

Section A: Short answer and extended response questions. Technology free.

Specific instructions to students

- Answer **all** questions in the spaces provided.
- A decimal approximation will not be accepted if an **exact** answer is required to a question.
- In questions where more than one mark is available, appropriate working **must** be shown.

QUESTION 1

Total 5 marks

Ten cards are numbered 1 to 10.

- a Write down the sample space for numbers that can be drawn from the 10 cards. **1 mark**

Sample space is {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}.

- b A card is drawn at random. What is the probability that the card is:
 i an odd number? **1 mark**

Odd numbers = {1, 3, 5, 7, 9}
 Number of odd numbers = 5
 $\text{Pr}(\text{odd}) = \frac{5}{10} = \frac{1}{2}$

- ii divisible by 5? **1 mark**

Divisible by 5 = {5, 10}. $\text{Pr}(\text{divisible by 5}) = \frac{2}{10} = \frac{1}{5}$.

- iii odd and divisible by 5? **1 mark**

Odd and divisible by 5 = {5}
 $\text{Pr}(\text{odd and divisible by 5}) = \frac{1}{10}$

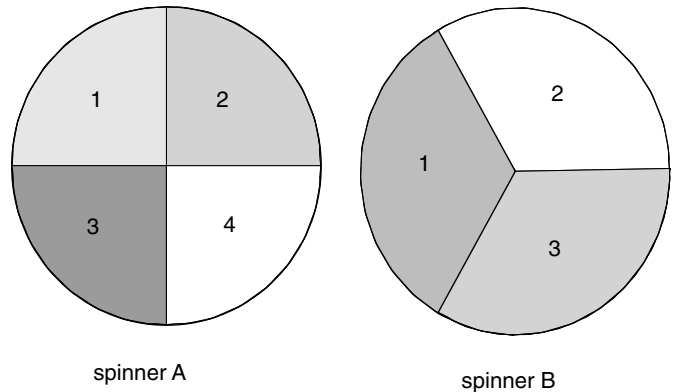
- iv odd or divisible by 5? **1 mark**

Odd or divisible by 5 = {1, 3, 5, 7, 9, 10}
 $\text{Pr}(\text{odd or divisible by 5}) = \frac{6}{10} = \frac{3}{5}$

QUESTION 2

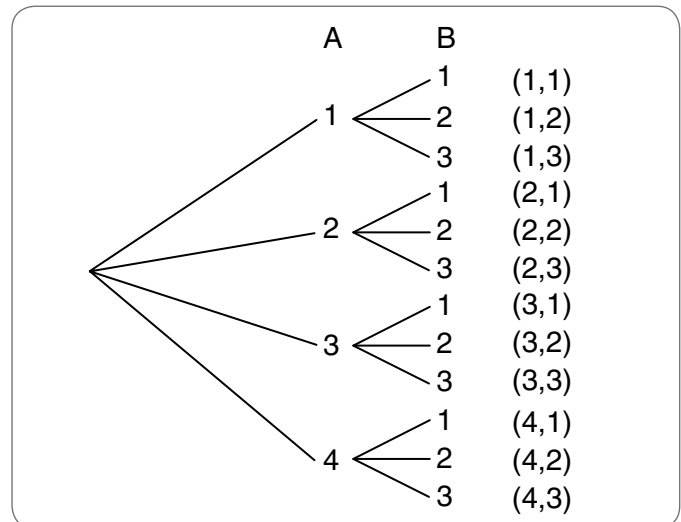
Total 5 marks

Spinner A is divided into 4 equal parts and spinner B into 3 equal parts. Each spinner is numbered as shown.



Each spinner is spun once and the number closest to the top of the page is recorded.

- a Set up a tree diagram of the sample space for this activity. **2 marks**



- b What is the probability that the number on spinner B is greater than the number on spinner A? **2 marks**

{(1, 2), (1, 3), (2, 3)}. Probability = $\frac{3}{12} = \frac{1}{4}$.

- c What is the probability of getting a 2 on spinner B if the number on spinner A is greater than 2? **1 mark**

Spinner A must be a 3 or 4.
 $\text{Pr}(2 \text{ on spinner B given a 3 or 4 on spinner A}) = \frac{2}{6} = \frac{1}{3}$.

