Extended answers to selected multiple-choice questions

Test 1

QUESTION 12

Using (2, 1), $B = -2$ and $C = 1$

$y = a(x - 2)^3 + 1$

Using (0, 5),

$5 = a(0 - 2)^3 + 1$

$a = -\frac{1}{2}$

$\therefore$ E

QUESTION 14

$(x - 3)^3 = \frac{1}{2}$

$x - 3 = \sqrt[3]{\frac{1}{2}}$

$x = 3 + \sqrt[3]{\frac{1}{2}}$

$x = 3 + \frac{1}{\sqrt[3]{2}}$

$\therefore$ B

Test 2

QUESTION 11

The domain is $x \in [-2, 4)$, the range is $y \in [1, 4)$ and the gradient is $\frac{1}{2}$.

$\therefore$ B

QUESTION 12

Using CAS: graph $f(x)$, find minimum value of $-\frac{1}{4}$.

$\therefore$ E

QUESTION 13

Using CAS: define $f(x)$. $f(x - 1) - f(x)$

$\therefore$ A

QUESTION 14

For maximal domain,

$x + 1 \geq 0$

$x \geq -1$

The domain is $[-1, \infty)$.

$\therefore$ D

QUESTION 15

$y = \sqrt{5 - x^2}$ is a semi-circle

$x = \pm 1$, $y = 2$, a many-to-one function.

$\therefore$ B

Test 3

QUESTION 11

$PQ = [-2 \times -1 + 1 \times 0 + 3 \times 4]$

$= [14]$

$\therefore$ D

QUESTION 12

Dimension of AB = $(2 \times 2)(2 \times 3) = 2 \times 3$

$\therefore$ B

QUESTION 13

The dimension of the original matrices should be (3×3), (3×1) and (3×1). A is undefined, C gives (1×3), the LHS of D is (1×3) and RHS is (3×1). E does not give the correct equations.

$\therefore$ B

QUESTION 15

$\text{speed} = \frac{9 - 0}{0 - 3}$

$= -3$ m/s

$\therefore$ D

Test 4

QUESTION 11

There are 13 hearts in the pack, including the ace of hearts plus three more aces, making 16 cards.
Probability is $\frac{16}{52} = \frac{4}{13}$.

$\therefore$ B

QUESTION 12

There are five salad sandwiches in the box.
Probability is $\frac{5}{12}$.

$\therefore$ E

QUESTION 13

$\Pr(P \cup Q) = \Pr(P) + \Pr(Q) - \Pr(P \cap Q)$

$= 0.35 + 0.4 - 0.25$

$= 0.5$

$\therefore$ C

QUESTION 14

$\Pr(P|Q) = \frac{\Pr(P \cap Q)}{\Pr(Q)}$

$= \frac{0.25}{0.4}$

$= 0.625$

$\therefore$ C

QUESTION 15

Since $\Pr(P) \times \Pr(Q) \neq \Pr(P \cap Q)$, P and Q are not independent events.

Since $\Pr(P) + \Pr(Q) \neq \Pr(P \cup Q)$, P and Q are not mutually exclusive.

	P	P'	
Q	$\Pr(P \cap Q) = 0.25$	$\Pr(P' \cap Q) = 0.15$	$\Pr(Q) = 0.4$
Q'	$\Pr(P \cap Q') = 0.1$	$\Pr(P' \cap Q') = 0.5$	$\Pr(Q') = 0.6$
	$\Pr(P) = 0.35$	$\Pr(P') = 0.65$	

From Karnaugh table, E is correct.

Test 5

QUESTION 7

The first place can be filled in 7 ways, the second in 6, the third in 5 . . . thus $\frac{7!}{1!} = 7!$

$\therefore$ C

QUESTION 8

The fiction can be arranged in 3! ways and the non-fiction in $4! = 3! \times 4! = 144$

$\therefore$ D

QUESTION 9

Select 3 from 15 $= {}^{15}C_3 = \frac{15!}{3! \times 12!}$

$\therefore$ E

QUESTION 10

Arranging any of the five letters $= 8 \times 7 \times 6 \times 5 \times 4 = 6720$.

Ending in a vowel means the last place can be filled in three ways, and the other places in seven, then six, then five, then four ways $= 7 \times 6 \times 5 \times 4 \times 3 = 2520$.

Probability $= \frac{3}{8}$.

$\therefore$ C

QUESTION 11

If any four books are selected, $\binom{11}{4} = 330$.

If two of each are selected, $\binom{6}{2} \times \binom{5}{2} = 150$.

Probability $= \frac{150}{330} = \frac{5}{11}$.

$\therefore$ A

Test 6

QUESTION 10

Using CAS: 33°C.

$\therefore$ C

QUESTION 11

Least value when $x \to \infty$, thus $2 \times 10^{-x} \to 0$, $f(x) \to -1$ or, using technology, work from the graph.

$\therefore$ D

QUESTION 12

$3^{2x} = 3^1$

$2x = 1$

$x = \frac{1}{2}$

$\therefore$ B

QUESTION 13

Vertical asymptote is $x = a$, where a is positive. There are no reflections in either axis.

$\therefore$ E

QUESTION 14

$\log_2(1 - x) = 3$

$1 - x = 2^3$

$x = 1 - 2^3$

$= -7$

$\therefore$ D

Test 7

QUESTION 10

From the graph, four asymptotes.

