Section A: Technology free. 39 marks
Section B: CAS technology assumed. 26 marks
Suggested time: 80 minutes

## Section A: Short answer and extended response questions. Technology free.

**Specific instructions to students**

- Answer **all** questions in the spaces provided.
- A decimal approximation will not be accepted if an **exact** answer is required to a question.
- In questions where more than one mark is available, appropriate working **must** be shown.

**QUESTION 1** **2 marks**

Solve $\frac{3x-1}{4} - \frac{2x+3}{5} \geq x$ for $x$.

$5(3x-1) - 4(2x+3) \geq 20x$
$15x - 5 - 8x - 12 \geq 20x$
$-13x \geq 17$
$x \leq -\frac{17}{13}$

**QUESTION 2** **Total 4 marks**

**a** Transpose $V = \frac{2R}{R-r}$ to make $r$ the subject. **2 marks**

$V(R-r) = 2R$
$VR - Vr = 2R$
$Vr = VR - 2R$
$r = \frac{VR - 2R}{V}$ or $r = R\left(1 - \frac{2}{V}\right)$

**b** Find $r$ when $V = 4$ and $R = \sqrt{2}$ in simplest surd form. **2 marks**

$r = \sqrt{2}\left(1 - \frac{2}{4}\right)$
$= \sqrt{2}\left(1 - \frac{1}{2}\right)$
$= \frac{\sqrt{2}}{2}$

**QUESTION 3** **Total 7 marks**

Factorise the following:

**a** $x^4 - 5x^2 - 6$

**i** Over the rational numbers. **2 marks**

$(x^2)^2 - 5(x^2) - 6$
$(x^2 - 6)(x^2 + 1)$

**ii** Over the real numbers. **1 mark**

$(x - \sqrt{6})(x + \sqrt{6})(x^2 + 1)$

**b** $2x^3 + 54$ **2 marks**

$2(x^3 + 27)$
$2(x + 3)(x^2 - 3x + 9)$

**c** $4 - (2x + 1)^2$ **2 marks**

$[2 - (2x + 1)][2 + (2x + 1)]$
$[2 - 2x - 1][2 + 2x + 1]$
$(1 - 2x)(2x + 3)$

**QUESTION 4** **2 marks**

Solve $x^2(x^2 - 8x - 9) = 0$ for $x$.

$x^2(x - 9)(x + 1) = 0$
$x = 0, 9, -1$

**QUESTION 5** **2 marks**

Use the quadratic formula to solve $2x(x - 2) = 1$ for $x$, in simplest form.

$2x^2 - 4x - 1 = 0$

$x = \frac{4 \pm \sqrt{16 - 4 \times 2 \times -1}}{2 \times 2}$, where $a = 2$, $b = -4$, $c = -1$
$= \frac{4 \pm \sqrt{24}}{4}$
$= \frac{2(2 \pm \sqrt{6})}{4}$
$= \frac{2 \pm \sqrt{6}}{2}$ or $1 \pm \frac{\sqrt{6}}{2}$

**QUESTION 6** **Total 4 marks**

**a** Show that $P(x) = 2x^3 + x^2 - 5x + 2$ is exactly divisible by $x + 2$. **1 mark**

$P(-2) = 2 \times (-2)^3 + (-2)^2 - 5 \times (-2) + 2$
$= -16 + 4 + 10 + 2$
$= 0$
By the factor theorem, $x + 2$ is a factor of $P(x)$.

**b** Hence, find the linear factors of $P(x)$. **3 marks**

$$\begin{array}{r}
2x^2 - 3x + 1 \\
x + 2\overline{)2x^3 + x^2 - 5x + 2} \\
\underline{2x^3 + 4x^2} \qquad\quad \\
-3x^2 - 5x \qquad \\
\underline{-3x^2 - 6x} \qquad \\
x + 2 \\
\underline{x + 2} \\
\underline{0} \quad\;
\end{array}$$

$(x + 2)(2x^2 - 3x + 1)$

$(x + 2)(2x - 1)(x - 1)$

or

$2x^2 (x + 2) - 3x(x + 2) + 1(x + 2)$

$(x + 2)(2x^2 - 3x + 1)$

$(x + 2)(2x - 1)(x - 1)$

**QUESTION 7** **2 marks**

If $f(x) = -2x^3 + 3x + 7$, find $f(a-1)$, expressed in expanded form.

$$\begin{aligned}
f(a - 1) &= -2(a - 1)^3 + 3(a - 1) + 7 \\
&= -2(a^3 - 3a^2 + 3a - 1) + 3a - 3 + 7 \\
&= -2a^3 + 6a^2 - 3a + 6
\end{aligned}$$

**QUESTION 8** **Total 5 marks**

The point (5, 2) is dilated by 2 units from the $y$ axis, followed by a reflection in the line $y = x$, followed by a translation of 1 unit to the left and 3 units up.

