Test 7

Section A: Technology free. 47 marks
Section B: CAS technology assumed. 43 marks
Suggested time: 90 minutes

Section A: Short answer and extended response questions. Technology free.

Specific instructions to students

- Answer **all** questions in the spaces provided.
- A decimal approximation will not be accepted if an **exact** answer is required to a question.
- In questions where more than one mark is available, appropriate working **must** be shown.

QUESTION 1 — Total 2 marks

a Write 130° in radians, in terms of π. — 1 mark

$130 \times \frac{\pi}{180} = \frac{13\pi}{18}$

b Write $\frac{15\pi}{6}$ radians in degrees. — 1 mark

$\frac{15\pi}{6} \times \frac{180}{\pi} = 450°$

QUESTION 2

Give the exact value of the following: — 3 marks

a $\sin(210°)$

$\sin(180 + 30)°$
$= -\sin(30)° = -\frac{1}{2}$

b $\cos\left(-\frac{5\pi}{3}\right)$

$\cos\left(2\pi - \frac{\pi}{6}\right)$
$= \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

c $\tan\left(\frac{3\pi}{4}\right)$

$\tan\left(\pi - \frac{\pi}{4}\right)$
$= -\tan\left(\frac{\pi}{4}\right) = -1$

QUESTION 3

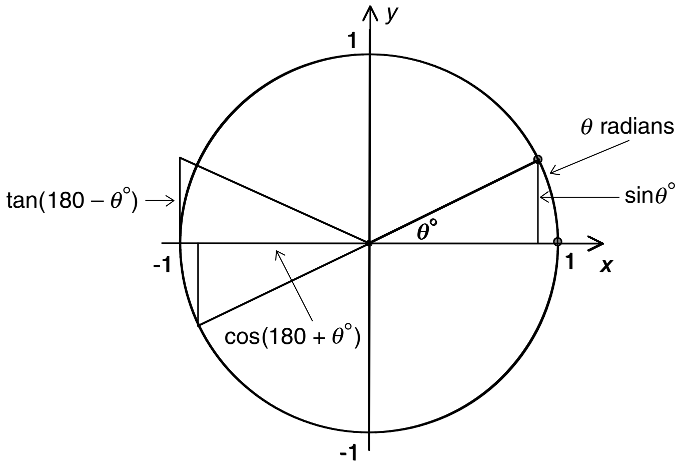
The diagram represents a unit circle with an angle $\theta°$ subtended at the centre, as shown. On the diagram mark the following: — 4 marks

a $\sin(\theta°)$

b $\cos(180° + \theta°)$

c $\tan(180° - \theta°)$

d θ radians



QUESTION 4 — Total 6 marks

a Given $\sin(x) = \frac{1}{3}$ and $\frac{\pi}{2} \le x \le \pi$, use the formula $\cos^2(x) + \sin^2(x) = 1$ to find $\cos(x)$. — 3 marks

$\cos^2(x) = 1 - \sin^2(x)$
$= 1 - \frac{1}{9} = \frac{8}{9}$

$\cos(x) = \pm\frac{\sqrt{8}}{3} = \pm\frac{2\sqrt{2}}{3}$

Since x is in the second quadrant, $\cos(x) = -\frac{2\sqrt{2}}{3}$.

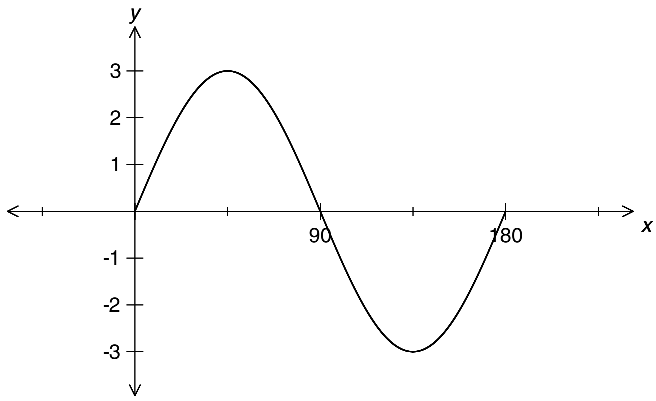
b Solve $\cos(x) = 1, 0 \le x \le 2\pi$. — 3 marks

