac{1}{2} = 0$$

Since $x \in \left[0, \frac{\pi}{2}\right]$, $x = 0$ or $\sin(x) = \frac{1}{2}$

$$x = \frac{\pi}{6}$$

The answer is C.

Question 22

$$\int_a^b g(x)\,dx = 4$$

so $\int_b^a g(x)\,dx = -4$

Now, $\int_b^a (3g(x)+5)dx$

$$\begin{aligned}&= 3\int_b^a g(x)dx + \int_b^a 5dx\\ &= 3\times -4 + [5x]_b^a\\ &= -12 + 5a - 5b\\ &= 5(a-b) - 12\end{aligned}$$

The answer is D.

SECTION 2

Question 1

a. $f(x)=2\log_e(x-1)$

Let $y=2\log_e(x-1)$

Swap *x* and *y*.

$x=2\log_e(y-1)$

Rearrange

$\frac{x}{2}=\log_e(y-1)$

$e^{\frac{x}{2}}=y-1$

$y=1+e^{\frac{x}{2}}$

Now, from the diagram,

$r_f=(-\infty,\infty)$ or R

So $d_{f^{-1}}=r_f=(-\infty,\infty)$ or R

The inverse function f^{-1} is defined by $f^{-1}:R\to R,\ f^{-1}(x)=1+e^{\frac{x}{2}}$

(1 mark) for swapping *x* and *y*

(1 mark) for $y=1+e^{\frac{x}{2}}$

(1 mark) for correct domain

b. d_h gives the values of *x* for which both *f* and *g* are defined.

Since $d_f=(1,\infty)$ and $d_g=R$, then $d_h=(1,\infty)$.

(1 mark)

c.

$h(x)=g(x)-f(x)$

$=x+1-2\log_e(x-1)$

$h'(x)=1-2\times\frac{1}{x-1}\times 1$

$=1-\frac{2}{x-1}$ **(1 mark)**

For a min/max $h'(x)=0$

$1-\frac{2}{x-1}=0$

$\frac{2}{x-1}=1$

$2=x-1$

$x=3$

(1 mark)

d. From part **c.** the minimum value of *h* occurs when $x=3$. Since $h(x)=g(x)-f(x)$, the minimum vertical distance between the graphs of $y=g(x)$ and $y=f(x)$ is given by $h(3)=3+1-2\log_e(3-1)$

$=4-2\log_e(2)$ units

(1 mark)

Total 7 marks

Question 2

a. Since the function h is a continuous function,

at $t = 10$, $\left|3\sin\frac{\pi}{20}(t-10)\right| + 1 = p$

becomes $\left|3\sin(0)\right| + 1 = p$

$\left|3 \times 0\right| + 1 = p$

$p = 1$

(1 mark)

(Check : at $t = 50$, $\left|3\sin\frac{\pi}{20}(40)\right| + 1$

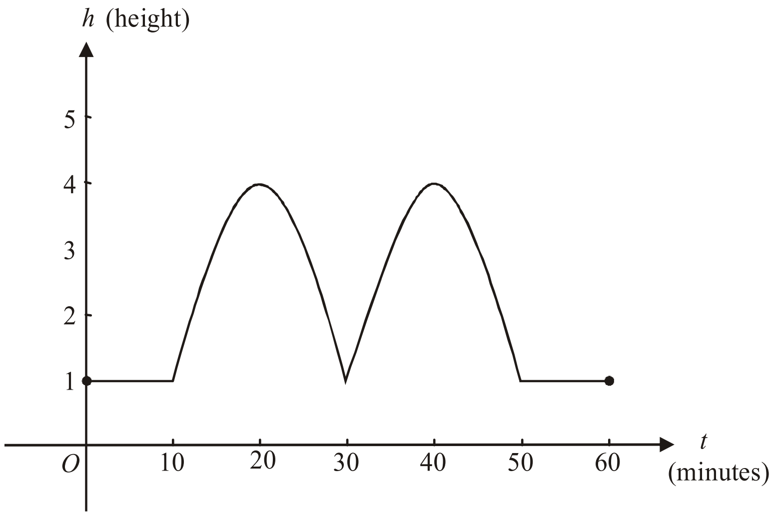
$= \left|3\sin(2\pi)\right| + 1$

$= \left|3 \times 0\right| + 1$

$= 1$

So $p = 1$)

b.