**a** Write a matrix for each transformation. **3 marks**

Dilation: $\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$ Reflection: $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ Translation: $\begin{bmatrix} -1 \\ 3 \end{bmatrix}$

**b** Find the coordinates of the image point of (5, 2) under the above transformations. **2 marks**

$$\begin{aligned}
\begin{bmatrix} x' \\ y' \end{bmatrix} &= \begin{bmatrix} -1 \\ 3 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 5 \\ 2 \end{bmatrix} \\
&= \begin{bmatrix} -1 \\ 3 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 10 \\ 2 \end{bmatrix} \\
&= \begin{bmatrix} -1 \\ 3 \end{bmatrix} + \begin{bmatrix} 2 \\ 10 \end{bmatrix} \\
&= \begin{bmatrix} 1 \\ 13 \end{bmatrix}
\end{aligned}$$

**QUESTION 9** **Total 7 marks**

For the function $f(x) = \dfrac{1}{x - 1} + 2$:

**a** write the equation of any asymptotes. **2 marks**

$x = 1$ and $y = 2$

**b** write $f(x)$ in the form $\dfrac{ax + b}{x - 1}$. **1 mark**

$$\begin{aligned}
f(x) &= \frac{1}{x - 1} + \frac{2(x - 1)}{x - 1} \\
&= \frac{1 + 2x - 2}{x - 1} \\
&= \frac{2x - 1}{x - 1}
\end{aligned}$$

**c** Evaluate any $x$ or $y$ intercepts of $f(x)$. **2 marks**

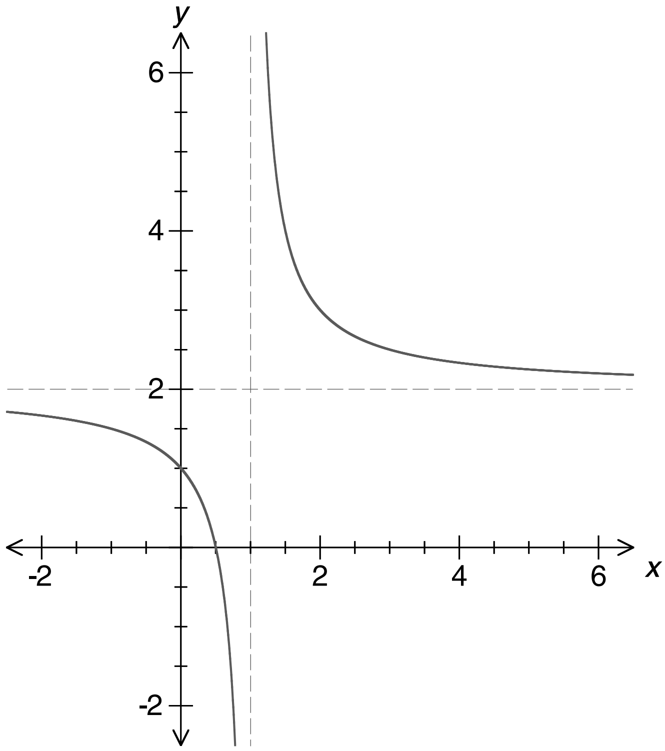
$y$ intercept: $y = \dfrac{-1}{-1}$

$= 1$

$x$ intercept: $2x - 1 = 0$

$x = \dfrac{1}{2}$

**d** Sketch the graph of $f(x)$. **2 marks**




**QUESTION 10** **4 marks**

Find the equation of the polynomial that applies to the following table.