QUESTION 3

Total 5 marks

A die is biased so that $\text{Pr}(\text{one}) = \text{Pr}(\text{three}) = \text{Pr}(\text{six}) = \frac{1}{6}$, $\text{Pr}(\text{two}) = \text{Pr}(\text{five}) = \frac{1}{12}$.

- a What is the probability of a 4 being rolled? **2 marks**

$\text{Pr}(\text{four}) = 1 - \left(\frac{3}{6} + \frac{2}{12}\right)$
 $= \frac{1}{3}$

- b The die is thrown twice. What is the probability of:
i throwing a 6 on the first throw and a 4 on the second throw? **1 mark**

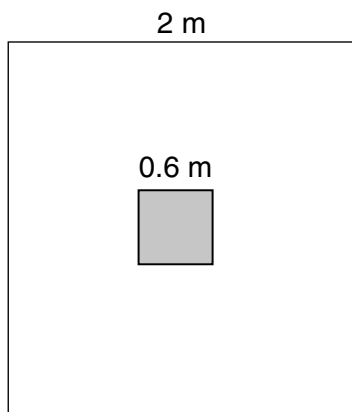
$$\frac{1}{6} \times \frac{1}{3} = \frac{1}{18}$$

- ii** a 6 and a 4 in any order on the two throws? **2 marks**

$$\frac{1}{18} + \frac{1}{18} = \frac{1}{9}$$

QUESTION 4 **Total 5 marks**

A game on a sports show consists of hitting a small square painted onto a larger square, as shown.



- a Evaluate the area of the each square. **2 marks**

Area of smaller square = 0.36 m^2
 Area of larger square = 4 m^2

- b Find the probability of hitting the smaller square. **2 marks**

$$\text{Pr}(\text{hitting the circle}) = \frac{0.36}{4} = 0.09$$

- c Find the probability of hitting the smaller square twice in 2 throws. **1 mark**

$$0.09 \times 0.09 = 0.0081$$

QUESTION 5 **Total 5 marks**

Given an experiment such that $\text{Pr}(A) = 0.4$, $\text{Pr}(B) = 0.5$ and $\text{Pr}(A \cup B) = 0.6$, find the following probabilities:

- a $\text{Pr}(A')$ **1 mark**

$$\text{Pr}(A') = 1 - \text{Pr}(A) = 0.6$$

- b $\text{Pr}(A \cap B)$ **2 marks**

$$\begin{aligned} \text{Pr}(A \cap B) &= \text{Pr}(A) + \text{Pr}(B) - \text{Pr}(A \cup B) \\ &= 0.4 + 0.5 - 0.6 \\ &= 0.3 \end{aligned}$$

- c Represent this data as a Karnaugh square. **1 mark**

	A	A'	
B	$\text{Pr}(A \cap B) = 0.3$	$\text{Pr}(A' \cap B) = 0.2$	$\text{Pr}(B) = 0.5$
B'	$\text{Pr}(A \cap B') = 0.1$	$\text{Pr}(A' \cap B') = 0.4$	$\text{Pr}(B') = 0.5$
	$\text{Pr}(A) = 0.4$	$\text{Pr}(A') = 0.6$	

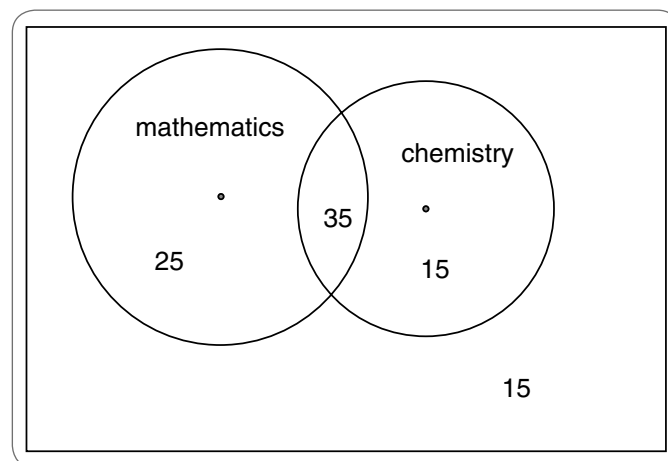
- d $\text{Pr}(A' \cap B)$ **1 mark**

From **c**, $\text{Pr}(A' \cap B) = 0.2$.