(1 mark) correct linear branches and endpoints
(1 mark) correct period of sine graph
(1 mark) correct amplitude and recognition of effect of absolute value

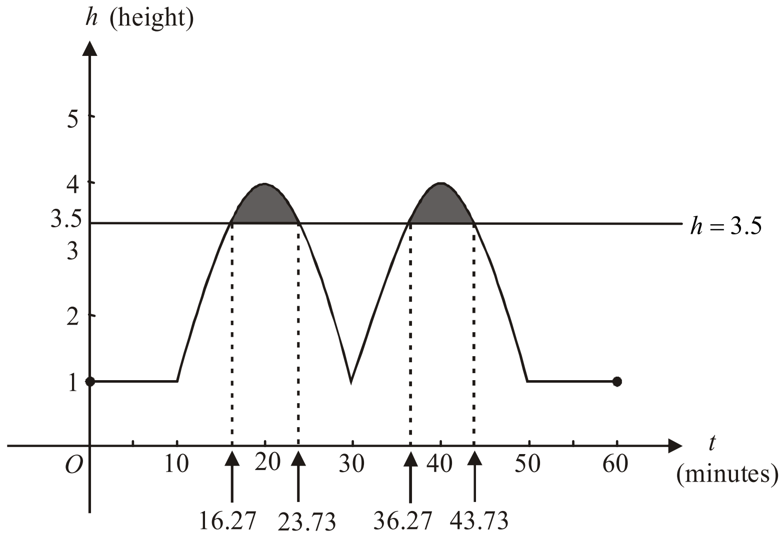
c. From the graph, and due to the symmetry of the sine curve, we see that the maximum occurs at $t = 20$ and at $t = 40$.

(1 mark)

d. We need to solve $h(t)=3\cdot 5$
Using a calculator to solve this we have $t=16\cdot 27141...$
Since Mal arrives at $t=13$ (i.e. 13 minutes after the hour, i.e. after 8 o'clock), he has to wait $3\cdot 2714...$ or $3\cdot 27$ minutes (correct to two decimal places).

(1 mark)

e. From part **d.** we found that Mal could first pass through the entrance at $t=16\cdot 27$. Using the graph below and bearing in mind the symmetry of the graph, we can find the total time Mal can gain entry.



$$\text{Total time} = 2\times(23\cdot 73 - 16\cdot 27)$$ **(1 mark)**

$$= 14\cdot 9 \text{ minutes (correct to 1 decimal place)}$$

(1 mark)

f. From part **d.**, the first time that Mal can pass through the entrance is at $t=16\cdot 27$.
Since the mass of his load is 2 tonnes, it will take him

$$T = m^2 + 12$$
$$= 2^2 + 12$$
$$= 16 \text{ minutes}$$ **(1 mark)**

to be back at the boomgate entrance.
At $t = 32\cdot 27$ however the boomgate is not high enough for Mal to pass through.
From part **e.** we see that he can pass through again at $t = 36\cdot 27$. In total, he will be inside for $36\cdot 27 - 16\cdot 27 = 20$ minutes.

(1 mark)
Total 10 marks

Question 3

a. The graph of $y = f(x)$ intersects with the *x*-axis when

$e^{2x}(x^2 - 2x) = 0$

$e^{2x} \neq 0 \qquad x^2 - 2x = 0$

$x(x-2) = 0$

$x = 0 \text{ or } x = 2$

So, $f(x) > 0$ for $x \in (-\infty, 0) \cup (2, \infty)$ or $x < 0 \cup x > 2$.