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| $y$ | $-1$ | 2 | 11 | 38 | 95 | 194 | 347 |

3, 9, 27, 57, 99, 153

6, 18, 30, 42, 54

12, 12, 12, 12

Thus, a cubic polynomial:

$y = ax^3 + bx^2 + cx + d$

$6a = 12$

$a = 2$

$(0, -1)$ implies $d = -1$

$y = 2x^3 + bx^2 + cx - 1$

$(1, 2)$: $2 = 2 + b + c - 1$

$b + c = 1$ equation 1

$(2, 11)$: $11 = 16 + 4b + 2c - 1$

$4b + 2c = -4$

$2b + c = -2$ equation 2

$b = -3$

$c = 4$

$\therefore$ The polynomial is $2x^3 - 3x^2 + 4x - 1$.

## Section B: Multiple-choice questions. CAS technology assumed.

**Specific instructions to students**

- A correct answer scores 1, and an incorrect answer scores 0.
- Marks are not deducted for incorrect answers.
- No marks are given if more than one answer is given.
- Choose the alternative which most correctly answers the question and mark your choice on the multiple-choice answer section at the bottom of each page, as shown in the example below.

1 A B C D E   USE PENCIL ONLY

- Use pencil only.

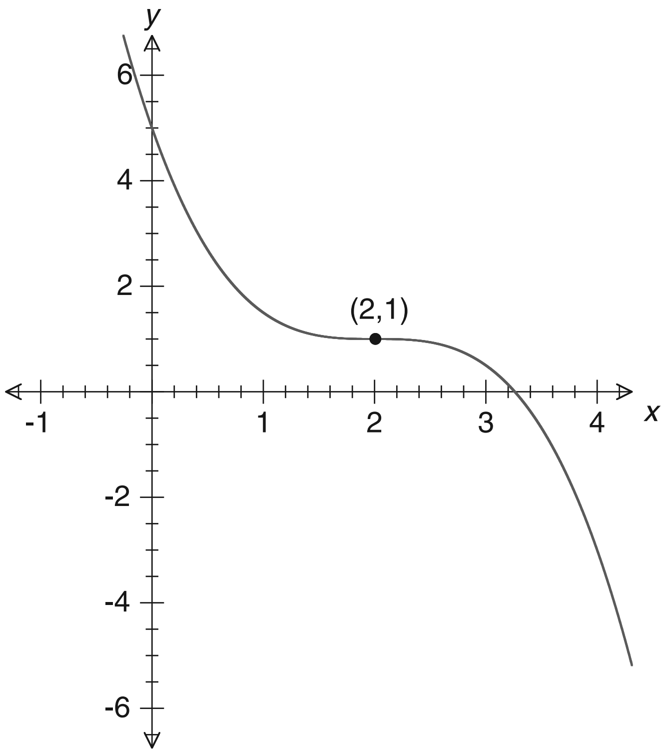
**QUESTION 11**

The graph of $f(x) = -(x - A)^2 + B$ has which of the following characteristics?

| | Turning point | $y$-intercept |
|---|---|---|
| **A** | $(-A, B)$ | $(0, A - B)$ |
| **B** | $(A, B)$ | $(0, A)$ |
| **C** | $(-A, -B)$ | $(0, B - A)$ |
| **D** | $(A, B)$ | $(0, B - A)$ |
| **E** | $(A, B)$ | $(0, B)$ |

**QUESTION 12**

The graph of $y = a(x + B)^3 + C$ is shown in the diagram.

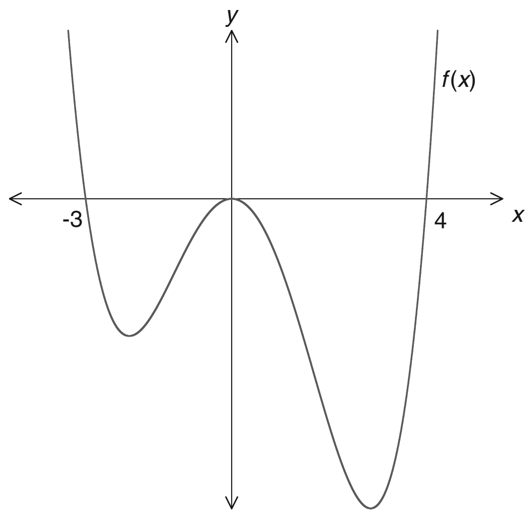



The values of $a$, $B$ and $C$ are:

| | $a$ | $B$ | $C$ |
|---|---|---|---|
| **A** | $\frac{1}{2}$ | $-2$ | 1 |
| **B** | $-\frac{1}{2}$ | $-2$ | 5 |
| **C** | $\frac{1}{2}$ | 2 | 5 |
| **D** | $-\frac{1}{2}$ | 2 | 1 |
| **E** | $-\frac{1}{2}$ | $-2$ | 1 |

**QUESTION 13**

The graph of $f(x)$ is shown in the diagram.