QUESTION 6 **Total 4 marks**

In a Year Eleven group of 90 students, 60 study mathematics, 50 study chemistry and 35 study both mathematics and chemistry.

- a Draw a Venn diagram to represent this information. **2 marks**



- b A student is selected at random. What is the probability that the student:

- i** studies mathematics but not chemistry? **1 mark**

$$\frac{25}{90} = \frac{5}{18}$$

- ii** studies neither mathematics nor chemistry? **1 mark**

$$\frac{15}{90} = \frac{1}{6}$$

QUESTION 7 **Total 8 marks**

To determine the insurance risk of a young married couple, Mary and Arthur, it is estimated that the probability Mary will be alive in 50 years time is 0.8 and the probability Arthur will be alive in 50 years time is 0.6. Their probabilities are independent. Find the probability that in 50 years' time:

- a both Mary and Arthur will be alive. **1 mark**

Probability Mary will be alive and Arthur will be alive = $0.8 \times 0.6 = 0.48$

- b only one will be alive. **2 marks**

$$\begin{aligned} &\text{Pr}(\text{Mary will be alive and Arthur will not be alive}) \\ &+ \text{Pr}(\text{Mary will not be alive and Arthur will be alive}) \\ &= 0.8 \times 0.4 + 0.2 \times 0.6 = 0.44 \end{aligned}$$

c at least one will be alive. 2 marks

$$\begin{aligned} & \text{Pr(at least one will be alive)} \\ &= 1 - \text{Pr(neither will be alive)} \\ &= 1 - 0.2 \times 0.4 = 0.92 \end{aligned}$$

d if one of them is alive it is Arthur. 3 marks

$$\begin{aligned} & \text{Pr(Arthur | one will be alive)} \\ &= \frac{\text{Pr(Arthur will be alive} \cap \text{one will be alive)}}{\text{Pr(one will be alive)}} \\ &= \frac{\text{Pr(Arthur will be alive and Mary will not be alive)}}{\text{Pr(one will be alive)}} \\ &= \frac{0.6 \times 0.2}{0.44} \\ &= \frac{3}{11} \end{aligned}$$

QUESTION 8 Total 5 marks

Factorise the following expressions.

a $(2x - 1)^2 - (3 - x)^2$ 1 mark

$$\begin{aligned} & [(2x - 1) - (3 - x)][(2x - 1) + (3 - x)] \\ &= (2x - 1 - 3 + x)(2x - 1 + 3 - x) \\ &= (3x - 4)(x + 2) \end{aligned}$$

b $6x^2 - 7x - 20$ 1 mark

$$(2x - 5)(3x + 4)$$

c $27 - (x + 2)^3$ 3 marks

$$\begin{aligned} & 3^3 - (x + 2)^3 \\ &= [3 - (x + 2)][3^2 + 3(x + 2) + (x + 2)^2] \\ &= [1 - x][9 + 3x + 6 + x^2 + 4x + 4] \\ &= (1 - x)(x^2 + 7x + 19) \end{aligned}$$

QUESTION 9 Total 3 marks

Given $f(x) = 2(x - 3)^3 - 1$,

a evaluate the following:

i $f(-2)$ 1 mark

$$f(-2) = 2(-2 - 3)^3 - 1 = -251$$

ii $f(3 - \sqrt{2})$. Express the answer in the form $a\sqrt{2} + b$, where a and b are integers. 2 marks

$$\begin{aligned} f(3 - \sqrt{2}) &= 2(3 - \sqrt{2} - 3)^3 - 1 \\ &= 2(-\sqrt{2})^3 - 1 \\ &= 2 \times -2\sqrt{2} - 1 \\ &= -4\sqrt{2} - 1 \end{aligned}$$

QUESTION 10 Total 5 marks

The straight line with the equation $ax + by = 10$ passes through the points $(-2, 2)$ and $(2, 8)$.

a Write two simultaneous equations that could be used to find the values of a and b . 1 mark

$$\begin{aligned} (-2, 2): -2a + 2b &= 10 && \text{Equation 1} \\ (2, 8): 2a + 8b &= 10 && \text{Equation 2} \end{aligned}$$

b Hence, find the values of a and b . 2 marks

Equation 1 + Equation 2:

$$10b = 20$$

$$b = 2$$

$$2a + 16 = 10$$

$$2a = -6$$

$$a = -3$$

c State the gradient and y intercept of the line. 2 marks

Equation is:

$$-3x + 2y = 10$$

$$y = \frac{3}{2}x + 5$$

Gradient is $\frac{3}{2}$, y intercept is 5.