$x = \cos^{-1}(1)$
$x = 0, 2\pi$

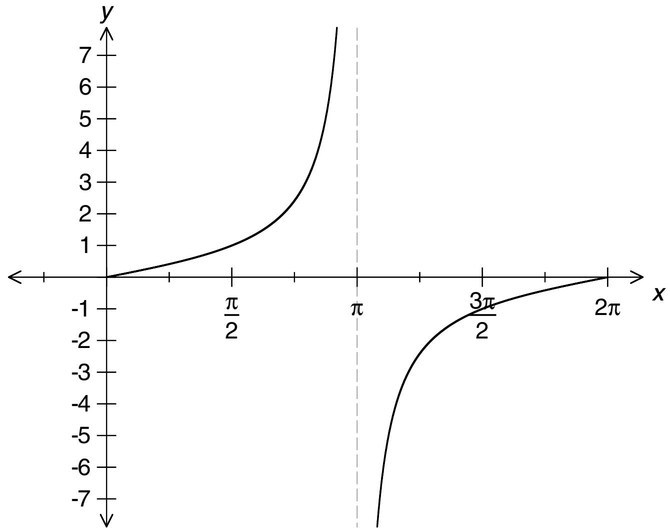
QUESTION 5 — Total 8 marks

Sketch the graph of each of the following for one cycle.

a $y = 3\sin(2x°)$ — 4 marks

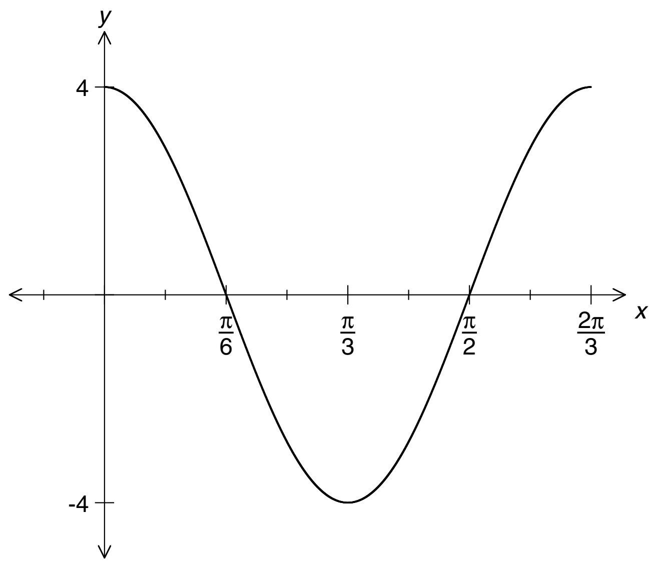


b $y = \tan\left(\frac{x}{2}\right)$ **4 marks**



QUESTION 6

The graph of $y = -4\sin\left(3\left(x - \frac{\pi}{6}\right)\right)$ is shown. By observing the shape of the graph, write its equation in the form $y = a\cos(bx)$. **4 marks**



The amplitude is 4. The period is $\frac{2\pi}{3}$.

Thus $b = \frac{2\pi}{\left(\frac{2\pi}{3}\right)} = 3$.

$\therefore y = 4\cos(3x)$

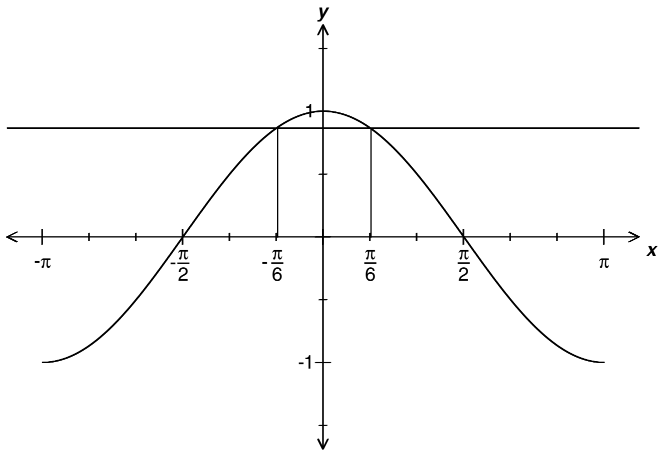
QUESTION 7 — Total 10 marks

a Solve $\cos(x) = \frac{\sqrt{3}}{2}, x \in \left[0, \frac{\pi}{2}\right]$. **2 marks**