$\therefore$ E

QUESTION 11

The amplitude is a; the vertical translation is c. The range is $[c - a, c + a]$.

$\therefore$ C

QUESTION 12

From the graph, there are eight solutions.

$\therefore$ D

QUESTION 13

From the graph: $0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}, 2\pi$.

$\therefore$ B

Test 8

QUESTION 10

From the graph, $x \leq 2\frac{1}{2}$.

$\therefore$ A

QUESTION 11

Instantaneous rate:

$V = 10t^2 - t^3$

$\frac{dV}{dt} = 20t - 3t^2$

$t = 5, \frac{dV}{dt} = 25$ L/min

$\therefore$ D

QUESTION 12

Equation of derivative: $\frac{dy}{dx} = 3x^2 - b$.

For stationary points: $\frac{dy}{dx} = 0$.

$3x^2 - b = 0$

$x = \pm\frac{\sqrt{3b}}{3}$

$\therefore$ D

QUESTION 13

$v = \frac{dx}{dt} = 4t - 1$

$\therefore$ C

QUESTION 14

$x < 0$, $\frac{dy}{dx}$ is positive;

$x = 0$, $\frac{dy}{dx} = 0$;

$0 < x < 3$,

$\frac{dy}{dx}$ is positive;

$x = 3$, $\frac{dy}{dx} = 0$,

$x > 3$, $\frac{dy}{dx}$ is negative.

$\therefore$ C

Test 9

QUESTION 11

Midpoint: $\frac{x-8}{2} = -1 \quad \frac{y+4}{2} = 8 \quad 6 + 12 = 18$

$x = 6 \quad y = 12$

$\therefore$ A

QUESTION 12

Time $= \frac{\text{distance}}{\text{speed}}$. Total time (hours) for 10 laps $= 7 \times \frac{x}{35} + 3 \times \frac{x}{15} = \frac{48}{60}$, where x km is the length of one circuit. $x = 2$ km.

$\therefore$ D

QUESTION 13

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

$\therefore$ E

QUESTION 14

Turning point is (k, k). Graph will touch x axis when $k = 0$.

$\therefore$ A

QUESTION 15

The centre is the midpoint between (2, 3) and (−4, 3): (−1, 3); the radius is midway between (2, 3) and (−4, 3): 3 units. The equation is $(x + 1)^2 + (y - 3)^2 = 9$.

$\therefore$ D

QUESTION 16

Since $f(x) = f(-x)$ for $f(x) = 1 - x^2$, it is not a one-one function.

$\therefore$ C

QUESTION 17

The reflection in the x axis is followed by a translation of −1 from the y axis.

$\therefore$ B

QUESTION 18

Four choose backstroke. Probability is $\frac{4}{20} = \frac{1}{5}$.

$\therefore$ E

QUESTION 19

General shape is $y = a - Mx^2$.

Using $(b, 0)$, $0 = a - b^2M$.

$M = \frac{a}{b^2}$

Thus $y = a - \frac{a}{b^2}x^2$

$= a\left(1 - \left(\frac{x}{b}\right)^2\right)$

$\therefore$ E

QUESTION 20

The equation of the image is $y = 2\sqrt{1 - x}$.

The y intercept is $y = 2\sqrt{1}$

$= 2$.

$\therefore$ A

Test 10

QUESTION 10

The equation is $y = -2^{-x} + 1$.

$\therefore$ B

QUESTION 11

$x - 1 = a^b$

$x = 1 + a^b$

$\therefore$ A

QUESTION 12

$x = \log_a (b - y)$

$a^x = b - y$

$y = b - a^x$

$\therefore$ E

QUESTION 13

$\cos(\theta) = \pm\sqrt{1 - \sin^2(\theta)}$

$= \pm\sqrt{1 - \frac{1}{9}}$

$= \pm\sqrt{\frac{8}{9}} = \pm\frac{2\sqrt{2}}{3}$

Since $\frac{\pi}{2} \le \theta \le \pi$, $\cos(\theta) = -\frac{2\sqrt{2}}{3}$.

$\therefore$ D

QUESTION 14

Find points of intersection between the graphs of $y = 2^{-x} - 1$ and $y = \sin(4x)$, $0 \le x \le 2\pi$. There are nine points of intersection and nine solutions.

$\therefore$ E

QUESTION 16

$f'(x) = 2x + \frac{2}{x^3}$

$f'(-1) = -2 + \frac{2}{-1} = -4$.

$\therefore$ C

QUESTION 17

Positive gradient from R to T

$\therefore$ B

QUESTION 18

Area from Q to S: $-\int_{Q}^{S} f(x)dx = \int_{S}^{Q} f(x)dx$.

Area from S to U: $\int_{S}^{U} f(x)dx$.

Total area: $\int_{S}^{Q} f(x)dx + \int_{S}^{U} f(x)dx$.

$\therefore$ B

QUESTION 19

The transition matrix is $T = \begin{bmatrix} 0.85 & 0.35 \\ 0.15 & 0.65 \end{bmatrix}$.

The equation is $T^4 S_0 = \begin{bmatrix} 0.85 & 0.35 \\ 0.15 & 0.65 \end{bmatrix}^4 \times \begin{bmatrix} 80 \\ 20 \end{bmatrix} = \begin{bmatrix} 71 \\ 29 \end{bmatrix}$.

Thus, 71 are expected to pass the test in four weeks.

$\therefore$ C