Section B: Multiple-choice questions. CAS technology assumed.

Specific instructions to students

- A correct answer scores 1, and an incorrect answer scores 0.
- Marks are not deducted for incorrect answers.
- No marks are given if more than one answer is given.
- Choose the alternative which most correctly answers the question and mark your choice on the multiple-choice answer section at the bottom of each page, as shown in the example below.

1 A B C D E

 **USE PENCIL ONLY**

- Use pencil only.

QUESTION 11

A card is taken at random from a shuffled pack of 52. The probability the card is either a heart or an ace is:

A $\frac{17}{52}$

B $\frac{4}{13}$

C $\frac{1}{13}$

D $\frac{15}{52}$

E $\frac{1}{4}$

QUESTION 12

John has the choice for lunch from a box containing 12 sandwiches. Four are meat, three are chicken and the remaining sandwiches are salad. If he selects one sandwich at random, the probability John chooses a salad sandwich is:

- A $\frac{1}{3}$
 B $\frac{7}{12}$
 C 1
 D $\frac{1}{12}$
 E $\frac{5}{12}$

The following information refers to Questions 13, 14 and 15:

$$\Pr(P) = 0.35, \Pr(Q) = 0.4 \text{ and } \Pr(P \cap Q) = 0.25$$

QUESTION 13

$\Pr(P \cup Q)$ is given by:

- A 0.3
 B 0.2
 C 0.5
 D 0.625
 E 1

QUESTION 14


$\Pr(P | Q)$ is given by:

- A 0.2
 B 0.3
 C 0.625
 D 0.714
 E 0.875

QUESTION 15

Which one of the following is true?

- A P and Q are independent events.
 B P and Q are mutually exclusive events.
 C $\Pr(P \cap Q') = 0.21$
 D $\Pr(Q') = 0.05$
 E $\Pr(P' \cap Q') = 0.5$

ONE ANSWER PER LINE					USE PENCIL ONLY 						
11	<input type="radio"/> A	<input checked="" type="radio"/> B	<input type="radio"/> C	<input type="radio"/> D	<input type="radio"/> E	14	<input type="radio"/> A	<input type="radio"/> B	<input checked="" type="radio"/> C	<input type="radio"/> D	<input type="radio"/> E
12	<input type="radio"/> A	<input type="radio"/> B	<input type="radio"/> C	<input type="radio"/> D	<input checked="" type="radio"/> E	15	<input type="radio"/> A	<input type="radio"/> B	<input type="radio"/> C	<input type="radio"/> D	<input checked="" type="radio"/> E
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Section B: Extended response questions. CAS technology assumed.

Specific instructions to students

- Answer **all** questions in the spaces provided.
- In questions where more than one mark is available, appropriate working **must** be shown.

QUESTION 16

Total 9 marks

A carton of 10 ice creams contains 3 defectively made ice creams. An ice cream is taken at random from the carton and replaced. A second ice cream is then taken at random.

- a What is the probability that the 2 ice creams are defective? 2 marks

$$\text{Probability of 2 defective ice creams} = \frac{3}{10} \times \frac{3}{10} = \frac{9}{100}$$

A second carton also containing 10 ice creams has 2 defectively made ice creams. Two ice creams are taken at random, with replacement, from this second carton.