(1 mark) lower branch
(1 mark) upper branch

b.

$$f(x) = e^{2x}(x^2 - 2x)$$

$$f'(x) = 2e^{2x}(x^2 - 2x) + e^{2x}(2x - 2) \qquad \text{(product rule)}$$

$$= 2e^{2x}(x^2 - 2x) + 2e^{2x}(x - 1)$$

$$= 2e^{2x}(x^2 - x - 1)$$

(1 mark)

$$= 2e^{2x}\left\{\left(x^2 - x + \frac{1}{4}\right) - \frac{1}{4} - 1\right\}$$

$$= 2e^{2x}\left\{\left(x - \frac{1}{2}\right)^2 - \frac{5}{4}\right\}$$

$$= 2e^{2x}\left(x - \frac{1}{2} - \frac{\sqrt{5}}{2}\right)\left(x - \frac{1}{2} + \frac{\sqrt{5}}{2}\right)$$

Given that

$f'(x) = 2e^{2x}(x - a - b)(x - a + b)$ then $a = \frac{1}{2}$ and $b = \frac{\sqrt{5}}{2}$ as required.

(1 mark)

c. Hence means use what you have already found.
A stationary point occurs when $f'(x) = 0$.

$$2e^{2x}\left(x - \frac{1}{2} - \frac{\sqrt{5}}{2}\right)\left(x - \frac{1}{2} + \frac{\sqrt{5}}{2}\right) = 0$$

$$2e^{2x} \neq 0, \qquad x = \frac{1}{2} \pm \frac{\sqrt{5}}{2}$$

From the graph, at the point where the graph has a minimum stationary point, $x > 0$.

so, $x = \frac{1}{2} + \frac{\sqrt{5}}{2}$

$= 1 \cdot 61803...$

(1 mark)

$f(1 \cdot 61803...) = -15 \cdot 7188...$

Coordinates of point where *f* is a minimum are $(1 \cdot 62, -15 \cdot 72)$ where each coordinate is correct to 2 decimal places.

(1 mark)

d. By observing the graph of $y = f(x)$ for $x \in [-3,3]$ we see that the maximum value of the function occurs at the right endpoint; that is at $x = 3$.

$$f(3) = e^{2\times 3}\left(3^2 - 2\times 3\right)$$
$$= 3e^6$$

So the maximum value over $x \in [-3,3]$ is $3e^6$.

(1 mark)

e. **i.** $f'(x) = 2e^{2x}\left(x^2 - x - 1\right)$ from part **b.**

$$f'(-1) = 2e^{-2}(1+1-1)$$
$$= 2e^{-2}$$

(1 mark)

ii. $f(x) = e^{2x}\left(x^2 - 2x\right)$

$$f(-1) = e^{-2}(1+2)$$
$$= 3e^{-2}$$

$f'(-1) = 2e^{-2}$ from part **e. i.**

So the tangent passes through the point $\left(-1, 3e^{-2}\right)$ and has a gradient of $2e^{-2}$.

The equation of the tangent is

$$y - 3e^{-2} = 2e^{-2}(x+1)$$
$$y = 2e^{-2}x + 2e^{-2} + 3e^{-2}$$
$$y = 2e^{-2}x + 5e^{-2}$$

(1 mark)

When $x = 0$,

$$y = 5e^{-2}$$

The *y*-intercept of the tangent is therefore $5e^{-2}$.