$x = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{\pi}{6}$

b Sketch the graphs of $y = \cos(x), x \in [-\pi, \pi]$ and $y = \frac{\sqrt{3}}{2}$. **4 marks**



c Hence, find: **2 marks**

i $\left\{x : \cos(x) = \frac{\sqrt{3}}{2}, x \in [-\pi, \pi]\right\}$

By symmetry of the graph, $x = \pm\frac{\pi}{6}$.

ii $\left\{x : \cos(x) > \frac{\sqrt{3}}{2}, x \in [-\pi, \pi]\right\}$ **2 marks**

From the graph, $-\frac{\pi}{6} < x < \frac{\pi}{6}$.

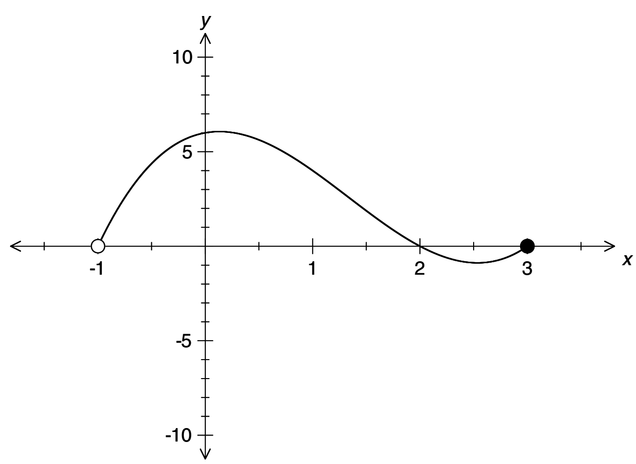
QUESTION 8 — Total 10 marks

a If $(x + 1)$ is a factor of $x^3 - 4x^2 + x + 6$, use long division to show that $(x - 2)$ and $(x - 3)$ are the other linear factors. **4 marks**

$$\begin{array}{r} x^2 - 5x + 6 \\ x + 1 \overline{)\, x^3 - 4x^2 + x + 6} \\ \underline{x^3 + x^2} \qquad\qquad \\ -5x^2 + x \qquad \\ \underline{-5x^2 - 5x} \qquad \\ 6x + 6 \\ \underline{6x + 6} \\ 0 \end{array}$$

Thus $(x + 1)(x^2 - 5x + 6) = (x + 1)(x - 2)(x - 3)$.

b Hence, sketch the graph of $f(x) = x^3 - 4x^2 + x + 6$ on the domain $x \in (1, 3]$. **4 marks**



c Find the average rate of change from $x = 0$ to $x = 2$.

2 marks

Average rate of change $= \dfrac{f(2) - f(0)}{2 - 0} = \dfrac{0 - 6}{2} = -3$

Section B: Multiple-choice questions. CAS technology assumed.

Specific instructions to students

- A correct answer scores 1, and an incorrect answer scores 0.
- Marks are not deducted for incorrect answers.
- No marks are given if more than one answer is given.
- Choose the alternative which most correctly answers the question and mark your choice on the multiple-choice answer section at the bottom of each page, as shown in the example below.

1 A B C D E **USE PENCIL ONLY**

- Use pencil only.

QUESTION 9

The amplitude and period of the graph $f: [0, 2\pi] \to R$, $f(x) = -3\sin(2\pi x) + 1$ are:

	Amplitude	Period
A	-2	2π
B	4	2π
C	3	2π
D	-3	1
E	3	1

QUESTION 10

The number of asymptotes for the graph of $y = -\tan(2x)$, $-\frac{3\pi}{4} \le x \le \frac{3\pi}{4}$ is:

A 0

B 1

C 2

D 3

E 4

QUESTION 11

The range of the function $f\,[0, 2\pi] \to R$ where $f(x) = a\cos(bx) + c$, where a, b and c are positive numbers, is:

A R

B $[-a, a]$

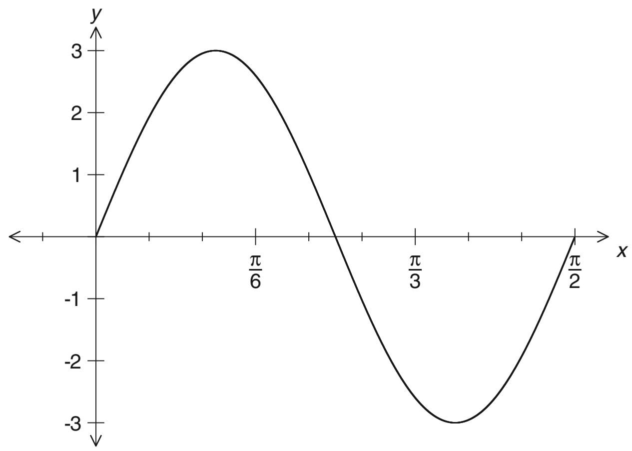
C $[c - a, c + a]$

D $[-a, 2\pi + a]$

E $[b - a, b + a]$

QUESTION 12

The graph of $y = 3\sin(4x)$ is shown for one cycle.



How many solutions are there for the equation $3\sin(4x) = 1$, $x \in [-\pi, \pi]$?

A 2

B 4

C 6

D 8

E 10

QUESTION 13

The graph of $y = 2\sin(3x)$, $0 \le x \le 2\pi$ has x intercepts at:

A $0, \pi, 2\pi$

B $0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}, 2\pi$

C $\frac{\pi}{4}, \frac{\pi}{2}$

D $0, \frac{\pi}{3}, \frac{\pi}{2}$

E $0, \frac{\pi}{4}, \frac{2\pi}{3}$

ONE ANSWER PER LINE **USE PENCIL ONLY**

9 A B C D E

10 A B C D E

11 A B C D E

12 A B C D E

13 A B C D E

Section B: Extended response questions. CAS technology assumed.

Specific instructions to students

- Answer **all** questions in the spaces provided.
- In questions where more than one mark is available, appropriate working **must** be shown.

QUESTION 14 **Total 15 marks**

a **i** Find the first quadrant (smallest positive) solution for $2\sin(x) = 1$. **1 mark**

$\sin(x) = \frac{1}{2}$

$x = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

ii Given that the general solution of $\sin(x) = a$ is $x = n\pi + (-1)^n \sin^{-1}(a)$, where $n \in \{0, \pm1, \pm2, \ldots\}$ and $a \in [-1, 1]$. Write the general solution for $2\sin(x) = 1$ and find x when $n = \{0, 1, 2\}$. **5 marks**

From part (a), $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$.

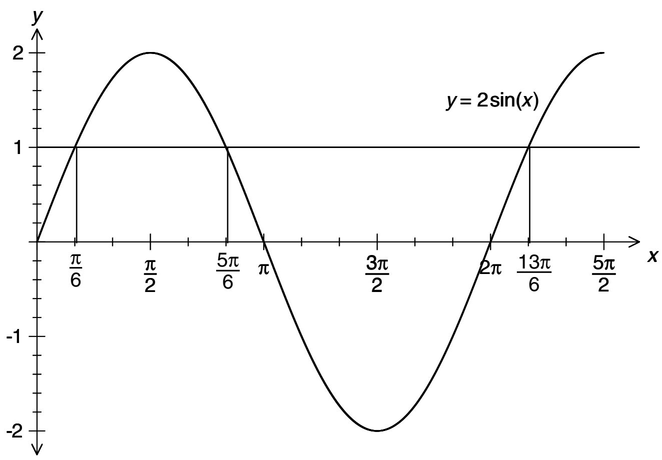
General solution is $x = n\pi + (-1)^n \times \frac{\pi}{6}$.

$n = 0$, $x = \frac{\pi}{6}$

$n = 1$, $x = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

$n = 2$, $x = 2\pi + \frac{\pi}{6} = \frac{13\pi}{6}$

iii Locate the solutions on the graph shown. **3 marks**



b Let $f(x) = 1 - 2\cos(2x)$. **2 marks**

i State the maximum and minimum values of $f(x)$.

Maximum when $\cos(2x) = -1 \Rightarrow f(x) = 1 + 2 = 3$.
$\therefore$ Maximum value is 3.