- b What is the probability that the 2 ice creams are defective? 1 mark

$$\frac{2}{10} \times \frac{2}{10} = \frac{1}{25}$$

- c What is the probability that the 2 ice creams taken at random, without replacement, are defective:
 i from the first carton? 2 marks

$$\frac{3}{10} \times \frac{2}{9} = \frac{6}{90} = \frac{1}{15}$$

- ii from the second carton? 1 mark

$$\frac{2}{10} \times \frac{1}{9} = \frac{2}{90} = \frac{1}{45}$$

A quality control inspector carries out the following procedure to check the production process. He selects a carton of 10 ice creams and takes a sample of 2, without replacement. If they are both defective he takes another carton and takes a sample of 2 ice creams from that carton, without replacement. If these are also defective he instructs the production plant to stop.

- d If the inspector selected the first carton and then the second carton, what is the probability that the plant will be shut down? 2 marks

$$\frac{1}{15} \times \frac{1}{45} = \frac{1}{675}$$

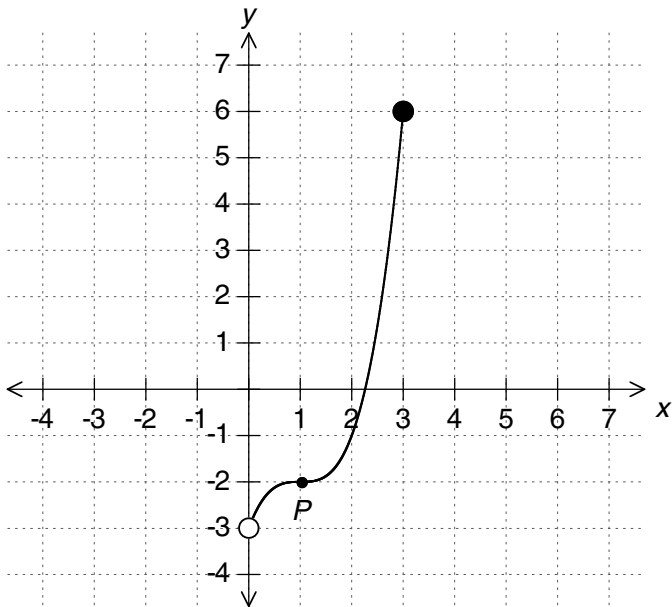
- e Would the order in which he selected the cartons change the probability of the plant being shut down? Give a brief explanation. **1 mark**

No difference. The probabilities would be the same because multiplication is commutative.

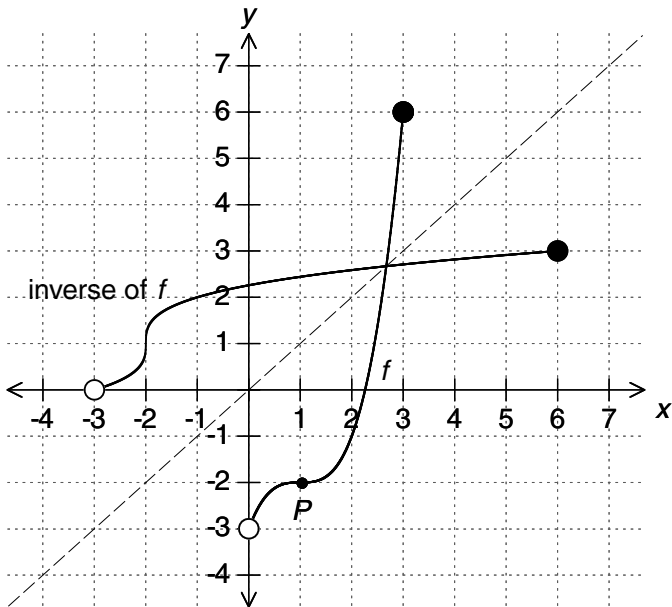
QUESTION 17

Total 14 marks

The graph of the function $f: S \rightarrow R$, $f(x) = a(x + B)^3 + C$ is shown in the diagram.



- a On the same set of axes, sketch the graph of f^{-1} , the inverse of f . **3 marks**



- b Find S . **1 mark**

S is the domain of function f : $S \in (0, 3]$

- c The point $P(1, -2)$ is a stationary point of inflection on f . Show that the values of a , B and C are 1, -1 and 2 respectively. **3 marks**

Since the SPI is $(1, -2)$, f is the image under the translation of 1 from the y axis and -2 from the x axis of $y = x^3$. Hence, $B = -1$ and $C = -2$.