(1 mark)

Total 10 marks

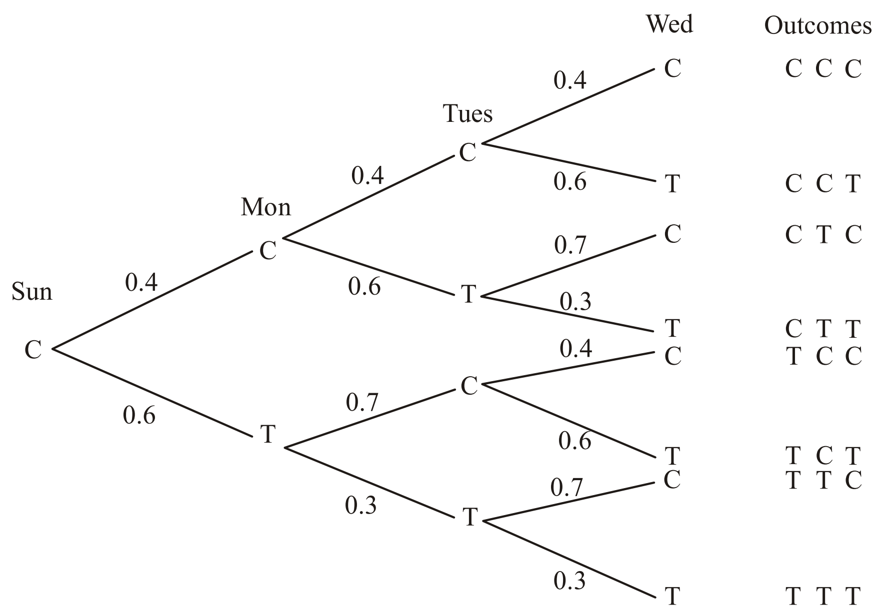
Question 4

a. **i.** Pr(cereal for next 2 days)
$= 0 \cdot 4 \times 0 \cdot 4$
$= 0 \cdot 16$

(1 mark)

ii. Method 1
Use a tree diagram.

Let C = cereal and T = toast.



(1 mark)

Pr(cereal just once over next 3 mornings)
$= 0.4 \times 0.6 \times 0.3 + 0.6 \times 0.7 \times 0.6 + 0.6 \times 0.3 \times 0.7$
$= 0.45$

(1 mark)

Method 2

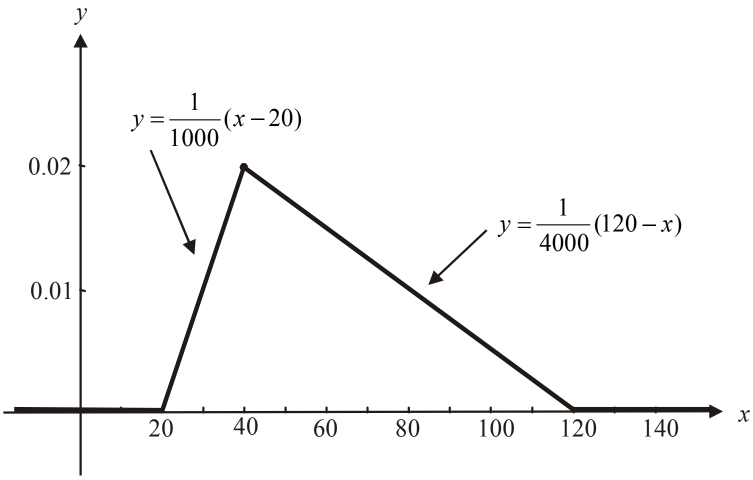
Pr(cereal just once over next 3 mornings)
$= \Pr(CTT, TCT, TTC)$ **(1 mark)**
$= 0.4 \times 0.6 \times 0.3 + 0.6 \times 0.7 \times 0.6 + 0.6 \times 0.3 \times 0.7$
$= 0.45$

(1 mark)

iii. Pr(toast on at least one occasion for next 3 mornings)
$= 1 - \Pr(\text{no toast for the next 3 mornings})$ **(1 mark)**
$= 1 - \Pr(\text{CCC})$
$= 1 - 0.4^3$
$= 0.936$

(1 mark)

b.