The values of $x$ for which $f(x) \geq 0$ are:

**A** $x < -3$ and $x > 4$

**B** $x \leq -3$ and $x \geq 4$

**C** $x = -3$ and $4$

**D** $x \leq -3$ and $x = 0$ and $x \geq 4$

**E** $-3 \leq x \leq 4$

**QUESTION 14**

If $2(x - 3)^3 - 1 = 0$, then $x$ is equal to:

**A** $3 - \frac{1}{\sqrt[3]{2}}$

**B** $3 + \frac{1}{\sqrt[3]{2}}$

**C** $3 \pm \sqrt[3]{\frac{1}{2}}$

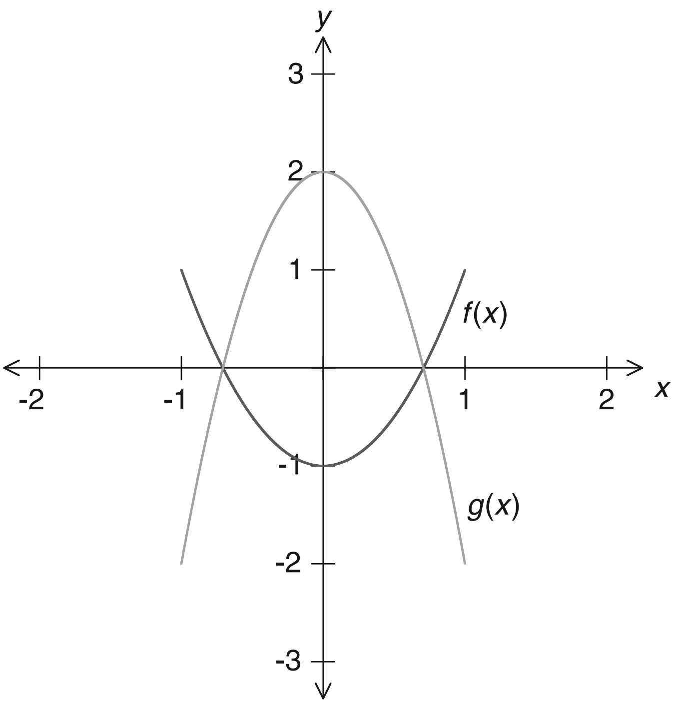
**D** $3 - \sqrt[3]{2}$

**E** $3 + \sqrt[3]{2}$

ONE ANSWER PER LINE   USE PENCIL ONLY

| | | | | | |
|---|---|---|---|---|---|
| 11 | A | B | C | D | E |
| 12 | A | B | C | D | E |
| 13 | A | B | C | D | E |
| 14 | A | B | C | D | E |

**QUESTION 15**
The diagram shows the graph of two parabolas, $f(x)$ and $g(x)$.




The graph of $y = f(x)$ is transformed into the graph of $y = g(x)$ by:

**A** a dilation by a factor of 2 units from the $x$ axis and a reflection in the $x$ axis

**B** a dilation by a factor of 2 units from the $y$ axis and a reflection in the $y$ axis

**C** a dilation by a factor of 2 units from the $x$ axis and a reflection in the $y$ axis

**D** a dilation by a factor of $\frac{1}{2}$ unit from the $x$ axis and a reflection in the $x$ axis

**E** a dilation by a factor of $\frac{1}{2}$ unit from the $y$ axis and a reflection in the $x$ axis

**ONE ANSWER PER LINE** **USE PENCIL ONLY**

15 [A] B C D E

## Section B: Extended response questions. CAS technology assumed.

**Specific instructions to students**

- Answer **all** questions in the spaces provided.
- In questions where more than one mark is available, appropriate working **must** be shown.

**QUESTION 16** **Total 6 marks**

A car hire company offers two hiring options, Standard and Premium. The Standard option costs \$35 per day plus \$0.40 per km travelled. The Premium option costs \$60 per day for unlimited travel.