Minimum value when $\cos(2x) = 1 \Rightarrow f(x) = 1 - 2 = -1$.
$\therefore$ Minimum value is -1.

ii Give the exact x value of the closest maximum point to the y axis. **2 marks**

Using CAS: SOLVE $1 - 2\cos(2x) = 3$

$x = \frac{\pi}{2}$

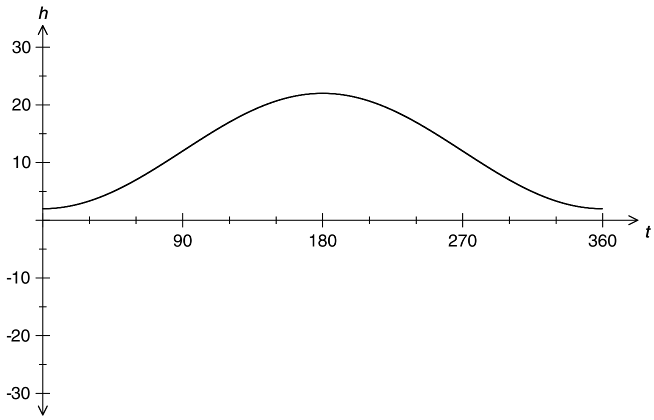
iii Give the value of the smallest positive x intercept, correct to four decimal places. **2 marks**

From graph (using CAS): $x = 0.5236$

QUESTION 15 **Total 11 marks**

At a certain town in the Arctic circle, the number of hours of sunlight in a day varies and is given by the formula $h(t) = 12 - 10\cos\left(\frac{\pi}{180}t\right)$, where h is the number of hours of sunlight on any day t. (Assume $h(t)$ is a continuous function.)

a Sketch one cycle of the graph of $h(t)$. **3 marks**



b What season of the year occurs when $t = 0$? **1 mark**

Winter

c What is the maximum and minimum amount of sunlight on any given day? **2 marks**

22 hours maximum; 2 hours minimum.

d Find values of t (to the nearest day) when there are 7 hours of sunlight. **2 marks**

From graph: $t = 60, 300$.

For safety reasons, the streetlights are left on for 24 hours a day when the daily sunlight falls below 5 hours.

e Find, to the nearest day, the number of days the streetlights are left on for 24 hours. **3 marks**

From the graph, find points of intersection between $h(t)$ and $h = 5$. $t = 46, 314$.

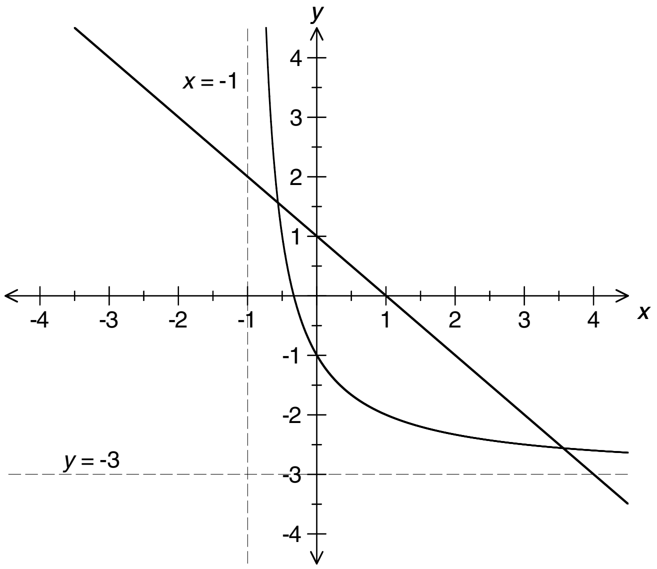
Number of days $= 46 + (360 - 314) = 92$ days

QUESTION 16 **Total 12 marks**

a State the transformations that give $y = \frac{2}{x+1} - 3$ as the image of $y = \frac{1}{x}$. **3 marks**

Dilation of 2 from the x axis; translation of -1 from the y axis and -3 from the x axis.

b Sketch the graph of $f: (-1, \infty) \to R, f(x) = \frac{2}{x+1} - 3$. Label any x and y intercepts and any asymptotes. **5 marks**



c On the same axes, sketch the graph of $y = 1 - x$. **1 mark**

See graph solution in part **b**.

d Hence, solve, correct to two decimal places, $\frac{2}{x+1} = 4 - x$. **3 marks**

Find points of intersection from graph.

CAS:

or

Solve $\frac{2}{x+1} - 3 = 1 - x$

$$\frac{2}{x+1} = 4 - x$$

$$x = -0.56, 3.56$$