(1 mark) correct branch for $20 \le x \le 40$
(1 mark) correct branch for $40 < x \le 120$
(1 mark) correct branch for $x < 20$ and $x > 120$

c. Method 1 – using a calculator

$$\Pr(x < 30) = \frac{1}{1000}\int_{20}^{30}(x-20)\,dx$$
$$= 0 \cdot 05$$

(1 mark)

Method 2 – graphically

$$\Pr(X < 30) = \frac{1}{2} \times 10 \times f(30)$$
$$= 5 \times \left(\frac{1}{1000}(30-20)\right)$$
$$= 0 \cdot 05$$

(1 mark)

d. This represents a binomial distribution.
From part **c.** $\Pr(X < 30) = 0 \cdot 05$; that is, the probability that Liv has less than one cup of cereal for breakfast is 0.05.
Method 1 – by hand
$${}^{10}C_2(0 \cdot 05)^2(0 \cdot 95)^8$$
$= 0 \cdot 075$ (correct to 3 decimal places)

(1 mark)

Method 2 – by calculator
$Bi(10,\ 0 \cdot 05,\ 2) = 0 \cdot 075$ (correct to 3 decimal places)

(1 mark)

e. Find $\Pr(X > 60 | X > 30)$

$$= \frac{P(X > 60 \cap X > 30)}{\Pr(X > 30)}$$

$$= \frac{\Pr(X > 60)}{1 - 0.05}$$ - using the diagram
- using part **c.**

$$= \frac{\Pr(X > 60)}{0.95}$$

Now $\Pr(X > 60) = \int_{60}^{120} \frac{1}{4000}(120 - x)dx$

$$= 0 \cdot 45$$

(1 mark)

So $\Pr(X > 60 | X > 30) = \frac{0 \cdot 45}{0 \cdot 95}$

$$= \frac{9}{19}$$

(1 mark)

f. $\Pr(X < n) = 0 \cdot 8$

$$\Pr(20 < X < 40) + \Pr(40 < X < n) = 0 \cdot 8$$

$$\frac{1}{1000}\int_{20}^{40}(x - 20)dx + \frac{1}{4000}\int_{40}^{n}(120 - x)dx = 0 \cdot 8$$

(1 mark)

$$0 \cdot 2 + \frac{1}{4000}\left[120x - \frac{x^2}{2}\right]_{40}^{n} = 0 \cdot 8$$

$$\frac{1}{4000}\left\{\left(120n - \frac{n^2}{2}\right) - (4800 - 800)\right\} = 0 \cdot 6$$

$$120n - \frac{n^2}{2} - 4000 = 2400$$

$$240n - n^2 - 12800 = 0$$

(1 mark)

$$n^2 - 240n + 12800 = 0$$

$$n = \frac{240 \pm \sqrt{240^2 - 4 \times 1 \times 12800}}{2}$$

$$= \frac{240 \pm \sqrt{6400}}{2}$$

$$= \frac{320}{2} \text{ or } \frac{160}{2}$$

$$n = 160 \text{ or } n = 80$$

Since $20 < n < 120$,

$$n = 80$$

(1 mark)
Total 15 marks

Question 5

a. At the point where $x = a$, the graph of $y = f(x)$ intersects with the graph of $y = w(x)$
.

So, $f(a) = w(a)$.

We need to show that $f(1) = w(1)$.

$$\begin{aligned} LS &= f(1) \\ &= -2 \times 1^2 + (\sqrt{3} - 1) \times 1 + 3 \\ &= -2 + \sqrt{3} - 1 + 3 \\ &= \sqrt{3} \end{aligned}$$

$$\begin{aligned} RS &= \sqrt{4 - 1^2} \\ &= \sqrt{3} \\ &= LS \end{aligned}$$

Have shown.

(1 mark)

b. i. From the graph, the two transformations could be a reflection in the *x*-axis followed by a reflection in the *y*-axis or vice versa.