While on holiday, Mary hires a car for $d$ days and drives for a total of $k$ kilometres.

**a** Write expressions for the total cost, \$C, for the Standard and Premium options, in terms of $d$ and $k$, for Mary's holiday. **2 marks**

Standard: $C = 35d + 0.4k$
Premium: $C = 60d$

**b** Mary's holiday is for 6 days. Which plan is the more economic if she plans to travel 500 kms? **1 mark**

Standard: $C = 35 \times 7 + 0.4 \times 500$
$= \$445$
Premium: $C = 60 \times 7$
$= \$420$
$\therefore$ Premium is the more economic.

**c** Mary decides to extend her holiday by an extra 3 days. Find the minimum kilometres Mary can travel so that the Premium option is cheaper than the Standard option. **3 marks**

Mary travels for a total of 10 days.
Premium: $C = 60 \times 10$
$= \$600$
For the Premium to be cheaper, the Standard must cost more than \$600.
Standard: $600 < 35 \times 10 + 0.4k$
$0.4k > 600 - 350$
$k > 625$ km
Alternatively, solve using CAS.
The Premium option is cheaper than the standard option if Mary travels more than 625 km.

## QUESTION 17 **Total 9 marks**

A family of graphs is represented by $f(x) = 2(x + 4)^2 + (3 - k)$, where $k$ is a real number.

**a** Find the $y$ intercept, in terms of $k$. **1 mark**

$y$ intercept: $f(0) = 2(0 + 4)^2 + (3 - k)$
$= 35 - k$

Or use CAS.

**b** Find the $x$ intercepts, in terms of $k$. Write the answer in the form $a \pm \frac{\sqrt{b(k-3)}}{2}$, where $a$ and $b$ are natural numbers. **2 marks**

$2(x + 4)^2 + (3 - k) = 0$

$(x + 4)^2 = \frac{k-3}{2}$

$x$ intercept: $x + 4 = \pm\sqrt{\frac{k-3}{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$= \pm\frac{\sqrt{2(k-3)}}{2}$

$x = -4 \pm \frac{\sqrt{2(k-3)}}{2}$ or

$\frac{-8 \pm \sqrt{2(k-3)}}{2}$

Or use CAS for working.

**c** Evaluate the $x$ and $y$ intercepts when $k = 5$. **2 marks**

$y$ intercept: $35 - 5 = 30$

$x = -4 \pm \frac{\sqrt{2(5-3)}}{2}$

$x$ intercept: $= -4 \pm \frac{\sqrt{4}}{2}$

$= -3, -5$

Or use CAS.

**d** Find the values of the $x$ and $y$ intercepts and the minimum value of $f(x)$, when $k = 8$. Give the value of any $x$ intercepts to four decimal places. **3 marks**

$y$ intercept is 27

$x$ intercepts are $-5.5811$ and $-2.4189$

minimum is $-5$

**e** For what values of $k$ will $f(x)$ have two $x$ intercepts? **1 mark**

When $f(x)$ has two $x$ intercepts, $k - 3 > 0$

$k > 3$

## QUESTION 18 **Total 6 marks**

A cubic graph passes through the points (1, 2), (−1, 3), (4, 2) and (0, 1).

**a** Using the general cubic equation, $f(x) = ax^3 + bx^2 + cx + d$, write equations in terms of $a$, $b$, $c$ and $d$ that can be used to find the equation of the cubic graph that passes through these four points. **2 marks**

(1, 2): $a + b + c + d = 2$

(−1, 3): $-a + b - c + d = 3$

(4, 2): $64a + 16b + 4c + d = 2$

(0, 1): $d = 1$

**b** Hence, or otherwise, find exact values for $a$, $b$, $c$ and $d$. Write the equation of the cubic function that passes through these four points. **1 mark**

CAS: $a = -\frac{7}{20}$, $b = \frac{3}{2}$, $c = -\frac{3}{20}$, $d = 1$

$f(x) = -\frac{7}{20}x^3 + \frac{3}{2}x^2 - \frac{3}{20}x + 1$

**c** Find the values of any $x$ intercepts and the coordinates of any stationary points. Give the answers correct to three decimal places. **3 marks**

CAS: The stationary points are (0.051, 0.996), (2.806, 4.657).

The $x$ intercept is 4.339.