The y intercept $(0, -3)$:

$$-3 = a(0 - 1)^3 - 2$$

$$-1 = -a$$

$$a = 1$$

- d State the range of f . **1 mark**

Range of $f = (-3, 6]$

- e State the domain and range of f^{-1} . **2 marks**

Domain of $f^{-1} = \text{range of } f = (-3, 6]$

Range of $f^{-1} = \text{domain of } f = (0, 3]$

- f Find the rule for f^{-1} . **3 marks**

Rule of inverse:

$$x = (y - 1)^3 - 2$$

$$x + 2 = (y - 1)^3$$

$$y - 1 = \sqrt[3]{x + 2}$$

$$y = \sqrt[3]{x + 2} + 1$$

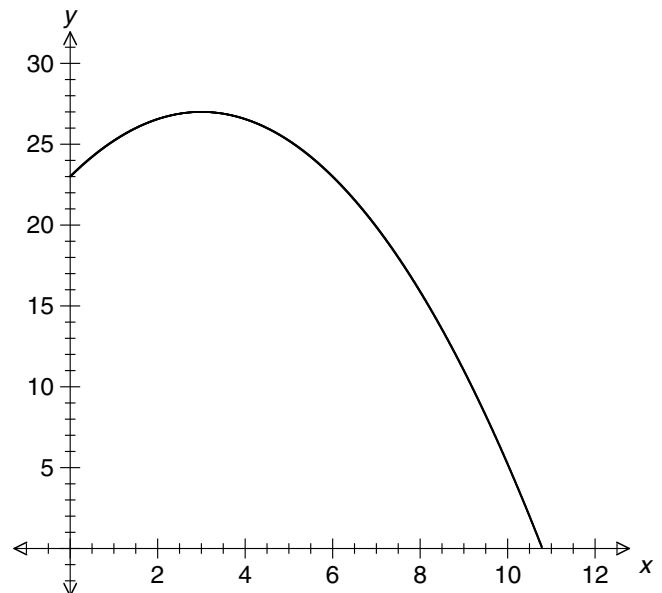
- g Find the point of intersection between f and f^{-1} , correct to four decimal places. **1 mark**

Using CAS technology gives $(2.6717, 2.6717)$ from the two graphs.

QUESTION 18

Total 7 marks

The path of a projectile from the side of a hill is given by $y = a(x - B)^2 + C$, where x m is the horizontal distance travelled and y m is the vertical height reached by the projectile. The diagram shows the path of the projectile when $a = -\frac{4}{9}$, $B = 3$ and $C = 27$.



- a What is the height at the point of projection? 1 mark

At the height of projection:

$$x = 0, y = -\frac{4}{9} \times 9 + 27 = 23 \text{ m.}$$

- b What is the maximum height reached by the projectile? 1 mark

Maximum height = maximum y value = 27 m (from equation of graph).

- c What is the horizontal distance travelled before the projectile hits the ground? Give the answer correct to the nearest centimetre. 2 marks

Using CAS: 10.79 m = 1079 cm.

- d The path of another projectile is also parabolic of the form $y = ax^2 + bx + c$. It is projected from a height of 10 m and must hit the ground 12 m horizontally from the point of projection and also clear a 9 m high tower which is 10 m from the base of projection.

- i Write a matrix equation to find the values of a , b and c . 2 marks

$$\begin{bmatrix} 0 & 0 & 1 \\ 144 & 12 & 1 \\ 100 & 10 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 9 \end{bmatrix}$$

- ii Find the values of a , b and c . 1 mark

Using CAS: $a = -\frac{11}{30}$, $b = \frac{107}{30}$ and $c = 10$

QUESTION 19 Total 5 marks

A rectangular box is open at the top. The height is h cm, the length is x cm and the width is 3 cm less than the length.

- a Write the surface area, S cm², and the volume, V cm³, in terms of x and h . 2 marks

$$S = 2xh + 2h(x - 3) + x(x - 3)$$
$$V = x(x - 3)h$$

- b If the height of the box is 10 cm and the maximum value for x is 10 cm, what would be a suitable domain for x ? 2 marks

From the graphs of S and V , the domain is $(3, 10]$.

- c For what value of x would the surface area be equal to the volume? 1 mark

From the graphs of S and V , $x = 6.40$ cm.