(1 mark) reflection in *x*-axis
(1 mark) reflection in *y*-axis

ii. If the graph of $y = f(x)$ is reflected in the *x*-axis the rule of the image function is

$$\begin{aligned} y &= -f(x) \\ &= -\left(-2x^2 + (\sqrt{3} - 1)x + 3\right) \\ &= 2x^2 - (\sqrt{3} - 1)x - 3 \end{aligned}$$ **(1 mark)**

If the graph of $y = -f(x)$ is then reflected in the *y*-axis it becomes the graph of

$$\begin{aligned} y &= -f(-x) \\ &= 2(-x)^2 - (\sqrt{3} - 1) \times -x - 3 \\ &= 2x^2 + (\sqrt{3} - 1)x - 3 \end{aligned}$$

So the rule for *g* is $g(x) = 2x^2 + (\sqrt{3} - 1)x - 3$.

(1 mark)

iii. From the graph, the domain of *g* is $x \in [-1, 0]$.

(1 mark)

c. **i.** The highest point will occur on the function *f*.

$$y = -2x^2 + \left(\sqrt{3} - 1\right)x + 3$$

$$\frac{dy}{dx} = -4x + \sqrt{3} - 1 = 0$$

(1 mark)

$$-4x = 1 - \sqrt{3}$$

$$x = \frac{\sqrt{3} - 1}{4}$$

(1 mark)

ii. $f\left(\frac{\sqrt{3}-1}{4}\right) = 3 \cdot 07$ (correct to 2 decimal places).

The maximum height of the machine is $5 + 3 \cdot 07 = 8 \cdot 07\text{m}$ above the ground (correct to 2 decimal places).

(1 mark)

d. From part **c.** the turning point of the inverted parabola described by $y = f(x)$ is $\left(\frac{\sqrt{3} - 1}{4}, 3 \cdot 07\right)$.

Hence means use what you have found so we use this rounded value of $3 \cdot 07$.

At the point where $x = a$, i.e. $x = 1$ (from part **a.**), $f(1) = \sqrt{3}$.

So $r_f = \left[\sqrt{3}, 3.07\right]$.

(1 mark) for $\sqrt{3}$ and correct bracket

(1 mark) for $3 \cdot 07$ and correct bracket

e. Cross-sectional area of top bucket

$$= \int_0^1 \left(f(x) - w(x)\right) dx$$

$$= \int_0^1 \left(-2x^2 + \left(\sqrt{3} - 1\right)x + 3 - \sqrt{4 - x^2}\right) dx$$

(1 mark) correct integrand

(1 mark) correct terminals

f. From part **i.**

$$\text{Area required} = \int_0^1 \left(f(x) - w(x)\right)dx$$

$$= \int_0^1 f(x)dx - \int_0^1 w(x)dx$$

$$= \int_0^1 \left(-2x^2 + \left(\sqrt{3}-1\right)x + 3\right)dx - \left(\frac{\sqrt{3}}{2} + \frac{\pi}{3}\right)$$ **(1 mark)**

$$= \left[\frac{-2x^3}{3} + \left(\sqrt{3}-1\right)\frac{x^2}{2} + 3x\right]_0^1 - \left(\frac{\sqrt{3}}{2} + \frac{\pi}{3}\right)$$ **(1 mark)**

$$= \left\{\left(\frac{-2}{3} + \left(\sqrt{3}-1\right)\times\frac{1}{2} + 3\right) - (0+0+0)\right\} - \left(\frac{\sqrt{3}}{2} + \frac{\pi}{3}\right)$$

$$= \frac{-2}{3} + \frac{\sqrt{3}-1}{2} + \frac{9}{3} - \frac{\sqrt{3}}{2} - \frac{\pi}{3}$$

$$= \frac{7}{3} + \frac{\sqrt{3}}{2} - \frac{1}{2} - \frac{\sqrt{3}}{2} - \frac{\pi}{3}$$

$$= \frac{11}{6} - \frac{\pi}{3} \text{ square units}$$

(1 mark)
Total 16 marks